Chapter 2

3. Since the trip consists of two parts, let the displacements during first and second parts of the motion be Δx_1 and Δx_2, and the corresponding time intervals be Δt_1 and Δt_2, respectively. Now, because the problem is one-dimensional and both displacements are in the same direction, the total displacement is $\Delta x = \Delta x_1 + \Delta x_2$, and the total time for the trip is $\Delta t = \Delta t_1 + \Delta t_2$. Using the definition of average velocity given in Eq. 2-2, we have

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}.$$

To find the average speed, we note that during a time Δt if the velocity remains a positive constant, then the speed is equal to the magnitude of velocity, and the distance is equal to the magnitude of displacement, with $d = |\Delta x| = v\Delta t$.

(a) During the first part of the motion, the displacement is Δx_1 = 40 km and the time interval is

$$t_1 = \frac{(40 \text{ km})}{(30 \text{ km/h})} = 1.33 \text{ h}.$$

Similarly, during the second part the displacement is Δx_2 = 40 km and the time interval is

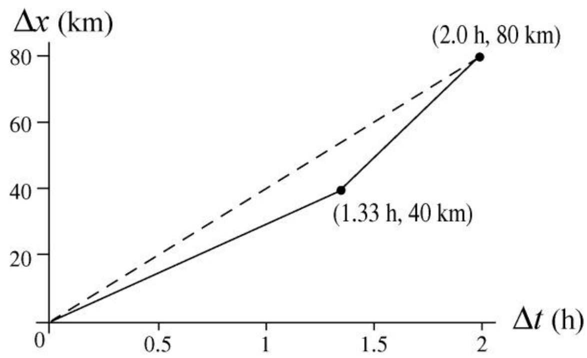
$$t_2 = \frac{(40 \text{ km})}{(60 \text{ km/h})} = 0.67 \text{ h}.$$

The total displacement is $\Delta x = \Delta x_1 + \Delta x_2$ = 40 km + 40 km = 80 km, and the total time elapsed is $\Delta t = \Delta t_1 + \Delta t_2$ = 2.00 h. Consequently, the average velocity is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{(80 \text{ km})}{(2.0 \text{ h})} = 40 \text{ km/h}.$$

(b) In this case, the average speed is the same as the magnitude of the average velocity: $s_{\text{avg}} = 40$ km/h.

(c) The graph of the entire trip is shown below; it consists of two contiguous line segments, the first having a slope of 30 km/h and connecting the origin to $(\Delta t_1, \Delta x_1)$ = $(1.33\text{ h}, 40\text{ km})$ and the second having a slope of 60 km/h and connecting $(\Delta t_1, \Delta x_1)$ to $(\Delta t, \Delta x)$ = (2.00 h, 80 km).



4. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the motion) we have speed = D/t. Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\frac{D}{v_{\text{up}}} + \frac{D}{v_{\text{down}}}}$$

which, after canceling D and plugging in v_{up} = 40 km/h and v_{down} = 60 km/h, yields 48 km/h for the average speed.

7. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h, the total time that elapses before they crash is t = (60 km)/(60 km/h) = 1.0 h. During this time, the bird travels a distance of $x = vt$ = (60 km/h)(1.0 h) = 60 km.

11. The values used in the problem statement make it easy to see that the first part of the trip (at 100 km/h) takes 1 hour, and the second part (at 40 km/h) also takes 1 hour. Expressed in decimal form, the time left is 1.25 hour, and the distance that remains is 160 km. Thus, a speed v = (160 km)/(1.25 h) = 128 km/h is needed.

17. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds.

(a) We plug into the given equation for x for t = 2.00 s and t = 3.00 s and obtain x_2 = 21.75 cm and x_3 = 50.25 cm, respectively. The average velocity during the time interval 2.00 ≤ t ≤ 3.00 s is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25\text{ cm} - 21.75\text{ cm}}{3.00\text{ s} - 2.00\text{ s}}$$

which yields v_{avg} = 28.5 cm/s.

(b) The instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$, which, at time t = 2.00 s, yields v = $(4.5)(2.00)^2$ = 18.0 cm/s.

(c) At t = 3.00 s, the instantaneous velocity is $v = (4.5)(3.00)^2$ = 40.5 cm/s.

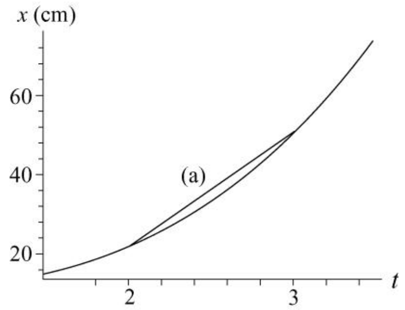
(d) At t = 2.50 s, the instantaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ cm/s.

(e) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (that is, when the particle is at $x_m = (x_2 + x_3)/2 = 36$ cm). Therefore,

$$x_m = 9.75 + 1.5t_m^3 \quad \Rightarrow \quad t_m = 2.596$$

in seconds. Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2 = 30.3$ cm/s.

(f) The answer to part (a) is given by the slope of the straight line between t = 2 and t = 3 in this x-vs-t plot. The answers to parts (b), (c), (d), and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



20. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{t} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

with SI units understood. These expressions are used in the parts that follow.

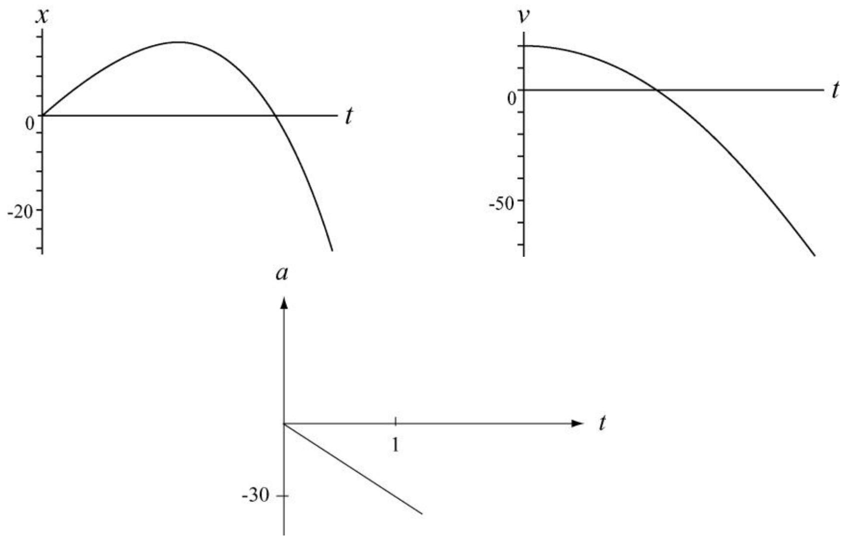
(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

(b) From 0 = – 30t, we find $a(0) = 0$ (that is, it vanishes at t = 0).

(c) It is clear that $a(t) = -30t$ is negative for $t > 0$.

(d) The acceleration $a(t) = -30t$ is positive for $t < 0$.

(e) The graphs are shown below. SI units are understood.



25. We separate the motion into two parts, and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed; we are given v_0 = 0; v = 20 m/s and a = 2.0 m/s^2. In part 2, the vehicle decelerates from its highest speed to a halt; we are given v_0 = 20 m/s; v = 0 and a = –1.0 m/s^2 (negative because the acceleration vector points opposite to the direction of motion).

(a) From Table 2-1, we find t_1 (the duration of part 1) from $v = v_0 + at$. In this way, $20 = 0 + 2.0t_1$ yields t_1 = 10 s. We obtain the duration t_2 of part 2 from the same equation. Thus, $0 = 20 + (-1.0)t_2$ leads to t_2 = 20 s, and the total is $t = t_1 + t_2$ = 30 s.

(b) For part 1, taking x_0 = 0, we use the equation $v^2 = v_0^2 + 2a(x - x_0)$ from Table 2-1 and find

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(20\text{ m/s})^2 - (0)^2}{2(2.0\text{ m/s}^2)} = 100\text{ m}.$$

This position is then the *initial* position for part 2, so that when the same equation is used in part 2 we obtain

$$x - 100\text{ m} = \frac{v^2 - v_0^2}{2a} = \frac{(0)^2 - (20\text{ m/s})^2}{2(-1.0\text{ m/s}^2)}.$$

Thus, the final position is x = 300 m. That this is also the total distance traveled should be evident (the vehicle did not "backtrack" or reverse its direction of motion).

28. We take $+x$ in the direction of motion, so v_0 = +24.6 m/s and a = – 4.92 m/s^2. We also take x_0 = 0.

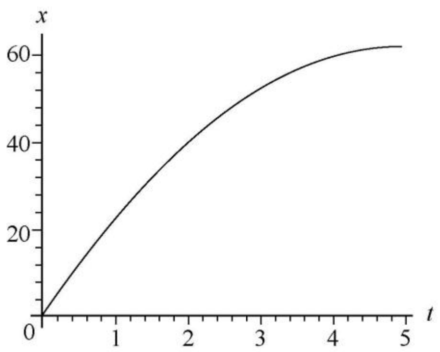
(a) The time to come to a halt is found using Eq. 2-11:

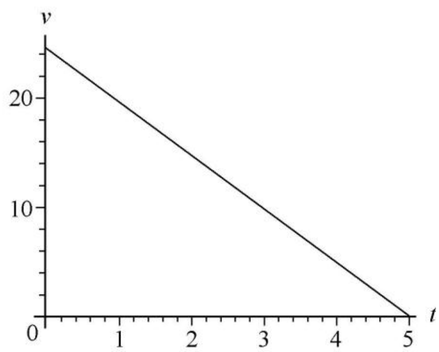
$$0 = v_0 + at \Rightarrow t = \frac{24.6\text{ m/s}}{-4.92\text{ m/s}^2} = 5.00\text{ s}.$$

(b) Although several of the equations in Table 2-1 will yield the result, we choose Eq. 2-16 (since it does not depend on our answer to part (a)).

$$0 = v_0^2 + 2ax \Rightarrow x = -\frac{(24.6\ \text{m/s})^2}{2\left(-4.92\ \text{m/s}^2\right)} = 61.5\ \text{m}.$$

(c) Using these results, we plot $v_0t + \frac{1}{2}at^2$ (the x graph, shown next, on the left) and $v_0 + at$ (the v graph, on the right) over $0 \le t \le 5$ s, with SI units understood.





46. Neglect of air resistance justifies setting $a = -g = -9.8$ m/s^2 (where *down* is our $-y$ direction) for the duration of the fall. This is constant acceleration motion, and we may use Table 2-1 (with Δy replacing Δx).

(a) Using Eq. 2-16 and taking the negative root (since the final velocity is downward), we have

$$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2(9.8\ \text{m/s}^2)(-1700\ \text{m})} = -183\ \text{m/s}.$$

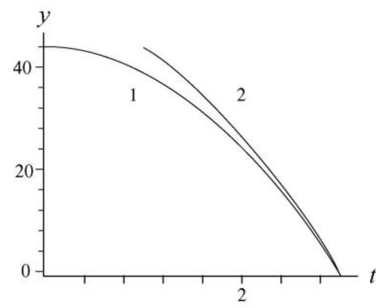
Its magnitude is therefore 183 m/s.

(b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so that its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

54. (a) We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Eq. 2-15 (with Δy replacing Δx) because this is constant acceleration motion. We use primed variables (except t) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial downward velocity $-v_0$, so that v_0 is being used for the initial *speed*). SI units are used throughout.

$$\Delta y' = 0(t) - \frac{1}{2}gt^2$$
$$\Delta y = (-v_0)(t-1) - \frac{1}{2}g(t-1)^2$$

Since the problem indicates $\Delta y' = \Delta y = -43.9$ m, we solve the first equation for t (finding $t = 2.99$ s) and use this result to solve the second equation for the initial speed of the second stone:



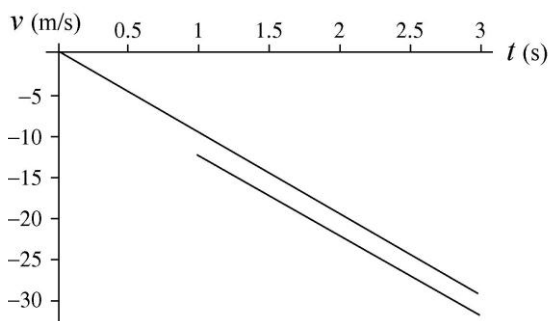
$$-43.9\text{ m} = (-v_0)(1.99\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(1.99\text{ s})^2$$

which leads to $v_0 = 12.3$ m/s.

(b) The velocity of the stones are given by

$$v'_y = \frac{d(\Delta y')}{dt} = -gt, \qquad v_y = \frac{d(\Delta y)}{dt} = -v_0 - g(t-1)$$

The plot is shown below:



62. The height reached by the player is $y = 0.76$ m (where we have taken the origin of the y axis at the floor and $+y$ to be upward).

(a) The initial velocity v_0 of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8\text{ m/s}^2)(0.76\text{ m})} = 3.86\text{ m/s}.$$

This is a consequence of Eq. 2-16 where velocity v vanishes. As the player reaches y_1

= 0.76 m – 0.15 m = 0.61 m, his speed v_1 satisfies $v_0^2 - v_1^2 = 2gy_1$, which yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{(3.86\ \text{m/s})^2 - 2(9.80\ \text{m/s}^2)(0.61\ \text{m})} = 1.71\ \text{m/s}\ .$$

The time t_1 that the player spends *ascending* in the top Δy_1 = 0.15 m of the jump can now be found from Eq. 2-17:

$$\Delta y_1 = \frac{1}{2}\left(v_1 + v\right)t_1 \;\Rightarrow\; t_1 = \frac{2\left(0.15\ \text{m}\right)}{1.71\ \text{m/s} + 0} = 0.175\ \text{s}$$

which means that the total time spent in that top 15 cm (both ascending and descending) is 2(0.175 s) = 0.35 s = 350 ms.

(b) The time t_2 when the player reaches a height of 0.15 m is found from Eq. 2-15:

$$0.15\ \text{m} = v_0 t_2 - \frac{1}{2}gt_2^2 = (3.86\ \text{m/s})t_2 - \frac{1}{2}(9.8\ \text{m/s}^2)t_2^2\ ,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots) t_2 = 0.041 s = 41 ms, which implies that the total time spent in that bottom 15 cm (both ascending and descending) is 2(41 ms) = 82 ms.

64. The graph shows y = 25 m to be the highest point (where the speed momentarily vanishes). The neglect of "air friction" (or whatever passes for that on the distant planet) is certainly reasonable due to the symmetry of the graph.

(a) To find the acceleration due to gravity g_p on that planet, we use Eq. 2-15 (with +y up)

$$y - y_0 = vt + \frac{1}{2}g_p t^2 \quad\Rightarrow\quad 25\ \text{m} - 0 = (0)(2.5\ \text{s}) + \frac{1}{2}g_p(2.5\ \text{s})^2$$

so that g_p = 8.0 m/s^2.

(b) That same (max) point on the graph can be used to find the initial velocity.

$$y - y_0 = \frac{1}{2}(v_0 + v)t \quad\Rightarrow\quad 25\ \text{m} - 0 = \frac{1}{2}\,(v_0 + 0)(2.5\ \text{s})$$

Therefore, v_0 = 20 m/s.