13

Vibrations and Waves

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The wavelength of a wave is the distance from crest to the following crest. Thus, the distance between a crest and the following trough is a half wavelength, giving $\lambda = 2(2\text{ m}) = 4\text{ m}$. The speed of the wave is then $v = \lambda f = (4\text{ m})(2\text{ Hz}) = 8\text{ m/s}$, and (c) is the correct choice.

5. The frequency of vibration is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Thus, increasing the mass by a factor of 9 will decrease the frequency to $\frac{1}{3}$ of its original value, and the correct answer is (b).

6. When the object is at its maximum displacement, the net force (directed back toward the equilibrium position) acting on it has magnitude

$$F_{\text{net}} = k\left|x_{\text{max}}\right| = (8.0\text{ N/m})(0.10\text{ m}) = 0.80\text{ N}$$

This force will give the mass an acceleration of $a = F_{\text{net}}/m = 0.80\text{ N}/0.40\text{ kg} = 2.0\text{ m/s}^2$, making (d) the correct choice.

8. The period of a simple pendulum is $T = 2\pi\sqrt{\ell/g}$, and its frequency is $f = 1/T = (1/2\pi)\sqrt{g/\ell}$. Thus, if the length is doubled so $\ell' = 2\ell$, the new frequency is

$$f' = \frac{1}{2\pi}\sqrt{\frac{g}{\ell'}} = \frac{1}{2\pi}\sqrt{\frac{g}{2\ell}} = \frac{1}{\sqrt{2}}\left(\frac{1}{2\pi}\sqrt{\frac{g}{\ell}}\right) = \frac{f}{\sqrt{2}}$$

and we see that (d) is the correct response.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

4. Each half-spring will have twice the spring constant of the full spring, as shown by the following argument. The force exerted by a spring is proportional to the separation of the coils as the spring is extended. Imagine that we extend a spring by a given distance and measure the distance between coils. We then cut the spring in half. If one of the half-springs is now extended by the same distance, the coils will be twice as far apart as they were for the complete spring. Thus, it takes twice as much force to stretch the half-spring, from which we conclude that the half-spring has a spring constant which is twice that of the complete spring.

8. Shorten the pendulum to decrease the period between ticks.

PROBLEM SOLUTIONS

13.2 The force compressing the spring is the weight of the object. Thus, the spring will be compressed a distance of

$$|x| = \frac{|F|}{k} = \frac{mg}{k} = \frac{(2.30\text{ kg})(9.80\text{ m/s}^2)}{1.46\times 10^3\text{ N/m}} = 1.54\times 10^{-2}\text{ m} = \boxed{1.54\text{ cm}}$$

13.10 (a) $k = \dfrac{F_{\max}}{x_{\max}} = \dfrac{230\text{ N}}{0.400\text{ m}} = \boxed{575\text{ N/m}}$

(b) $work\ done = PE_s = \dfrac{1}{2}kx^2 = \dfrac{1}{2}(575\text{ N/m})(0.400)^2 = \boxed{46.0\text{ J}}$

13.35 (a) The period is the time for one complete oscillation. Hence,

$$T = \frac{2.00 \text{ min}}{82}\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \frac{120 \text{ s}}{82.0} \qquad \text{or} \qquad T = \boxed{1.46 \text{ s}}$$

(b) The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\ell/g}$, so the local acceleration of gravity must be

$$g = \frac{4\pi^2\ell}{T^2} = \frac{4\pi^2(0.520 \text{ m})}{(120 \text{ s}/82.0)^2} = \boxed{9.59 \text{ m/s}^2}$$

13.43 (a) The speed of propagation for a wave is the product of its frequency and its wavelength, $v = \lambda f$. Thus, the frequency must be

$$f = \frac{v}{\lambda} = \frac{3.00\times10^8 \text{ m/s}}{5.50\times10^{-7} \text{ m}} = \boxed{5.45\times10^{14} \text{ Hz}}$$

(b) The period is $T = \frac{1}{f} = \frac{1}{5.45\times10^{14} \text{ Hz}} = \boxed{1.83\times10^{-15} \text{ s}}$