3

Vectors and Two-Dimensional Motion

ANSWERS TO MULTIPLE CHOICE QUESTIONS

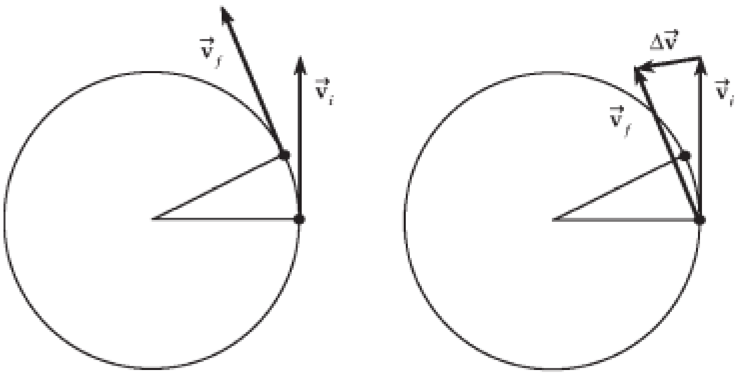
2. The skier has zero initial velocity in the vertical direction $(v_{0y} = 0)$ and undergoes a vertical displacement of $\Delta y = -3.20$ m. The constant acceleration in the vertical direction is $a_y = -g$, so we use $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ to find the time of flight as

$$-3.20 \text{ m} = 0 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)t^2 \qquad \text{or} \qquad t = \sqrt{\frac{2(-3.20 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.808 \text{ s}$$

During this time, the object moves with constant horizontal velocity $v_x = v_{0x} = 22.0$ m/s. The horizontal distance travelled during the flight is

$$\Delta x = v_x t = (22.0 \text{ m/s})(0.808 \text{ s}) = 17.8 \text{ m},$$ which is choice (d).

6. Consider any two very closely spaced points on a circular path and draw vectors of the same length (to represent a constant velocity magnitude or speed) tangent to the path at each of these points as shown in the leftmost diagram below. Now carefully move the velocity vector $\vec{\mathbf{v}}_f$ at the second point down so its tail is at the first point as shown in the rightmost diagram. Then, draw the vector difference $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i$, and observe that if the start of this vector were located on the circular path midway between the two points, its direction would be inward toward the center of the circle.



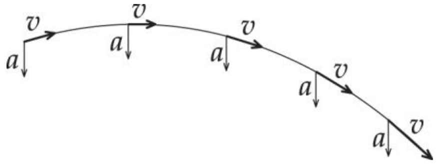
Thus, for an object following the circular path at constant speed, its instantaneous acceleration, $\vec{\mathbf{a}} = \lim_{\Delta t \to 0}\left(\Delta\vec{\mathbf{v}}/\Delta t\right)$, at the point midway between your initial and end points is directed toward the center of the circle, and the only correct choice for this question is (d).

10. While in the air, the baseball is a projectile whose velocity always has a constant horizontal component $(v_x = v_{0x})$ and a vertical component that changes at a constant rate $(\Delta v_y/\Delta t = a_y = -g)$. At the highest point on the path, the vertical velocity of the ball is momentarily zero. Thus, at this point, the resultant velocity of the ball is horizontal and its acceleration continues to be directed downward $(a_x = 0,\ a_y = -g)$. The only correct choice given for this question is (c).

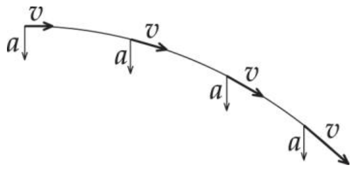
13. Of the choices listed, the quantities which have magnitude or size, but no direction, associated with them (i.e., scalar quantities) are (b) temperature, (c) volume, and (e) height. The other quantities, (a) velocity of a sports car and (d) displacement of a tennis player who moves from the court's backline to the net, have both magnitude and direction associated with them, and are both vector quantities.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

4. **(A)**

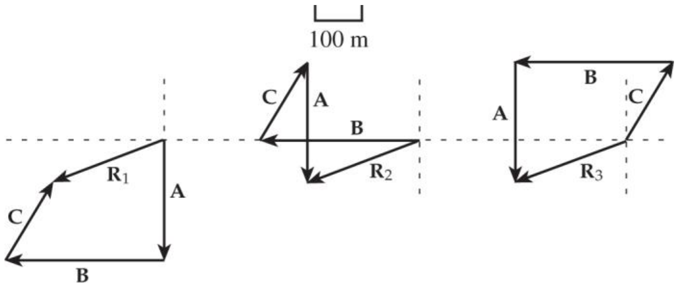


(B)



PROBLEM SOLUTIONS

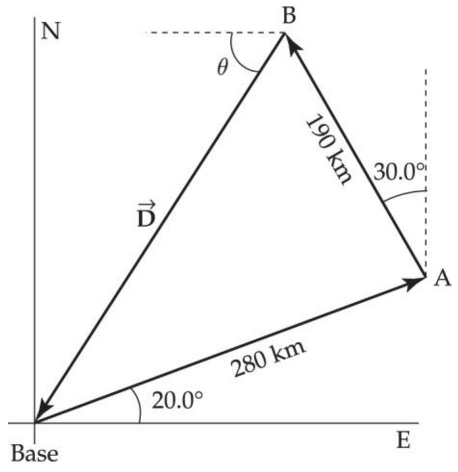
3.4 (a) The three diagrams shown below represent the graphical solutions for the three vector sums: $\vec{\mathbf{R}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$, $\vec{\mathbf{R}}_2 = \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{A}}$, and $\vec{\mathbf{R}}_3 = \vec{\mathbf{C}} + \vec{\mathbf{B}} + \vec{\mathbf{A}}$.



(b) We observe that $\vec{\mathbf{R}}_1 = \vec{\mathbf{R}}_2 = \vec{\mathbf{R}}_3$, illustrating that the sum of a set of vectors is not affected by the order in which the vectors are added.

3.7 Using a vector diagram, drawn to scale, like that shown at the right, the displacement from Lake B back to base camp is given by the vector $\vec{\mathbf{D}}$. Measuring the length of this vector and multiplying by the chosen scale factor should give the magnitude of this displacement as 310 km. Measuring the angle θ should yield a value of 57°. Thus, the displacement from B to the base camp is

$$\vec{\mathbf{D}} = \boxed{310 \text{ km at } \theta = 57° \text{ S of W}}$$



3.13 (a) Her net x (east-west) displacement is $-3.00+0+6.00=+3.00$ blocks, while her net y (north-south) displacement is $0+4.00+0=+4.00$ blocks. The magnitude of the resultant displacement is

$$R=\sqrt{(\Sigma x)^2+(\Sigma y)^2}=\sqrt{(3.00)^2+(4.00)^2}=5.00\text{ blocks}$$

and the angle the resultant makes with the x-axis (eastward direction) is

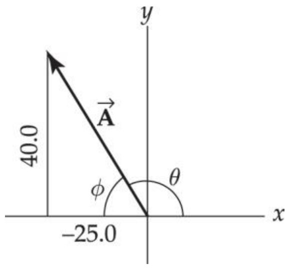
$$\theta=\tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)=\tan^{-1}\left(\frac{4.00}{3.00}\right)=\tan^{-1}(1.33)=53.1^\circ$$

The resultant displacement is then $\boxed{5.00\text{ blocks at }53.1^\circ\text{ N of E}}$.

(b) The total distance travelled is $3.00+4.00+6.00=\boxed{13.0\text{ blocks}}$.

3.15 $A_x=-25.0$ $A_y=40.0$

$$A=\sqrt{A_x^2+A_y^2}=\sqrt{(-25.0)^2+(40.0)^2}=47.2\text{ units}$$



From the triangle, we find that

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan^{-1}\left(\frac{|40.0|}{25.0}\right) = 58.0°, \text{ so } \theta = 180° - \phi = 122°$$

Thus, $\boxed{\vec{\mathbf{A}} = 47.2 \text{ units at } 122° \text{ counterclockwise from the } +x\text{-axis}}$.

3.22 $v_{0x} = (101 \text{ mi/h})\left(\frac{0.447 \text{ m/s}}{1 \text{ mi/h}}\right) = 45.1 \text{ m/s}$ and $\Delta x = (60.5 \text{ ft})\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 18.4 \text{ m}$

The time to reach home plate is $t = \frac{\Delta x}{v_{0x}} = \frac{18.4 \text{ m}}{45.1 \text{ m/s}} = 0.408 \text{ s}$.

In this time interval, the vertical displacement is

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.408 \text{ s})^2 = -0.817 \text{ m}$$

Thus, the ball drops vertically $0.817 \text{ m}(3.281 \text{ ft/1 m}) = \boxed{2.68 \text{ ft}}$.