8

Rotational Equilibrium and Rotational Dynamics

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Assuming a uniform, solid disk, its moment of inertia about a perpendicular axis through its center is $I = MR^2/2$, so $\tau = I\alpha$ gives

$$\alpha = \frac{2\tau}{MR^2} = \frac{2(40.0\ \text{N}\cdot\text{m})}{(25.0\ \text{kg})(0.800\ \text{m})^2} = 5.00\ \ \text{rad/s}^2$$

and the correct answer is (b).

3. $\tau = rF\sin\theta = (0.500\ \text{m})(80.0\ \text{N})\sin(60.0°) = 34.6\ \text{N}\cdot\text{m}$ which is choice (a).

7. The moment of inertia of a body is determined by its mass and the way that mass is distributed about the rotation axis. Also, the location of the body's center of mass is determined by how its mass is distributed. As long as these qualities do not change, both the moment of inertia and the center of mass are constant. From $\tau = I\alpha$, we see that when a body experiences a constant, non-zero torque, it will have a constant, non-zero angular acceleration. However, since the angular acceleration is non-zero, the angular velocity ω (and hence the angular momentum, $L = I\omega$) will vary in time. The correct responses to this question are then (b) and (e).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The moment of inertia depends on the distribution of mass with respect to a given axis. If the axis is changed, then each bit of mass that makes up the object is at a different distance from the axis than before. Compare the moments of inertia of a uniform rigid rod about axes perpendicular to the rod, first passing through its center of mass, then passing through an end. For example, if you wiggle repeatedly a meter stick back and forth about an axis passing through its center of mass, you will find it does not take much effort to reverse the direction of rotation. However, if you move the axis to an end, you will find it more difficult to wiggle the stick back and forth. The moment of inertia about the end is much larger, because much of the mass of the stick is farther from the axis.

12. (a) Consider two people, at the ends of a long table, pushing with equal magnitude forces directed in opposite directions perpendicular to the length of the table. The net force will be zero, yet the net torque is not zero.

(b) Consider a falling body. The net force acting on it is its weight, yet the net torque about the center of gravity is zero.

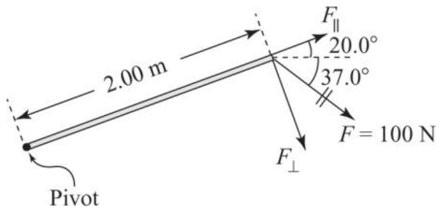
PROBLEM SOLUTIONS

8.1 Resolve the 100-N force into components parallel to and perpendicular to the rod, as

$$F_{\parallel} = F\cos\left(20.0^\circ + 37.0^\circ\right) = F\cos 57.0^\circ$$

and

$$F_{\perp} = F\sin\left(20.0^\circ + 37.0^\circ\right) = F\sin 57.0^\circ$$



The lever arm of $F_{\perp}$ about the indicated pivot is 2.00 m, while that of $F_{\parallel}$ is zero. The torque due to the 100-N force may be computed as the sum of the torques of its components, giving

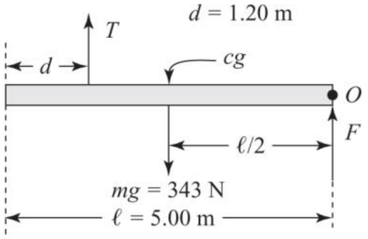
$$\tau = F_{\parallel}(0) - F_{\perp}(2.00\ \text{m}) = 0 - \left[(100\ \text{N})\sin 57.0°\right](2.00\ \text{m}) = -168\ \text{N}\cdot\text{m}$$

or $\boxed{\tau = 168\ \text{N}\cdot\text{m clockwise}}$

8.4 The lever arm is $d = \left(1.20\times10^{-2}\ \text{m}\right)\cos 48.0° = 8.03\times10^{-3}$ m, and the torque is

$$\tau = Fd = (80.0\ \text{N})\left(8.03\times10^{-3}\ \text{m}\right) = \boxed{0.642\ \text{N}\cdot\text{m counterclockwise}}$$

8.8 Since the beam is uniform, its center of gravity is at its geometric center.



Requiring that $\Sigma\tau = 0$ about an axis through point *O* and perpendicular to the page gives

$$F(0) + mg(\ell/2) - T(\ell - d) = 0$$

(a) The tension in the rope must then be

$$T = \frac{mg(\ell/2)}{\ell - d} = \frac{(343\text{ N})(2.50\text{ m})}{5.00\text{ m} - 1.20\text{ m}} = \boxed{226\text{ N}}$$

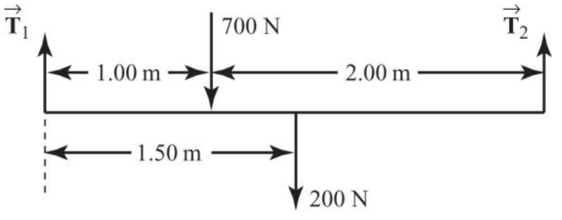
(b) The force the column exerts is found from

$$\Sigma F_y = 0 \quad \Rightarrow \quad F + T - mg = 0$$

or $$F = mg - T = 343\text{ N} - 226\text{ N} = \boxed{717\text{ N upward}}$$

8.20 Consider the torques about an axis perpendicular to the page through the left end of the scaffold.

$$\Sigma\tau = 0 \Rightarrow T_1(0) - (700\text{ N})(1.00\text{ m}) - (200\text{ N})(1.50\text{ m}) + T_2(3.00\text{ m}) = 0$$



From which, $T_2 = \boxed{333\text{ N}}$

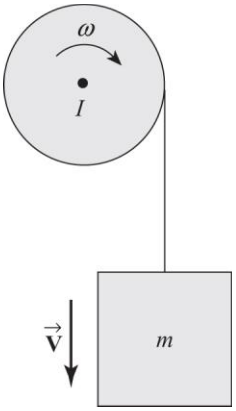
Then, from $\Sigma F_y = 0$, we have

$$T_1 + T_2 - 700\text{ N} - 200\text{ N} = 0$$

or

$$T_1 = 900\text{ N} - T_2 = 900\text{ N} - 333\text{ N} = \boxed{567\text{ N}}$$

8.52 As the bucket drops, it loses gravitational potential energy. The spool gains rotational kinetic energy and the bucket gains translational kinetic energy. Since the string does not slip on the spool, $v = r\omega$ where r is the radius of the spool. The moment of inertia of the spool is $I = \frac{1}{2}Mr^2$, where M is the mass of the spool. Conservation of energy gives



$$\left(KE_t + KE_r + PE_g\right)_f = \left(KE_t + KE_r + PE_g\right)_i$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy_f = 0 + 0 + mgy_i$$

or $$\frac{1}{2}m(r\omega)^2 + \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega^2 = mg\left(y_i - y_f\right)$$

This gives

$$\omega = \sqrt{\frac{2mg\left(y_i - y_f\right)}{\left(m + \frac{1}{2}M\right)r^2}} = \sqrt{\frac{2(3.00\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m})}{\left[3.00\text{ kg} + \frac{1}{2}(5.00\text{ kg})\right](0.600\text{ m})^2}} = \boxed{10.9\text{ rad/s}}$$

8.54 (a) $L = I\omega = \left(MR^2\right)\omega = (2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{2.72\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}(2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{1.36\text{ kg}\cdot\text{m}^2/\text{s}}$

(c) $L = I\omega = \left(\frac{2}{5}MR^2\right)\omega = \frac{2}{5}(2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{1.09\text{ kg}\cdot\text{m}^2/\text{s}}$

(d) $L = I\omega = \left(\frac{2}{3}MR^2\right)\omega = \frac{2}{3}(2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{1.81\text{ kg}\cdot\text{m}^2/\text{s}}$