2

Motion in One Dimension

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Once the arrow has left the bow, it has a constant downward acceleration equal to the free-fall acceleration, g. Taking upward as the positive direction, the elapsed time required for the velocity to change from an initial value of $15.0\ \text{m/s upward}\ (v_0 = +15.0\ \text{m/s})$ to a value of $8.00\ \text{m/s downward}\ (v_f = -8.00\ \text{m/s})$ is given by

$$\Delta t = \frac{\Delta v}{a} = \frac{v_f - v_0}{-g} = \frac{-8.00\ \text{m/s} - (+15.0\ \text{m/s})}{-9.80\ \text{m/s}^2} = 2.35\ \text{s}$$

Thus, the correct choice is (d).

2. In Figure MCQ2.2, there are five spaces separating adjacent oil drops, and these spaces span a distance of $\Delta x = 600\ \text{meters}$. Since the drops occur every 5.0 s, the time span of each space is 5.0 s and the total time interval shown in the figure is $\Delta t = 5(5.0\ \text{s}) = 25\ \text{s}$. The average speed of the car is then

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{600\ \text{m}}{25\ \text{s}} = 24\ \text{m/s}$$

making (b) the correct choice.

5. The initial velocity of the car is $v_0 = 0$ and the velocity at time t is v. The constant acceleration is therefore given by $a = \Delta v/\Delta t = (v - v_0)/t = (v - 0)/t = v/t$ and the average velocity of the car is $\bar{v} = (v + v_0)/2 = (v + 0)/2 = v/2$. The distance traveled in time t is $\Delta x = \bar{v}t = vt/2$. In the special case where $a = 0\ (\text{and hence } v = v_0 = 0)$, we see that statements (a), (b), (c), and (d) are all correct. However, in the general case $(a \neq 0,\ \text{and hence } v \neq 0)$, only statements (b) and (c) are true. Statement (e) is not true in either case.

6. The motion of the boat is very similar to that of a object thrown straight upward into the air. In both cases, the object has a constant acceleration which is directed opposite to the direction of the initial velocity. Just as the object thrown upward slows down and stops momentarily before it starts speeding up as it falls back downward, the boat will continue to move northward for some time, slowing uniformly until it comes to a momentary stop. It will then start to move in the southward direction, gaining speed as it goes. The correct answer is (c).

8. From $\Delta x = v_0 t + \frac{1}{2}at^2$, the distance travelled in time *t*, starting from rest $(v_0 = 0)$ with constant acceleration *a*, is $\Delta x = \frac{1}{2}at^2$. Thus, the ratio of the distances travelled in two individual trials, one of duration $t_1 = 6\text{ s}$ and the second of duration $t_2 = 2\text{ s}$, is

$$\frac{\Delta x_2}{\Delta x_1} = \frac{\frac{1}{2}at_2^2}{\frac{1}{2}at_1^2} = \left(\frac{t_2}{t_1}\right)^2 = \left(\frac{2\text{ s}}{6\text{ s}}\right)^2 = \frac{1}{9}$$

and the correct answer is (c).

11. At ground level, the displacement of the rock from its launch point is $\Delta y = -h$, where *h* is the height of the tower and upward has been chosen as the positive direction. From $v^2 = v_o^2 + 2a(\Delta y)$, the speed of the rock just before hitting the ground is found to be

$$|v| = \left|\pm\sqrt{v_0^2 + 2a(\Delta y)}\right| = \sqrt{v_0^2 + 2(-g)(-h)} = \sqrt{(12\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(40.0\text{ m})} = 30\text{ m/s}$$

Choice (b) is therefore the correct response to this question.

12. Once the ball has left the thrower's hand, it is a freely falling body with a constant, non-zero, acceleration of $a = -g$. Since the acceleration of the ball is not zero at any point on its trajectory, choices (a) through (d) are all false and the correct response is (e).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Yes. The particle may stop at some instant, but still have an acceleration, as when a ball thrown straight up reaches its maximum height.

PROBLEM SOLUTIONS

2.1 We assume that you are approximately 2 m tall and that the nerve impulse travels at uniform speed. The elapsed time is then

$$\Delta t = \frac{\Delta x}{v} = \frac{2\text{ m}}{100\text{ m/s}} = 2\times10^{-2}\text{ s} = \boxed{0.02\text{ s}}$$

2.2 (a) At constant speed, $c = 3\times10^{8}\text{ m/s}$, the distance light travels in 0.1 s is

$$\Delta x = c(\Delta t) = \left(3\times10^{8}\text{ m/s}\right)(0.1\text{ s})$$
$$= \left(3\times10^{7}\ \cancel{\text{m}}\right)\left(\frac{1\text{ mi}}{1.609\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{km}}}{10^{3}\ \cancel{\text{m}}}\right) = \boxed{2\times10^{4}\text{ mi}}$$

(b) Comparing the result of part (a) to the diameter of the Earth, D_E, we find

$$\frac{\Delta x}{D_E} = \frac{\Delta x}{2R_E} = \frac{3.0\times10^{7}\ \cancel{\text{m}}}{2\left(6.38\times10^{6}\ \cancel{\text{m}}\right)} \approx \boxed{2.4} \qquad (\text{with } R_E = \text{Earth's radius})$$

2.3 Distances travelled between pairs of cities are

$$\Delta x_1 = v_1(\Delta t_1) = (80.0\text{ km/h})(0.500\text{ h}) = 40.0\text{ km}$$

$$\Delta x_2 = v_2(\Delta t_2) = (100\text{ km/h})(0.200\text{ h}) = 20.0\text{ km}$$

$$\Delta x_3 = v_3(\Delta t_3) = (40.0\text{ km/h})(0.750\text{ h}) = 30.0\text{ km}$$

Thus, the total distance travelled is $\Delta x = (40.0 + 20.0 + 30.0)\text{ km} = 90.0\text{ km}$, and the elapsed time is $\Delta t = 0.500\text{ h} + 0.200\text{ h} + 0.750\text{ h} + 0.250\text{ h} = 1.70\text{ h}$.

(a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{90.0\text{ km}}{1.70\text{ h}} = \boxed{52.9\text{ km/h}}$

(b) $\Delta x = \boxed{90.0\text{ km}}$ (see above)

2.7 (a) $\text{Displacement} = \Delta x = (85.0\text{ km/h})(35.0\text{ min})\left(\dfrac{1\text{ h}}{60.0\text{ min}}\right) + 130\text{ km} = \boxed{180\text{ km}}$

(b) The total elapsed time is $\Delta t = (35.0\text{ min} + 15.0\text{ min})\left(\dfrac{1\text{ h}}{60.0\text{ min}}\right) + 2.00\text{ h} = 2.83\text{ h}$

so, $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{180 \text{ km}}{2.84 \text{ h}} = \boxed{63.6 \text{ km/h}}$$

2.9 The plane starts from rest $(v_0 = 0)$ and maintains a constant acceleration of $a = +1.3 \text{ m/s}^2$. Thus, we find the distance it will travel before reaching the required takeoff speed $(v = 75 \text{ m/s})$, from $v^2 = v_0^2 + 2a(\Delta x)$, as

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{(75 \text{ m/s})^2 - 0}{2(1.3 \text{ m/s}^2)} = 2.2 \times 10^3 \text{ m} = 2.2 \text{ km}$$

Since this distance is less than the length of the runway, $\boxed{\text{the plane takes off safely.}}$

2.21 We choose the positive direction to point away from the wall. Then, the initial velocity of the ball is $v_i = -25.0 \text{ m/s}$ and the final velocity is $v_f = +22.0 \text{ m/s}$. If this change in velocity occurs over a time interval of $\Delta t = 3.50 \text{ ms}$ (i.e., the interval during which the ball is in contact with the wall), the average acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{+22.0 \text{ m/s} - (-25.0 \text{ m/s})}{3.50 \times 10^{-3} \text{ s}} = \boxed{1.34 \times 10^4 \text{ m/s}^2}$$

2.23 From $a = \dfrac{\Delta v}{\Delta t}$, we have $\Delta t = \dfrac{\Delta v}{a} = \dfrac{(60 - 55) \text{ mi/h}}{0.60 \text{ m/s}^2}\left(\dfrac{0.447 \text{ m/s}}{1 \text{ mi/h}}\right) = \boxed{3.7 \text{ s}}$

2.24 **(i)** (a) From $t = 0$ to $t = 5.0$ s,

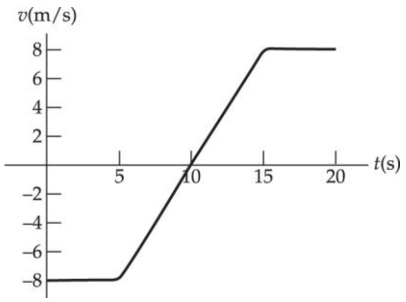
$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{-8.0 \text{ m/s} - (-8.0 \text{ m/s})}{5.0 \text{ s} - 0} = \boxed{0}$$

(b) From $t = 5.0$ s to $t = 15$ s,

$$\bar{a} = \frac{8.0 \text{ m/s} - (-8.0 \text{ m/s})}{15 \text{ s} - 5.0 \text{ s}} = \boxed{1.6 \text{ m/s}^2}$$

(c) From $t = 0$ to $t = 20$ s,

$$\bar{a} = \frac{8.0 \text{ m/s} - (-8.0 \text{ m/s})}{20 \text{ s} - 0} = \boxed{0.80 \text{ m/s}^2}$$



(ii) At any instant, the instantaneous acceleration equals the slope of the line tangent to the *v* vs. *t* graph at that point in time.

(a) At t = 2.0 s, the slope of the tangent line to the curve is $\boxed{0}$.

(b) At t = 10 s, the slope of the tangent line is $\boxed{1.6\ \text{m/s}^2}$.

(c) At t = 18 s, the slope of the tangent line is $\boxed{0}$.

2.29 (a) $\Delta x = \bar{v}(\Delta t) = \left[(v + v_0)/2\right]\Delta t$ becomes

$$40.0\ \text{m} = \left(\frac{2.80\,\text{m/s} + v_0}{2}\right)(8.50\ \text{s})$$

which yields $v_0 = \dfrac{2}{8.50\ \text{s}}(40.0\ \text{m}) - 2.80\,\text{m/s} = \boxed{6.61\ \text{m/s}}$

(b) $a = \dfrac{v - v_0}{\Delta t} = \dfrac{2.80\ \text{m/s} - 6.61\ \text{m/s}}{8.50\ \text{s}} = \boxed{-0.448\ \text{m/s}^2}$

2.32 (a) From $v_f^2 = v_i^2 + 2a(\Delta x)$, with $v_i = 6.00\ \text{m/s}$ and $v_f = 12.0\ \text{m/s}$, we find

$$\Delta x = \frac{v_f^2 - v_i^2}{2a} = \frac{(12.0\ \text{m/s})^2 - (6.00\ \text{m/s})^2}{2(4.00\ \text{m/s}^2)} = \boxed{13.5\ \text{m}}$$

(b) In this case, the object moves in the same direction for the entire time interval and the total distance traveled is simply the magnitude or absolute value of the displacement. That is,

$$d = |\Delta x| = \boxed{13.5 \text{ m}}$$

(c) Here, $v_i = -6.00 \text{ m/s}$ and $v_f = 12.0 \text{ m/s}$, and we find

$$\Delta x = \frac{v_f^2 - v_i^2}{2a} = \boxed{13.5 \text{ m}} \qquad \text{[the same as in part (a)]}$$

(d) In this case, the object initially slows down as it travels in the negative *x*-direction, stops momentarily, and then gains speed as it begins travelling in the positive *x*-direction. We find the total distance traveled by first finding the displacement during each phase of this motion.

While coming to rest $(v_i = -6.00 \text{ m/s},\ v_f = 0)$,

$$\Delta x_1 = \frac{v_f^2 - v_i^2}{2a} = \frac{(0)^2 - (-6.00 \text{ m/s})^2}{2(4.00 \text{ m/s}^2)} = -4.50 \text{ m}$$

After reversing direction $(v_i = 0,\ v_f = 12.0 \text{ m/s})$,

$$\Delta x_2 = \frac{v_f^2 - v_i^2}{2a} = \frac{(12.0 \text{ m/s})^2 - (0)^2}{2(4.00 \text{ m/s}^2)} = 18.0 \text{ m}$$

Note that the net displacement is $\Delta x = \Delta x_1 + \Delta x_2 = -4.50 \text{ m} + 18.0 \text{ m} = 13.5 \text{ m}$, as found in part (c) above. However, the total distance travelled in this case is

$$d = |\Delta x_1| + |\Delta x_2| = |-4.50 \text{ m}| + |18.0 \text{ m}| = \boxed{22.5 \text{ m}}$$

2.45 (a) From $v^2 = v_0^2 + 2a(\Delta y)$ with $v = 0$, we have

$$(\Delta y)_{\max} = \frac{v^2 - v_0^2}{2a} = \frac{0 - (25.0 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{31.9 \text{ m}}$$

(b) The time to reach the highest point is

$$t_{\text{up}} = \frac{v - v_0}{a} = \frac{0 - 25.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = \boxed{2.55 \text{ s}}$$

(c) The time required for the ball to fall 31.9 m, starting from rest, is found from

$$\Delta y = (0)t + \frac{1}{2}at^2 \text{ as } t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-31.9 \text{ m})}{-9.80 \text{ m/s}^2}} = \boxed{2.55 \text{ s}}$$

(d) The velocity of the ball when it returns to the original level (2.55 s after it starts to fall from rest) is

$$v = v_0 + at = 0 + \left(-9.80 \text{ m/s}^2\right)(2.55 \text{ s}) = \boxed{-25.0 \text{ m/s}}$$

2.50 (a) After 2.00 s, the velocity of the mailbag is

$$v_{\text{bag}} = v_0 + at = -1.50 \text{ m/s} + \left(-9.80 \text{ m/s}^2\right)(2.00 \text{ s}) = -21.1 \text{ m/s}$$

The negative sign tells us that the bag is moving downward and the magnitude of the velocity gives the speed as $\boxed{21.1 \text{ m/s}}$.

(b) The displacement of the mailbag after 2.00 s is

$$(\Delta y)_{\text{bag}} = \left(\frac{v + v_0}{2}\right)t = \left[\frac{-21.1 \text{ m/s} + (-1.50 \text{ m/s})}{2}\right](2.00 \text{ s}) = -22.6 \text{ m}$$

During this time, the helicopter, moving downward with constant velocity, undergoes a displacement of

$$(\Delta y)_{\text{copter}} = v_0 t + \frac{1}{2}at^2 = (-1.5 \text{ m/s})(2.00 \text{ s}) + 0 = -3.00 \text{ m}$$

The distance separating the package and the helicopter at this time is then

$$d = \left|(\Delta y)_p - (\Delta y)_h\right| = |-22.6 \text{ m} - (-3.00 \text{ m})| = |-19.6 \text{ m}| = \boxed{19.6 \text{ m}}$$

(c) Here, $(v_0)_{\text{bag}} = (v_0)_{\text{copter}} = +1.50\ \text{m/s}$ and $a_{bag} = -9.80\ \text{m/s}^2$ while $a_{\text{copter}} = 0$. After 2.00 s, the velocity of the mailbag is

$$v_{\text{bag}} = 1.50\ \frac{\text{m}}{\text{s}} + \left(-9.80\ \frac{\text{m}}{\text{s}^2}\right)(2.00\ \text{s}) = -18.1\ \frac{\text{m}}{\text{s}}$$ and its speed is $\left|v_{\text{bag}}\right| = \boxed{18.1\ \frac{\text{m}}{\text{s}}}$

In this case, the displacement of the helicopter during the 2.00 s interval is

$$\Delta y_{\text{copter}} = (+1.50\ \text{m/s})(2.00\ \text{s}) + 0 = +3.00\ \text{m}$$

Meanwhile, the mailbag has a displacement of

$$(\Delta y)_{\text{bag}} = \left(\frac{v_{\text{bag}} + v_0}{2}\right)t = \left[\frac{-18.1\ \text{m/s} + 1.50\ \text{m/s}}{2}\right](2.00\ \text{s}) = -16.6\ \text{m}$$

The distance separating the package and the helicopter at this time is then

$$d = \left|(\Delta y)_p - (\Delta y)_h\right| = \left|-16.6\ \text{m} - (+3.00\ \text{m})\right| = \left|-19.6\ \text{m}\right| = \boxed{19.6\ \text{m}}$$

(b) The required time is $t = \boxed{0.782\ \text{s}}$ as calculated above.