6

Momentum and Collisions

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The magnitude of the impulse is

$$I = \Delta p = p_f - p_i = mv_f - mv_i = m\left(v_f - v_i\right)$$

or $$I = \left(0.450\text{ kg}\right)\left(12.8\text{ m/s} - 3.20\text{ m/s}\right) = 4.32\text{ kg}\cdot\text{m/s}$$

making (b) the correct choice.

2. The mass in motion after the rice ball is added to the bowl is twice the original moving mass. Therefore, to conserve momentum, the speed of the (rice ball + bowl) after the event must be one half of the initial speed of the bowl (i.e., $v_f = v_i/2$). The final kinetic energy is then

$$KE_f = \frac{1}{2}\left(m_{\text{ball}} + m_{\text{bowl}}\right)v_f^2 = \frac{1}{2}\left(2m_{\text{bowl}}\right)\left(\frac{v_i}{2}\right)^2 = \frac{1}{2}\left(\frac{1}{2}m_{\text{bowl}}v_i^2\right) = \frac{E}{2}$$

and the correct choice is (c).

3. Assuming that the collision was head-on so that, after impact, the wreckage moves in the original direction of the car's motion, conservation of momentum during the impact gives

$$\left(m_c + m_t\right)v_f = m_c v_{0c} + m_t v_{0t} = m_c v + m_t\left(0\right)$$

or $v_f = \left(\dfrac{m_c}{m_c + m_t}\right)v = \left(\dfrac{m}{m+2m}\right)v = \dfrac{v}{3}$

showing that (c) is the correct choice.

4. The impulse given to the ball is $I = F_{av}(\Delta t) = mv_f - mv_i = m(v_f - v_i)$. Choosing the direction of the final velocity of the ball as the positive direction, this gives

$$F_{av} = \frac{m(v_f - v_i)}{(\Delta t)} = \frac{(57.0\times10^{-3}\ \text{kg})[+25.0\ \text{m/s} - (-21.0\ \text{m/s})]}{0.060\ \text{s}} = 43.7\ \text{kg}\cdot\text{m/s}^2 = 43.7\ \text{N}$$

and the correct choice is (c).

6. We choose the original direction of motion of the cart as the positive direction. Then, $v_i = 6\ \text{m/s}$ and $v_f = -2\ \text{m/s}$. The change in the momentum of the cart is

$$\Delta p = mv_f - mv_i = m(v_f - v_i) = (5\ \text{kg})(-2\ \text{m/s} - 6\ \text{m/s}) = -40\ \text{kg}\cdot\text{m/s}$$

and choice (c) is the correct answer.

7. The requirements of conserving both momentum and kinetic energy in a head-on elastic collision are summarized by the equations

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \qquad \text{and} \qquad v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

With $m_1 = 2$ kg, $v_{1i} = +4\ \text{m/s}$, $m_2 = 1$ kg, and $v_{2i} = 0$, these requirements become

$$8\text{ m/s}+0=2v_{1f}+v_{2f} \qquad \text{and} \qquad 4\text{ m/s}-0=-\left(v_{1f}-v_{2f}\right)$$

Solving these equations simultaneously yields

$$v_{1f}=+\frac{4}{3}\text{ m/s} \qquad \text{and} \qquad v_{2f}=+\frac{16}{3}\text{ m/s}$$

Thus, choice (a) is the correct answer.

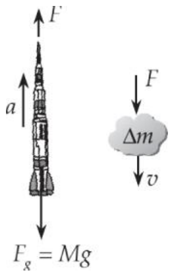
9. Expressing the kinetic energy as $KE=p^2/2m$ (see Questions 10 and 11), we see that the ratio of the magnitudes of the momenta of two particles is

$$\frac{p_2}{p_1}=\frac{\sqrt{2m_2\,(KE)_2}}{\sqrt{2m_1\,(KE)_1}}=\sqrt{\left(\frac{m_2}{m_1}\right)\frac{(KE)_2}{(KE)_1}}$$

Thus, we see that if the particles have equal kinetic energies, the magnitudes of their momenta are equal only if the masses are also equal. However, momentum is a *vector quantity* and we can say the two particles have equal momenta only it both the magnitudes and directions are equal, making choice (d) the correct answer.

12. Consider the sketches at the right. The leftmost sketch shows the rocket immediately after the engine is fired (while the rocket's velocity is still essentially zero). It has two forces acting on it, an upward thrust *F* exerted by the burnt fuel being ejected from the engine, and a downward force of gravity. These forces produce the upward acceleration *a* of the rocket according to Newton's second law:

$$\Sigma F_y=F-F_g=Ma$$



Since $F_g = Mg$, the thrust exerted on the rocket by the ejected fuel is

$$F = F_g + Ma = M\left(a+g\right)$$

The rightmost part of the sketch shows a quantity of burnt fuel that was initially at rest within the rocket, but a very short time Δt later is moving downward at speed v. As this material is ejected, it exerts the upward thrust *F* on the rocket. By Newton's third law, the rocket exerts a downward force of equal magnitude on this burnt fuel. This force imparts an impulse $I = F(\Delta t) = \Delta p = \Delta m(v-0)$ to the ejected material. Thus, the rate the rocket is burning and ejecting fuel must be

$$\frac{\Delta m}{\Delta t} = \frac{F}{v-0} = \frac{M\left(a+g\right)}{v} = \frac{\left(3.00\times10^5\text{ kg}\right)\left[(36.0+9.80)\text{ m/s}^2\right]}{4.50\times10^3\text{ m/s}} = 3.05\times10^3\text{ kg/s}$$

and we see that choice (a) is the correct response.

Note: Failure to include the gravitational force in this analysis will lead some students to incorrectly select choice (b) as their answer.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

4. The glass, concrete, and steel were part of a rigid structure that shattered upon impact of the airplanes with the towers and upon collapse of the buildings as the steel support structures weakened due to high temperatures of the burning fuel. The sheets of paper floating down were probably not in the vicinity of the direct impact, where they would have burned after being exposed to very high temperatures. The papers were most likely situated on desktops or open file cabinets and were blown out of the buildings as they collapsed.

10. A certain impulse is required to stop the egg. But, if the time during which the momentum change of the egg occurs is increased, the resulting force on the egg is reduced. The time is increased when the sheet billows out as the egg is brought to a stop. The force is reduced low enough so that the egg will not break.

14. Its speed decreases as its mass increases. No external horizontal forces act on the box-rainwater system, so its horizontal momentum cannot change as the box moves along the surface. Because the product mv_x must be constant, and because the mass of the box (m) is increasing as it slowly fills with water, the horizontal speed of the box must decrease.

PROBLEM SOLUTIONS

6.4 (a) We find the maximum height from $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$, with $v_y = 0$ at $\Delta y = h_{\text{max}}$.

$$h_{\text{max}} = \frac{0 - v_0^2}{2(-g)} = \boxed{v_0^2/2g}$$

(b) We use conservation of energy to find the velocity of the ball at $y_f = h_{\text{max}}/2 = v_0^2/4g$. Taking $PE_g = 0$ at $y_i = 0$, we have

$$KE_f + PE_{g,f} = KE_i + PE_{g,i}$$

or $$\frac{1}{2}\cancel{m}v^2 + \cancel{m}\cancel{g}\left(\frac{v_0^2}{4\cancel{g}}\right) = \frac{1}{2}\cancel{m}v_0^2 + 0$$

giving $v^2 = v_0^2/2$, or $v = v_0/\sqrt{2}$. The momentum at this height is then

$$p = mv = \boxed{mv_0/\sqrt{2}}$$

6.7 (a) $\frac{KE}{p} = \frac{\frac{1}{2}mv^2}{mv} = \frac{v}{2}$ so $v = \frac{2(KE)}{p} = \frac{2(275\ \text{J})}{25.0\ \text{kg}\cdot\text{m/s}} = \boxed{22.0\ \text{m/s}}$

(b) $m = \frac{p}{v} = \frac{25.0\ \text{kg}\cdot\text{m/s}}{22.0\ \text{m/s}} = \boxed{1.14\ \text{kg}}$

6.11 The velocity of the ball just before impact is found from $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ as

$$v_1 = -\sqrt{v_{0y}^2 + 2a_y\Delta y} = -\sqrt{0 + 2\left(-9.80\ \text{m/s}^2\right)\left(-1.25\ \text{m}\right)} = -4.95\ \text{m/s}$$

and the rebound velocity with which it leaves the floor is

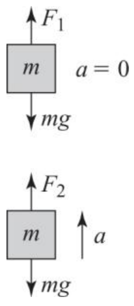
$$v_2 = +\sqrt{v_f^2 - 2a_y\Delta y} = +\sqrt{0 - 2\left(-9.80\ \text{m/s}^2\right)\left(+0.960\ \text{m}\right)} = +4.34\ \text{m/s}$$

The impulse given the ball by the floor is then

$$\begin{aligned}\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t = \Delta\left(m\vec{\mathbf{v}}\right) = m\left(\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1\right)\\ = \left(0.150\ \text{kg}\right)\left[+4.34\ \text{m/s} - \left(-4.95\ \text{m/s}\right)\right] = +1.39\ \text{N}\cdot\text{s} = \boxed{1.39\ \text{N/s upward}}\end{aligned}$$

6.14 (a) Choose upward as the positive direction:

$$I = m\left(v_f - v_i\right) = \left(65.0\ \text{kg}\right)\left(+1.80\ \text{m/s} - 0\right) = +117\ \text{kg}\cdot\text{m/s} = \boxed{117\ \text{kg}\cdot\text{m/s}\quad\text{upward}}$$



(b) Before the jump, the player is in equilibrium, so

$$\Sigma F_y = ma_y = 0 \quad \Rightarrow \quad F_1 = mg$$

or $$F_1 = (65.0\ \text{kg})(9.80\ \text{m/s}^2) = +637\ \text{N} = \boxed{637\ \text{N upward}}$$

(c) From the impulse-momentum theorem, the net force the player experiences during the jump is

$$F_{\text{net}} = \frac{I}{\Delta t} = \frac{+117\ \text{kg}\cdot\text{m/s}}{0.450\ \text{s}} = +260\ \text{N}$$

But $F_{\text{net}} = F_2 - mg = F_2 - F_1$, where F_2 is the upward force the floor exerts on the player during the jump and F_1 is the force exerted by the floor before the jump. Thus,

$$F_2 = F_{\text{net}} + F_1 = +260\ \text{N} + 637\ \text{N} = +897\ \text{N} = \boxed{897\ \text{N upward}}$$

6.15 (a) The impulse equals the area under the *F* versus *t* graph. This area is the sum of the area of the rectangle plus the area of the triangle. Thus,

$$I = (2.0\ \text{N})(3.0\ \text{s}) + \frac{1}{2}(2.0\ \text{N})(2.0\ \text{s}) = \boxed{8.0\ \text{N}\cdot\text{s}}$$

(b) $I = F_{av}(\Delta t) = \Delta p = m(v_f - v_i)$

$$8.0\ \text{N}\cdot\text{s} = (1.5\ \text{kg})v_f - 0, \text{ giving } v_f = \boxed{5.3\ \text{m/s}}$$

(c) $I = F_{av}(\Delta t) = \Delta p = m(v_f - v_i)$, so $v_f = v_i + \dfrac{I}{m}$

$$v_f = -2.0\ \text{m/s} + \frac{8.0\ \text{N}\cdot\text{s}}{1.5\ \text{kg}} = \boxed{3.3\ \text{m/s}}$$

6.22 (a) The mass of the rifle is

$$m = \frac{w}{g} = \frac{30\ \text{N}}{9.80\ \text{m/s}^2} = \left(\frac{30}{9.8}\right)\ \text{kg}$$

We choose the direction of the bullet's motion to be negative. Then, conservation of momentum gives

$$(m_{\text{rifle}}v_{\text{rifle}} + m_{\text{bullet}}v_{\text{bullet}})_f = (m_{\text{rifle}}v_{\text{rifle}} + m_{\text{bullet}}v_{\text{bullet}})_i$$

or $$[(30/9.8)\ \text{kg}]v_{\text{rifle}} + (5.0\times10^{-3}\ \text{kg})(-300\ \text{m/s}) = 0 + 0$$

and $$v_{\text{rifle}} = \frac{9.8(5.0\times10^{-3}\ \text{kg})(300\ \text{m/s})}{30\ \text{kg}} = \boxed{0.49\ \text{m/s}}$$

(b) The mass of the man plus rifle is

$$m = \frac{730\ \text{N}}{9.80\ \text{m/s}^2} = 74.5\ \text{kg}$$

We use the same approach as in (a), to find

$$v = \left(\frac{5.0\times10^{-3}\ \text{kg}}{74.5\ \text{kg}}\right)(300\ \text{m/s}) = \boxed{2.0\times10^{-2}\ \text{m/s}}$$

6.26 The boat and fisherman move as a single unit having mass

$$m_{BF} = m_B + m_F = 125\ \text{kg} + 75\ \text{kg} = 200\ \text{kg}$$

Before the package is thrown, all parts of the system, (boat + fisherman) and package, are at rest, so the total initial momentum is zero. Neglecting water resistance, the final momentum of the system must also be zero, or

$$m_{BF}v_{BF} + m_p v_p = 0$$

giving $$v_{BF} = -\left(\frac{m_p}{m_{BF}}\right)v_p = -\left(\frac{15\ \text{kg}}{200\ \text{kg}}\right)(+4.5\ \text{m/s})$$

and $$v_{BF} = -0.34\ \text{m/s} \quad \text{or} \quad \boxed{0.34\ \text{m/s toward the left}}$$