Alternating Current Circuits
Electromagnetic Waves

The output of an AC generator has a sinusoidal form, it has an instantaneous voltage value as well as a maximum alternating voltage value. $\Delta v = \Delta V_{\max} \sin(2\pi f t)$

(I) When a circuit has an ac generator and a resistor, the current is given by: $I = I_{\max} \sin(2\pi f t)$
hence the voltage and the current reach their maximum at the same time.
(1) They are said to be in phase.
(2) Observe that the average value of the current over one cycle is zero.
(3) The maximum value of the current occurs only in an instant of time during the cycle.
(4) Heat dissipation is related to: Power = $I^2 R$ hence whether or not the current I is positive or negative the square of its value will be a positive value.
(5) Since the I^2 varies as the $\sin^2$(2 pi f t) one finds:

The average value that is looked at upon is given by: $I^2 = \frac{I_{\max}{}^2}{2} = I_{rms}{}^2$

Where " rms " = root mean square. Hence: $I_{rms} = \frac{I_{\max}}{\sqrt{2}}$ = 0.707 $\mathbf{I}_{max}$ then: $P_{avg.} = I_{rms}{}^2 R$

The alternating voltage is also studied in terms of the rms voltages given by: $\Delta V_{rms} = \frac{\Delta V_{\max}}{\sqrt{2}}$

Hence $\Delta V_{rms} = 0.7071\ \Delta V_{max}$
By Ohm's Law $\Delta V_{R,rms} = I_{rms} R$
Consequently across the resistor: $\Delta V_{R,max} = I_{max} R$
The voltage across the resistor is then given by: $V_R = I R$

(II) When a circuit has an ac generator and a capacitor, upon closing the switch, the current is the maximum and decreases with time to a value of zero as the capacitor charges to its max, while the voltage starts from zero. $\boxed{I = I_{\max} \sin(2\pi f t + 90^o)}$ $\Delta v = \Delta V_{\max} \sin(2\pi f t)$

Hence we conclude that the voltage lags the current by 90^o across the capacitor.

The capacitor's **impeding** effect on the current is called the **Capacitance Reactance X_C**

$$X_C = \frac{1}{2\pi f C}$$

Units of C: farads
Units of f: hertz
Units of X_C:: ohms
Units of (2 pi f): units of angular frequency.

The voltage across the Capacitor is then given by: $V_{C,rms} = I_{rms} X_C$

(III) When a circuit has an ac generator and an inductor, upon closing the switch:

(1) "the changing current output of the generator produces **a back emf** that **impedes** the current in the circuit. "
(2) the metallic material of the inductor presents a resistance (r_L) in the wire forming the coil.

The magnitude of the back emf is given by: $\Delta v_L = L\frac{\Delta I}{\Delta t}$

Observe that the back emf increases as L increases, the larger the inductance is, larger the back emf.

The inductor's **impeding** effect on the current is called the **Inductive Reactance X_L**

$$X_L = 2\pi f L$$

Units of L: henries
Units of f: hertz
Units of X_L:: ohms
Units of (2 pi f): units of angular frequency.

Observe that the Inductive Reactance increases as f increases.

The back emf and the inductive reactance are direct functions of L.

Ohm's Law for the voltage across the inductor is then given by: $\Delta V_{L,rms} = I_{rms} X_L$

Because of the presence of the back emf affecting the current, the current has the following equation

$I = I_{max} \sin(2\pi f t - 90^o)$ while the voltage keeps its form: $\Delta v = \Delta V_{max} \sin(2\pi f t)$

In summary:
For a resistor: The voltage and the current are in phase $I = I_{max} \sin(2\pi f t)$

For a capacitor: The voltage lags the current $I = I_{max} \sin(2\pi f t + 90^o)$

$\Delta v = \Delta V_{max} \sin(2\pi f t)$

For an inductor: The voltage leads the current. $I = I_{max} \sin(2\pi f t - 90^o)$

To calculate the maximum voltages across the elements:

$\Delta V_{R,max} = I_{max} R$ $\Delta V_{R,max} = I_{max} R$
$V_R = I R$

$X_C = \frac{1}{2\pi f C}$ $\Delta V_{C,rms} = I_{rms} X_C$ $\Delta V_{C,max} = I_{max} X_C$

$X_L = 2\pi f L$ $\Delta V_{L,rms} = I_{rms} X_L$ $\Delta V_r = I r_L$ $\Delta V_{L,max} = I_{max} X_L$

The magnitude of the back emf is given by: $\Delta v_L = L\frac{\Delta I}{\Delta t}$

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The RLC Series Circuit

The net instantaneous voltage across all the circuit elements is the sum of the instantaneous voltages across the separate elements. As a result of this, the net voltage is " out-of-step" with the instantaneous current by an amount called the phase angle. ϕ

$\Delta V_{max}{}^2 = \Delta V_R{}^2 + (\Delta V_L - \Delta V_C)^2$ $\tan\phi = \dfrac{\Delta V_L - \Delta V_C}{\Delta V_R}$

The phasor can be viewed as a vector of magnitude ΔV_{max} rotating at a constant frequency F so that its projection is in the y-axis (Δv) and it is the instantaneous voltage in the circuit.

$$\Delta v = \Delta V_{max} \sin(\phi)$$

Then the reactance triangle that yields the impedence (Z) relation becomes:

Z= impedence
The units of Z: ohms

$$Z^2 = R^2 + (X_L - X_C)^2 \qquad \tan\phi = \frac{X_L - X_C}{R}$$

Therefore Ohm's Law yields: $\Delta V_{max} = I_{max}\sqrt{R^2 + (X_L - X_C)^2}$ $\Delta V_{max} = I_{max} Z$

Power in an AC Circuit

Energy stored in a capacitor: At the maximum voltage value the energy stored is PE_C .

$PE_C = \dfrac{1}{2}C(\Delta V_{max})^2$ " No power losses occur in a capacitor in an AC circuit. "

The maximum energy stored in the inductor: At the maximum current the energy stored:

$PE_L = \dfrac{1}{2}LI_{max}^2$ " When the current begins to decrease in the circuit, this stored energy is returned to the source.

The average power delivered to a resistor in an RLC circuit is: $P_{avg} = I_{max}^2 R$

This is a conversion of electrical power converted to internal energy in the resistor, as a consequence the temperature of the resistor is to increase and if its temperature is higher than the room temperature, then conversion to heat occurs as a result of the transmission of energy to the room environment.

Another equation that reflects the average power loss in an AC circuit is found in: $P_{avg} = I_{rms}\Delta V_{R,rms}$

Since: $\Delta V_{R,rms} = \Delta V_{rms}\cos\phi$ then average power delivered by the generator in an AC circuit is given by: $P_{av} = I_{max}\Delta V_{rms}\cos\phi$

$\cos\phi =$ is called the **power factor**.

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RLC Circuit connected in series: Its resonance

The rms current in a series RLC circuit is given by: $I_{rms} = \frac{\Delta V_{rms}}{Z}$ Z = impedence of the circuit

The maximum value of the I_{rms} occurs when the value of Z is minimum and the minimum value of Z occura when Z = R and consequently $X_L - X_C = 0$; that is, $X_L = X_C$

That is: $2\pi fL = \frac{1}{2\pi fC}$ and $2^2 \pi^2 f^2 LC = 1$ then $f_0 = \frac{1}{2\pi\sqrt{LC}}$

where f_o = resonance frequency of the circuit.

Hence at f_o the $I_{rms} = \Delta V_{rms} / R$ since Z = R If R = 0 ohms, then I_{rms} = infinite value but real circuits always have some resistance including r_L (resistance of the inductor) which automatically presents limitations to this occurrence.

See figure 21.13 on page 708

An **Electric Transformer** is made out of 2 coils wound around a core of soft iron. As AC current flows through the primary coil, a magnetic field becomes present in the center of the coil and in the core. An AC current system creates a variable magnetic field, hence the secondary coil experiences a change in the magnetic field is equivalent to the crossing of magnetic field lines by the coil of the of the secondary; therefore an AC current is induced in the secondary coil.

The AC voltage of the primary coil is given by: $\Delta V_1 = -N_1 \frac{\Delta \Phi_B}{\Delta t}$ N_1 = Number of turns in the primary coil.

The magnetic flux through the turns of the primary coil, equals to the magnetic flux through the secondary.

The induced voltage in the secondary coil is given by: $\Delta V_2 = -N_2 \frac{\Delta \Phi_B}{\Delta t}$ therefore $\Delta V_2 = \frac{N_2}{N_1} \Delta V_1$

Since N_1 is larger than N_2 then ΔV_2 is smaller than ΔV_1. When N_2 is larger than N_1 then the voltage in the secondary will be much larger than the voltage in the primary. If the number of turns in the secondary couil is 100 times larger than the number of turns in the primary, then the voltage of the secondary coil would be 100 times larger.

Likewise, the power input equals the power output in the secondary, hence $I_1 \Delta V_1 = I_2 \Delta V_2$

When $\Delta V_2 > \Delta V_1$ then current in the secondary would be smaller by the same factor than the current in the primary coil of the transformer.

Electromagnetic Waves

Acceleration of electric charges generate through radiation electromagnetic waves.
Electromagnetic waves are generated through radiation when electric charges accelerate.
These waves are made of oscillating electric and magnetic fields which are perpendicular to each other and also perpendicular to the direction of propagation of the wave.

Maxwell's equations are the fundamental laws of how the electric and magnetic fields behave.
Maxwell's Fundamental Laws yield the existence of the electromagnetic waves.
1839 - 1879

Maxwell derived the speed of light as $c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$ (permeability and permittivity of free space) Maxwell showed that the electromagnetic waves travel at the speed of light when traveling through empty space.

Maxwell also showed that: $c = \frac{E}{B}$ E and B = Electric and magnetic fields of the wave.

However, the speed of the electromagnetic waves in a medium different than free space is affected and reduced; the speed is given by " v " and the ratio of c / v = n is known as the index of refraction of the medium.

The electromagnetic waves carry energy; and they are classified as transverse waves since the wave front is perpendicular to the direction of propagation.

Also, the electromagnetic waves obey the Principle of Superposition.

Heinrich Hertz (1857 - 1894) in 1887 was the first one to generate and detect electromagnetic waves in a laboratory. LC circuits were used in the process; the capacitor stores energy in the electric field, given by: $Q^2_{max}/(2C)$, that is, (1/2) C $(\Delta V_{max})^2$ at its max voltage. The inductor stores its electrical energy ($LI^2/2$) in the magnetic field of the inductor as the current increases. In LC circuits, oscillations of transfer of energy occur between the capacitor and the inductor; this occurrence is used for the generation of electromagnetic waves then transfer via a connection from the inductor to the two . spheres making a transmitter. The receiver of the electromagnetic waves is nothing more than a single loop of wire connected to two spheres.

The average Power per Unit Area is defined as the intensity of the wave = I $I = \frac{E_{max} B_{max}}{2\mu_o}$ $c = \frac{E}{B}$

Therefore: $I = \frac{E^2_{max}}{2\mu_o c} = \frac{c}{2\mu_o} B^2_{max}$ this is the average intensity of the electromagnetic wave. The energy carried is shared equally by the E and B fields.

The energy transferred to a surface in a given amount of time is given by: U = I A Δt ; consequently momentum **p** is transferred to that surface.

Maxwell showed that the total momentum transferred in a time Δ t for normal incidence of the wave on the surface for a complete absorption is given by: p = U / c

When the surface is a reflector and in the event of a perfect reflector due to the change in momentum Δ p = 2 mv , hence for a complete reflection surface p = 2 U / c

The speed of light is also given by: $c = f\lambda$ $\lambda =$ wavelength f = frequency of the wave. Therefore the product of the wavelength and the frequency in vacuum is constant and equal to c. The electromagnetic spectrum shows the numerous ratios of frequency and wavelength with the corresponding value of " c " speed of light = 3.00×10^8 m/s

Wavelength of:		frequencies	Applications	
2.00×10^2 m	radio & tv waves	1.50×10^6 s^{-1}	tv	a range of values
1.00×10^{-2} m to +	Microwaves	3.00×10^{10} s^{-1}	Microwaves	a range of values
7.00×10^{-6} m	Infrared waves	4.28×10^{13} s^{-1}	radar navigation systems	a range of values
4.00×10^{-7} m to	5.60×10^{-7} m (red to yellow green).	7.50×10^{14} s^{-1}	Visible light	a range of values

Doppler effect frequency for electromagnetic waves is given by: $f_o = f_s(1 \pm \frac{u}{c})$ with u << c

Chapter 22

Reflection and Refraction of Light

(1600 to early 1800) I. Newton and Ch. Huygens
The initial model used to describe light was that light was a stream of particles emitted by a source. This initial model was appropriate to describe a set of properties of light. However as physics developed, it was found that the wave theory also explained the laws of reflection and refraction of light.

As an electromagnetic wave associated with light was presented as photons corpuscles or discontinuous quanta of energy maintain the particle like nature initially postulated. The energy of light
The energy of light was presented as: $E = h f$ where $h = 6.63 \times 10^{-34}$ J s (Planck's constant)
Hence the theory of light has the features of waves and particles. "A. Einstein"

Huygens: Every point on the wave front of a wave, can be considered to a point source
Light travels in straight line in a medium until the ray encounters a surface and its direction can change according to the material and angle of incidence.

Reflection of light occurs according to the conditions of the surface it encounters. The surface can be smooth or rough. In a smooth surface one finds that the angle of incidence is equal to the angle of reflection. This is known as the Law of Reflection.

$$\theta_{incidence} = \theta_{reflected}$$

The angles are measured between the normal to the surface and the ray of light.
The normal to the surface is a perpendicular line to the surface.

$$\theta_1 = \theta_1'$$

In a rough surface, the beam of light can reflect on any direction in a non-uniform manner.

A ray of light when reaching a surface can also be partially reflected and partially refracted. The partially reflected beam follows the law of reflection, while the partially refracted beam follows the law of refraction, that is, the refracted ray makes an angle with the extended normal into the new medium and it follows Snell's Law of refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$

therefore:

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} \qquad = \frac{\lambda_1}{\lambda_2}$$

Each medium is characterized by a quantity known as the index of refraction " n " where n = c / v
c = speed of light in vacuum and v = speed of light in the medium

Note that the frequency of the wave is characteristic of the source, consequently as the ray of light travels from one medium to another, its velocity and wavelength change but its frequency remains constant. velocity = wavelength x frequency

Smaller velocity implies smaller wavelength.

We can then express the index of refraction as the ratio of the wavelength in the vacuum to the wavelength in the medium.

$$n = \frac{\lambda_{vacuum}}{\lambda_{medium}} = \frac{\lambda_o}{\lambda_n}$$

Therefore the angle of refraction is directly proportional to the wavelength of the ray in the medium.

When the incident ray of light is perpendicular to the surface, no change in direction occurs, but the velocity in the new medium is different than the velocity in the incident medium

A light source can be a source of light rays of different frequencies, hence of different wavelengths, but they are produced and propagate simultaneously.

A monochromatic ray is a ray light of one frequency.

White light is made out of rays of light of different frequencies. The light spectrum shows the colors and rays at different frequencies.

Note that the path of a light ray trough a refracting surface is reversible.

Total internal reflection will occur at an specific angle called the "the critical angle" when a ray of light

travels from a denser medium (large index of refraction) to a lighter medium (smaller index of refraction).

The refracted beams in this case are bent away from the normal because $n_1 > n_2$ and at a particular angle of incidence called the critical angle, "the beam is entirely reflected at the boundary."
This is possible only when a ray of light goes from a medium of higher index of refraction into a medium of a lower index of refraction. However, for the incident rays with angles of incidence larger that the critical angle they all obey the law of reflection. The critical angle is given by:

with $n_1 > n_2$ $$\sin\theta_c = \frac{n_2}{n_1}$$

This characteristic lead the development of fiber optics. This is an entirely total reflection inside a solid glass or a transparent plastic rod. " Very little intensity is lost as a result of reflections on the sides of fiber."