Chapter 3

4. The angle described by a full circle is 360° = 2π rad, which is the basis of our conversion factor.

(a) $20.0° = (20.0°)\dfrac{2\pi \text{ rad}}{360°} = 0.349 \text{ rad}$.

(b) $50.0° = (50.0°)\dfrac{2\pi \text{ rad}}{360°} = 0.873 \text{ rad}$.

(c) $100° = (100°)\dfrac{2\pi \text{ rad}}{360°} = 1.75 \text{ rad}$.

(d) $0.330 \text{ rad} = (0.330 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 18.9°$.

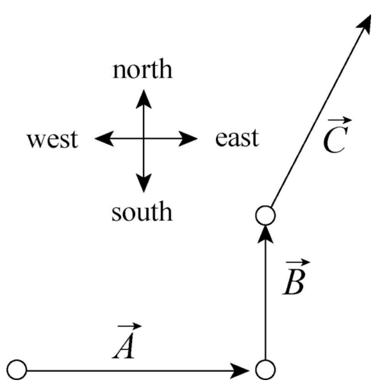
(e) $2.10 \text{ rad} = (2.10 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 120°$.

(f) $7.70 \text{ rad} = (7.70 \text{ rad})\dfrac{360°}{2\pi \text{ rad}} = 441°$.

6. (a) The height is $h = d \sin\theta$, where $d = 12.5$ m and $\theta = 20.0°$. Therefore, $h = 4.28$ m.

(b) The horizontal distance is $d \cos\theta = 11.7$ m.

12. We label the displacement vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\text{i}}$ direction (+*x* direction) and *north* as the $\hat{\text{j}}$ direction (+*y* direction). We note that the angle between $\vec{C}$ and the *x* axis is 60°. Thus,



$$\vec{A} = (50 \text{ km})\,\hat{\text{i}}$$
$$\vec{B} = (30 \text{ km})\,\hat{\text{j}}$$
$$\vec{C} = (25 \text{ km})\cos(60°)\,\hat{\text{i}} + (25 \text{ km})\sin(60°)\,\hat{\text{j}}$$

(a) The total displacement of the car from its initial position is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = (62.5 \text{ km})\,\hat{\text{i}} + (51.7 \text{ km})\,\hat{\text{j}}$$

24. As a vector addition problem, we express the situation (described in the problem statement) as $\vec{A} + \vec{B} = (3A)\,\hat{j}$, where $\vec{A} = A\,\hat{i}$ and $B = 7.0$ m. Since $\hat{i} \perp \hat{j}$ we may use the Pythagorean theorem to express B in terms of the magnitudes of the other two vectors:

$$B = \sqrt{(3A)^2 + A^2} \qquad \Rightarrow \qquad A = \frac{1}{\sqrt{10}}B \;=\; 2.2 \text{ m} .$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{(62.5\,\text{km})^2 + (51.7\,\text{km})^2} = 81 \text{ km}.$$

(b) The angle (counterclockwise from +x axis) is $\tan^{-1}$ (51.7 km/62.5 km) = 40°, which is to say that it points 40° *north of east*. Although the resultant $\vec{r}$ is shown in our sketch, it would be a direct line from the “tail” of $\vec{A}$ to the “head” of $\vec{C}$.

39. From the definition of the dot product between $\vec{A}$ and $\vec{B}$, $\vec{A}\cdot\vec{B} = AB\cos\theta$, we have

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

With $A = 6.00$, $B = 7.00$ and $\vec{A}\cdot\vec{B} = 14.0$, $\cos\theta = 0.333$, or $\theta = 70.5°$.

41. Since $ab\cos\phi = a_x b_x + a_y b_y + a_z b_z$,

$$\cos\phi = \frac{a_x b_x \;+\; a_y b_y \;+\; a_z b_z}{ab}.$$

The magnitudes of the vectors given in the problem are

$$a \;= |\vec{a}| = \sqrt{(3.00)^2 \;+\; (3.00)^2 \;+\; (3.00)^2} = 5.20$$

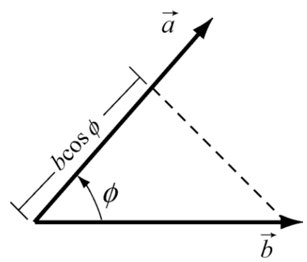
$$b \;= |\vec{b}| = \sqrt{(2.00)^2 \;+\; (1.00)^2 \;+\; (3.00)^2} = 3.74.$$

The angle between them is found from

$$\cos\phi \;=\; \frac{(3.00)(2.00) \;+\; (3.00)(1.00) \;+\; (3.00)(3.00)}{(5.20)(3.74)} = 0.926.$$

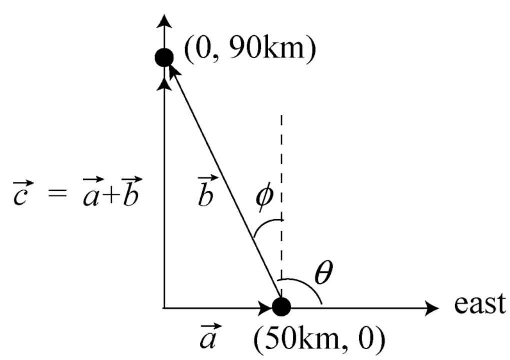
The angle is $\phi = 22°$.

As the name implies, the scalar product (or dot product) between two vectors is a scalar quantity. It can be regarded as the product between the magnitude of one of the vectors and the scalar component of the second vector along the direction of the first one, as illustrated below (see also in Fig. 3-18 of the text):



$$\vec{a}\cdot\vec{b} = ab\cos\phi = (a)(b\cos\phi)$$

49. The situation is depicted in the figure below.



Let $\vec{a}$ represent the first part of his actual voyage (50.0 km east) and $\vec{c}$ represent the intended voyage (90.0 km north). We are looking for a vector $\vec{b}$ such that $\vec{c} = \vec{a} + \vec{b}$.

(a) Using the Pythagorean theorem, the distance traveled by the sailboat is $b = \sqrt{(50.0\text{ km})^2 + (90.0\text{ km})^2} = 103\text{ km}$.

(b) The direction is

$$\phi = \tan^{-1}\left(\frac{50.0\text{ km}}{90.0\text{ km}}\right) = 29.1°$$

west of north (which is equivalent to 60.9° north of due west).

Note that this problem could also be solved by first expressing the vectors in unit-vector notation: $\vec{a} = (50.0\text{ km})\hat{\text{i}}, \vec{c} = (90.0\text{ km})\hat{\text{j}}$. This gives

$$\vec{b} = \vec{c} - \vec{a} = -(50.0\text{ km})\hat{\text{i}} + (90.0\text{ km})\hat{\text{j}}$$

The angle between $\vec{b}$ and the +x-axis is

$$\theta = \tan^{-1}\left(\frac{90.0 \text{ km}}{-50.0 \text{ km}}\right) = 119.1°$$

The angle θ is related to ϕ by $\theta = 90° + \phi$.

58. The vector can be written as $\vec{d} = (2.5 \text{ m})\hat{j}$, where we have taken $\hat{j}$ to be the unit vector pointing north.

(a) The magnitude of the vector $\vec{a} = 4.0\,\vec{d}$ is (4.0)(2.5 m) = 10 m.

(b) The direction of the vector $\vec{a} = 4.0\vec{d}$ is the same as the direction of $\vec{d}$ (north).

(c) The magnitude of the vector $\vec{c} = -3.0\vec{d}$ is (3.0)(2.5 m) = 7.5 m.

(d) The direction of the vector $\vec{c} = -3.0\vec{d}$ is the opposite of the direction of $\vec{d}$. Thus, the direction of $\vec{c}$ is south.

62. We choose +*x* east and +*y* north and measure all angles in the "standard" way (positive ones counterclockwise from +*x*, negative ones clockwise). Thus, vector $\vec{d}_1$ has magnitude d_1 = 3.66 (with the unit meter and three significant figures assumed) and direction θ_1 = 90°. Also, $\vec{d}_2$ has magnitude d_2 = 1.83 and direction θ_2 = –45°, and vector $\vec{d}_3$ has magnitude d_3 = 0.91 and direction θ_3 = –135°. We add the *x* and *y* components, respectively:

$$x:\; d_1 \cos\theta_1 + d_2 \cos\theta_2 + d_3 \cos\theta_3 = 0.65 \text{ m}$$

$$y:\; d_1 \sin\theta_1 + d_2 \sin\theta_2 + d_3 \sin\theta_3 = 1.7 \text{ m}.$$

(a) The magnitude of the direct displacement (the vector sum $\vec{d}_1 + \vec{d}_2 + \vec{d}_3$) is $\sqrt{(0.65 \text{ m})^2 + (1.7 \text{ m})^2} = 1.8$ m.

(b) The angle (understood in the sense described above) is $\tan^{-1}$ (1.7/0.65) = 69°. That is, the first putt must aim in the direction 69° north of east.

64. (a) The vectors should be parallel to achieve a resultant 7 m long (the unprimed case shown below),

(b) anti-parallel (in opposite directions) to achieve a resultant 1 m long (primed case shown),

(c) and perpendicular to achieve a resultant $\sqrt{3^2 + 4^2} = 5\,\text{m}$ long (the double-primed case shown).

In each sketch, the vectors are shown in a "head-to-tail" sketch but the resultant is not shown. The resultant would be a straight line drawn from beginning to end; the beginning is indicated by A (with or without primes, as the case may be) and the end is indicated by B.