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Rotational Motion and the Law of Gravity

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Earth moves 2π radians around the Sun in 1 year. The average angular speed is then

$$\omega_{\text{av}} = \frac{2\pi \text{ rad}}{1 \text{ y}}\left(\frac{1 \text{ y}}{3.156\times 10^{7} \text{ s}}\right) = 1.99\times 10^{-7} \text{ rad/s}$$

which is choice (e).

3. The wheel has a radius of 0.500 m and made 320 revolutions. The distance traveled is

$$s = r\theta = (0.500 \text{ m})\left(3.2\times 10^{2} \text{ rev}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 1.0\times 10^{3} \text{ m} = 1.0 \text{ km}$$

so choice (c) is the correct answer.

4. The angular displacement will be

$$\Delta\theta = \omega_{\text{av}}\cdot\Delta t = \left(\frac{\omega_f + \omega_i}{2}\right)\Delta t = \left(\frac{12.00 \text{ rad/s} + 4.00 \text{ rad/s}}{2}\right)(4.00 \text{ s}) = 32.0 \text{ rad}$$

which matches choice (d).

5. According to Newton's law of universal gravitation, the gravitational force one body exerts on the other decreases as the distance separating the two bodies increases. When on Earth's surface, the astronaut's distance from the center of the Earth is Earth's radius $r_0 = R_E$. If h is the altitude at which the station orbits above the surface, her distance from Earth's center when on the station is $r' = R_E + h > r_0$. Thus, she experiences a smaller force while on the space station and (c) is the correct choice.

7. The required centripetal force is $F_c = ma_c = mv^2/r = mr\omega^2$. When *m* and ω are both constant, the centripetal force is directly proportional to the radius of the circular path. Thus, as the rider moves toward the center of the merry-go-round, the centripetal force decreases and the correct choice is (c).

9. The satellite experiences a gravitational force, always directed toward the center of its orbit, and supplying the centripetal force required to hold it in its orbit. This force gives the satellite a centripetal acceleration, even if it is moving with constant angular speed. At each point on the circular orbit, the gravitational force is directed along a radius line of the path, and is perpendicular to the motion of the satellite, so this force does no work on the satellite. Therefore, the only true statement among the listed choices is (d).

PROBLEM SOLUTIONS

7.1 (a) Earth rotates 2π radians (360°) on its axis in 1 day. Thus,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{1 \text{ day}}\left(\frac{1 \text{ day}}{8.64\times10^4 \text{ s}}\right) = \boxed{7.27\times10^{-5} \text{ rad/s}}$$

(b) Because of its rotation about its axis, $\boxed{\text{Earth bulges at the equator.}}$

7.5 (a) $\alpha = \dfrac{\left(2.51\times10^4 \text{ rev/min} - 0\right)}{3.20 \text{ s}}\left(\dfrac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60.0 \text{ s}}\right) = \boxed{821 \text{ rad/s}^2}$

(b) $\theta = \bar{\omega} t = \left(\dfrac{\omega_f + \omega_0}{2}\right)t = \left[\dfrac{\left(2.51\times10^4 \text{ rev/min}\right)\left(2\pi \text{ rad/1 rev}\right)\left(1 \text{ min/60.0 s}\right) + 0}{2}\right](3.20 \text{ s})$

$= \boxed{4.21\times10^3 \text{ rad}}$

7.21 (a) From $\Sigma F_r = ma_c$, we have

$$T = m\left(\frac{v_t^2}{r}\right) = \frac{(55.0\ \text{kg})(4.00\ \text{m/s})^2}{0.800\ \text{m}} = 1.10\times10^3\ \text{N} = \boxed{1.10\ \text{kN}}$$

(b) The tension is $\boxed{\text{larger than her weight}}$ by a factor of

$$\frac{T}{mg} = \frac{1.10\times10^3\ \text{N}}{(55.0\ \text{kg})(9.80\ \text{m/s}^2)} = \boxed{2.04\ \text{times}}$$

7.23 Friction between the tires and the roadway is capable of giving the truck a maximum centripetal acceleration of

$$a_{c,\,\text{max}} = \frac{v_{t,\,\text{max}}^2}{r} = \frac{(32.0\ \text{m/s})^2}{150\ \text{m}} = 6.83\ \text{m/s}^2$$

If the radius of the curve changes to 75.0 m, the maximum safe speed will be

$$v_{t,\,\text{max}} = \sqrt{r\, a_{c,\,\text{max}}} = \sqrt{(75.0\ \text{m})(6.83\ \text{m/s}^2)} = \boxed{22.6\ \text{m/s}}$$

7.33 At the half-way point the spaceship is 1.92×10^8 m from both bodies. The force exerted on the ship by the Earth is directed toward the Earth and has magnitude

$$F_E = \frac{Gm_E m_s}{r^2}$$
$$= \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(3.00\times10^{4}\ \text{kg}\right)}{\left(1.92\times10^{8}\ \text{m}\right)^2} = 325\ \text{N}$$

The force exerted on the ship by the Moon is directed toward the Moon and has a magnitude of

$$F_M = \frac{Gm_M m_s}{r^2}$$
$$= \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.36\times10^{22}\ \text{kg}\right)\left(3.00\times10^{4}\ \text{kg}\right)}{\left(1.92\times10^{8}\ \text{m}\right)^2} = 4.00\ \text{N}$$

The resultant force is $\left(325\ \text{N} - 4.00\ \text{N}\right) = \boxed{321\ \text{N directed toward Earth}}$.

7.34 The radius of the satellite's orbit is

$$r = R_E + h = 6.38\times10^{6}\ \text{m} + 2.00\times10^{6}\ \text{m} = 8.38\times10^{6}\ \text{m}$$

(a) $PE_g = -\dfrac{GM_E m}{r}$

$$= -\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{8.38\times10^{6}\ \text{m}} = \boxed{-4.76\times10^{9}\ \text{J}}$$

(b) $F = \dfrac{GM_E m}{r^2} = \left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\dfrac{\left(5.98\times10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{\left(8.38\times10^{6}\ \text{m}\right)^2} = \boxed{568\ \text{N}}$