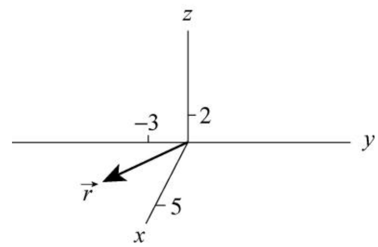
# Chapter 4




8. Our coordinate system has $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north. The first displacement is $\vec{r}_{AB} = (483 \text{ km})\hat{\text{i}}$ and the second is $\vec{r}_{BC} = (-966 \text{ km})\hat{\text{j}}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = (483 \text{ km})\hat{\text{i}} - (966 \text{ km})\hat{\text{j}}$$

which yields $|\vec{r}_{AC}| = \sqrt{(483 \text{ km})^2 + (-966 \text{ km})^2} = 1.08\times10^3 \text{ km}$.

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{-966 \text{ km}}{483 \text{ km}}\right) = -63.4°.$$

We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(c) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives

$$\vec{v}_{\text{avg}} = \frac{(483 \text{ km})\hat{\text{i}} - (966 \text{ km})\hat{\text{j}}}{2.25 \text{ h}} = (215 \text{ km/h})\hat{\text{i}} - (429 \text{ km/h})\hat{\text{j}}$$

with a magnitude $|\vec{v}_{\text{avg}}| = \sqrt{(215 \text{ km/h})^2 + (-429 \text{ km/h})^2} = 480 \text{ km/h}$.

(d) The direction of $\vec{v}_{\text{avg}}$ is 26.6° east of south, same as in part (b). In magnitude-angle notation, we would have $\vec{v}_{\text{avg}} = (480 \text{ km/h} \angle -63.4°)$.

(e) Assuming the *AB* trip was a straight one, and similarly for the *BC* trip, then $|\vec{r}_{AB}|$ is the distance traveled during the *AB* trip, and $|\vec{r}_{BC}|$ is the distance traveled during the *BC* trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483 \text{ km} + 966 \text{ km}}{2.25 \text{ h}} = 644 \text{ km/h}.$$

20. The acceleration is constant so that use of Table 2-1 (for both the $x$ and $y$ motions) is permitted. Where units are not shown, SI units are to be understood. Collision between particles $A$ and $B$ requires two things. First, the $y$ motion of $B$ must satisfy (using Eq. 2-15 and noting that $\theta$ is measured from the $y$ axis)

$$y = \frac{1}{2}a_y t^2 \quad \Rightarrow \quad 30\text{ m} = \frac{1}{2}\left[(0.40\text{ m/s}^2)\cos\theta\right]t^2.$$

Second, the $x$ motions of $A$ and $B$ must coincide:

$$vt = \frac{1}{2}a_x t^2 \quad \Rightarrow \ (3.0\text{ m/s})t = \frac{1}{2}\left[(0.40\text{ m/s}^2)\sin\theta\right]t^2.$$

We eliminate a factor of $t$ in the last relationship and formally solve for time:

$$t = \frac{2v}{a_x} = \frac{2(3.0\text{ m/s})}{(0.40\text{ m/s}^2)\sin\theta}.$$

This is then plugged into the previous equation to produce

$$30\text{ m} = \frac{1}{2}\left[(0.40\text{ m/s}^2)\cos\theta\right]\left(\frac{2(3.0\text{ m/s})}{(0.40\text{ m/s}^2)\sin\theta}\right)^2$$

which, with the use of $\sin^2\theta = 1 - \cos^2\theta$, simplifies to

$$30 = \frac{9.0}{0.20}\,\frac{\cos\theta}{1-\cos^2\theta} \quad \Rightarrow \ 1-\cos^2\theta = \frac{9.0}{(0.20)(30)}\cos\theta.$$

We use the quadratic formula (choosing the positive root) to solve for $\cos\theta$:

$$\cos\theta = \frac{-1.5+\sqrt{1.5^2-4(1.0)(-1.0)}}{2} = \frac{1}{2}$$

which yields $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$.

21. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 10$ m/s.

(a) With the origin at the initial point (where the dart leaves the thrower's hand), the $y$ coordinate of the dart is given by $y = -\frac{1}{2}gt^2$, so that with $y = -PQ$ we have $PQ = \frac{1}{2}\left(9.8\ \text{m/s}^2\right)\left(0.19\ \text{s}\right)^2 = 0.18\ \text{m}.$

(b) From $x = v_0t$ we obtain $x = (10\ \text{m/s})(0.19\ \text{s}) = 1.9\ \text{m}$.

22. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the initial point (edge of table), the $y$ coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. If $t$ is the time of flight and $y = -1.20$ m indicates the level at which the ball hits the floor, then

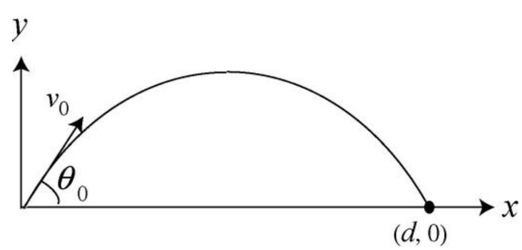
$$t = \sqrt{\frac{2\left(-1.20\ \text{m}\right)}{-9.80\ \text{m/s}^2}} = 0.495\text{s}.$$

(b) The initial (horizontal) velocity of the ball is $\vec{v} = v_0\,\hat{\text{i}}$. Since $x = 1.52$ m is the horizontal position of its impact point with the floor, we have $x = v_0t$. Thus,

$$v_0 = \frac{x}{t} = \frac{1.52\ \text{m}}{0.495\ \text{s}} = 3.07\ \text{m/s}.$$

35. **THINK** This problem deals with projectile motion of a bullet. We're interested in the firing angle that allows the bullet to strike a target at some distance away.

**EXPRESS** We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let $\theta_0$ be the firing angle. If the target is a distance $d$ away, then its coordinates are $x = d$, $y = 0$.




The projectile motion equations lead to

$$d = (v_0 \cos\theta_0)t, \qquad 0 = v_0 t \sin\theta_0 - \frac{1}{2}gt^2$$

where $\theta_0$ is the firing angle. The setup of the problem is shown in the figure above (scale exaggerated).

**ANALYZE** The time at which the bullet strikes the target is given by $t = d/(v_0 \cos\theta_0)$. Eliminating $t$ leads to $2v_0^2 \sin\theta_0 \cos\theta_0 - gd = 0$. Using $\sin\theta_0 \cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

$$v_0^2 \sin\ (2\theta_0) = gd \ \Rightarrow\ \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80\ \text{m/s}^2)(45.7\ \text{m})}{(460\ \text{m/s})^2}$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$, or $\theta_0$ = 0.0606°. If the gun is aimed at a point a distance $\ell$ above the target, then tan $\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = (45.7\ \text{m})\tan(0.0606°) = 0.0484\ \text{m} = 4.84\ \text{cm}.$$

**LEARN** Due to the downward gravitational acceleration, in order for the bullet to strike the target, the gun must be aimed at a point slightly above the target.

42. (a) Using the fact that the person (as the projectile) reaches the maximum height over the middle wheel located at $x = 23\ \text{m} + (23/2)\ \text{m} = 34.5\ \text{m}$, we can deduce the initial launch speed from Eq. 4-26:

$$x = \frac{R}{2} = \frac{v_0^2 \sin 2\theta_0}{2g} \quad \Rightarrow \quad v_0 = \sqrt{\frac{2gx}{\sin 2\theta_0}} = \sqrt{\frac{2(9.8\ \text{m/s}^2)(34.5\ \text{m})}{\sin(2\cdot 53°)}} = 26.5\ \text{m/s}\ .$$

Upon substituting the value to Eq. 4-25, we obtain

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\ \text{m} + (23\ \text{m})\tan 53° - \frac{(9.8\ \text{m/s}^2)(23\ \text{m})^2}{2(26.5\ \text{m/s})^2(\cos 53°)^2} = 23.3\ \text{m}.$$

Since the height of the wheel is $h_w = 18$ m, the clearance over the first wheel is $\Delta y = y - h_w = 23.3\ \text{m} - 18\ \text{m} = 5.3\ \text{m}$.

(b) The height of the person when he is directly above the second wheel can be found by solving Eq. 4-24. With the second wheel located at $x = 23\ \text{m} + (23/2)\ \text{m} = 34.5\ \text{m}$, we have

$$y = y_0 + x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} = 3.0\ \text{m} + (34.5\ \text{m})\tan 53° - \frac{(9.8\ \text{m/s}^2)(34.5\ \text{m})^2}{2(26.52\ \text{m/s})^2(\cos 53°)^2}$$
$$= 25.9\ \text{m}.$$

Therefore, the clearance over the second wheel is $\Delta y = y - h_w = 25.9\ \text{m} - 18\ \text{m} = 7.9\ \text{m}$.

(c) The location of the center of the net is given by

$$0 = y - y_0 = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} \quad\Rightarrow\quad x = \frac{v_0^2\sin 2\theta_0}{g} = \frac{(26.52\text{ m/s})^2\sin(2\cdot 53°)}{9.8\text{ m/s}^2} = 69\text{ m.}$$

44. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 161\text{ km/h}$. Converting to SI units, this is $v_0$ = 44.7 m/s.

(a) With the origin at the initial point (where the ball leaves the pitcher's hand), the $y$ coordinate of the ball is given by $y = -\frac{1}{2}gt^2$, and the $x$ coordinate is given by $x = v_0t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if $x$ = 18.3/2 m, then $t$ = (18.3/2 m)/(44.7 m/s) = 0.205 s.

(b) And the time to travel the next 18.3/2 m must also be 0.205 s. It can be useful to write the horizontal equation as $\Delta x = v_0\Delta t$ in order that this result can be seen more clearly.

(c) Using the equation $y = -\frac{1}{2}gt^2$, we see that the ball has reached the height of $|-\frac{1}{2}\left(9.80\text{ m/s}^2\right)\left(0.205\text{ s}\right)^2| = 0.205\text{ m}$ at the moment the ball is halfway to the batter.

(d) The ball's height when it reaches the batter is $-\frac{1}{2}\left(9.80\text{ m/s}^2\right)\left(0.409\text{ s}\right)^2 = -0.820\text{m}$, which, when subtracted from the previous result, implies it has fallen another 0.615 m. Since the value of $y$ is not simply proportional to $t$, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial $y$-velocity for the first half of the motion is not the same as the "initial" $y$-velocity for the second half of the motion.