Final Review

Intermediate Algebra / MAT135 2014 FALL

1. Solve for \( c \).

\[ a = \frac{1}{2} (b + c - d) \]

2. Solve for \( d \).

\[ a = \frac{b + 4}{c + d} \]

3. Solve for \( c \).

\[ \frac{9}{b} = d - \frac{5}{c} \]

4. Jenny, Abdul, and Frank sent a total of 99 text messages during the weekend. Abdul sent 9 more messages than Jenny. Frank sent 4 times as many messages as Jenny. How many messages did they each send?
5. Three consecutive integers have a sum of 51. Find the integers.

6. Linda works as a tutor for $9 an hour and as a waitress for $14 an hour. This month, she worked a combined total of 36 hours at her two jobs.

   Let \( t \) be the number of hours Linda worked as a tutor this month. Write an expression for the combined total dollar amount she earned this month.

7. A Web music store offers two versions of a popular song. The size of the standard version is 2.7 megabytes (MB). The size of the high-quality version is 4.2 MB. Yesterday, the high-quality version was downloaded three times as often as the standard version. The total size downloaded for the two versions was 4437 MB. How many downloads of the standard version were there?

8. A tortoise is walking in the desert. It takes 6 minutes to walk 9.6 meters. What is its speed?

9. Two trains leave the station at the same time, one heading east and the other west. The eastbound train travels at 75 miles per hour. The westbound train travels at 95 miles per hour. How long will it take for the two trains to be 442 miles apart?

   Do not do any rounding.

   \( \square \) hours

10. The perimeter of a rectangular painting is 360 centimeters. If the length of the painting is 96 centimeters, what is its width?
11. An alloy is a mixture of metals. Suppose that a certain alloy is made by mixing 180 grams of an alloy containing 24% copper with 140 grams of an alloy containing 34% copper.

Answer the questions below. Do not do any rounding.

(a) How many grams of copper are in the resulting mixture?
_____ grams

(b) What percentage of the resulting mixture is copper?
_____ %

12. Graph the set \( \{ x \mid -3 < x \leq 4 \} \) on the number line.

Then, write the set using interval notation.

13. Solve the inequality for \( y \).

\( 6 - 9y > 30 - 5y \)

Simplify your answer as much as possible.
14. For each inequality, choose the statement that describes its solution. If applicable, give the solution.

\[-4(x+3)+31 \geq 4(5-x)\]

- No solution
- \(x \leq \)
- \(x \geq \)
- All real numbers are solutions

\[4(5-y)+6y < 14\]

- No solution
- \(y < \)
- \(y > \)
- All real numbers are solutions

15. Solve the compound inequality.

\[-5 < 2x - 3 \leq 3\]

Graph the solution on the number line.

16. Solve the compound inequality.

\[4y + 6 \leq 30 \quad \text{and} \quad 3y - 4 < -1\]

Write the solution in interval notation. If there is no solution, enter \(\emptyset\).
17. Solve for \( w \).
\[ |w| + 7 = 15 \]

18. Solve for \( y \).
\[ 4 |y - 6| = 40 \]

19. Solve for \( w \).
\[ |4w - 7| = |4w - 5| \]

20. Graph the solution to the inequality on the number line.
\[ |y - 2| \geq 7 \]

21. Graph the solution to the inequality on the number line.
\[ |5y - 10| \geq 5 \]
22. Solve.

\[ 3 |y - 8| + 2 > 11 \]

23. A line passes through the point \((8, -3)\) and has a slope of \(\frac{5}{4}\).

Write an equation in slope-intercept form for this line.

24. A line passes through the point \((8, -3)\) and has a slope of \(\frac{3}{4}\).

Write an equation in point-slope form for this line.

25. Write an equation of the line below.
26. Find an equation for the line below.

27. Write equations for the horizontal and vertical lines passing through the point \((-9, 3)\).

   horizontal line:

   vertical line:

28. The equations of three lines are given below.

   Line 1: \(4x - 6y = -2\)
   
   Line 2: \(y = \frac{3}{2}x - 6\)
   
   Line 3: \(-3y = 2x + 5\)

   For each pair of lines, determine whether they are parallel, perpendicular, or neither.
29. Consider the line $y = -\frac{5}{3}x + 5$.

(a) Find the equation of the line that is perpendicular to this line and passes through the point $(-5, -4)$.

(b) Find the equation of the line that is parallel to this line and passes through the point $(-5, -4)$.

30. Suppose that the height (in centimeters) of a candle is a linear function of the amount of time (in hours) it has been burning. After 13 hours of burning, a candle has a height of 20.5 centimeters. After 27 hours of burning, its height is 13.5 centimeters. What is the height of the candle after 16 hours?
31. For each graph below, state whether it represents a function.

- **Function?**
  - Yes    No

- **Function?**
  - Yes    No

- **Function?**
  - Yes    No

- **Function?**
  - Yes    No

- **Function?**
  - Yes    No

- **Function?**
  - Yes    No
32. The functions \( f \) and \( g \) are defined as follows.

\[
f(x) = -4x - 3 \quad g(x) = 4x^2 - x
\]

Find \( f'(6) \) and \( g(-5) \).

Simplify your answers as much as possible.

33. The entire graph of the function \( h \) is shown in the figure below. Write the domain and range of \( h \) using interval notation.

![Graph of the function h](image)

34. Find the domain of the function.

\[
v(x) = \sqrt{-x+5} \]

Write your answer using interval notation.

35. For each of the following equations, determine whether \( y \) is a function of \( x \).

<table>
<thead>
<tr>
<th>( y = 3x^2 - 1 )</th>
<th>Function</th>
<th>Not a function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y^2 = 4x )</td>
<td>Function</td>
<td>Not a function</td>
</tr>
<tr>
<td>( x = 8y^2 + 5 )</td>
<td>Function</td>
<td>Not a function</td>
</tr>
<tr>
<td>( 3x - y = 9 )</td>
<td>Function</td>
<td>Not a function</td>
</tr>
</tbody>
</table>
36. For each system of linear equations shown below, classify the system as "consistent dependent," "consistent independent," or "inconsistent." Then, answer the question about its solutions.

L1: \( y = x - 3 \)
L2: \( y = x - 2 \)

This system of equations is:
- consistent dependent - consistent independent - inconsistent
This means the system has:
- a unique solution:
  Solution: \( ( , ) \)
- no solution
- infinitely many solutions

L1: \( y = \frac{1}{2} x - 3 \)
L2: \( y = -x \)

This system of equations is:
- consistent dependent - consistent independent - inconsistent
This means the system has:
- a unique solution:
  Solution: \( ( , ) \)
- no solution
- infinitely many solutions
This system of equations is:
- consistent dependent - consistent independent - inconsistent
This means the system has:
- a unique solution:
  Solution: \(( , )\)
- no solution
- infinitely many solutions

37. Solve the following system of equations.

\[
\begin{align*}
5x + 3y &= 5 \\
5x + 7y &= 25
\end{align*}
\]

38. Solve the following system of equations.

\[
\begin{align*}
-3x - 4y &= 11 \\
-5x - 3y &= 0
\end{align*}
\]
39. A party rental company has chairs and tables for rent. The total cost to rent 3 chairs and 5 tables is $47. The total cost to rent 12 chairs and 2 tables is $35. What is the cost to rent each chair and each table?

40. Two mechanics worked on a car. The first mechanic worked for 20 hours, and the second mechanic worked for 5 hours. Together they charged a total of $1675. What was the rate charged per hour by each mechanic if the sum of the two rates was $140 per hour?

   First mechanic:
   Second mechanic:

41. Ashley bought a desktop computer and a laptop computer. Before finance charges, the laptop cost $300 less than the desktop. She paid for the computers using two different financing plans. For the desktop the interest rate was 7% per year, and for the laptop it was 6% per year. The total finance charges for one year were $396. How much did each computer cost before finance charges?

42. Factor the following expression.

   \(3v^7w^2x^4 + 21v^2x^3\)

43. Factor by grouping.

   \(5y^3 + 7y^2 + 15y + 21\)
44. Factor by grouping.

\[ yx + 6y - 5x - 30 \]

45. Factor.

\[ x^2 - 10xy + 16y^2 \]

46. Factor completely.

\[ 2y^2 - 26y - 60 \]

47. Factor.

\[ 2z^2 - 15z + 7 \]
48. Factor by grouping (sometimes called the ac-method).

\[ 14x^2 + x - 4 \]

First, choose a form with appropriate signs. Then, fill in the blanks with numbers to be used for grouping. Finally, show the factorization.

<table>
<thead>
<tr>
<th>Form:</th>
<th>Factorization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 14x^2 + _x + _x - 4 ]</td>
<td>[ _ _ _ _ \_ _ _ _ ]</td>
</tr>
<tr>
<td>[ 14x^2 + _x - _x - 4 ]</td>
<td>[ _ _ _ _ _ _ _ _ ]</td>
</tr>
<tr>
<td>[ 14x^2 - _x + _x - 4 ]</td>
<td>[ _ _ _ _ _ _ _ _ ]</td>
</tr>
<tr>
<td>[ 14x^2 - _x - _x - 4 ]</td>
<td>[ _ _ _ _ _ _ _ _ ]</td>
</tr>
</tbody>
</table>

49. Factor:

\[ 2x^2 + 9xy - 18y^2 \]

50. Factor:

\[ 81w^2 + 36w + 4 \]
51. Factor completely.

\[ 75v^2 - 12v^4 \]

52. Solve the equation

\[ (y + 5)^2 = 2y^2 + 4y + 30 \]

for \( y \).

53. For the following right triangle, find the side length \( x \). Round your answer to the nearest hundredth.

54. Simplify.

\[ \frac{32w^2x^2}{8w^2x^2} \]
55. Find all excluded values for the expression.
   That is, find all values of \( \nu \) for which the expression is undefined.

\[
\frac{\nu - 6}{\nu^2 - 81}
\]

If there is more than one value, separate them with commas.

56. Simplify.

\[
\frac{5w^2 - 9w}{3w^2 - 18w}
\]

57. Simplify.

\[
\frac{\nu - 4}{\nu^2 - 8\nu + 16}
\]

58. Simplify.

\[
\frac{\nu^2 - 4\nu + 4}{3\nu^2 - 12}
\]
59. Simplify.
\[
\frac{27 - 3w^2}{w^2 - 8w + 15}
\]

60. Multiply.
\[
\frac{x - 3}{2x - 18} \cdot \frac{4x - 36}{3x - 9}
\]
Simplify your answer as much as possible.

61. Multiply.
\[
\frac{x + 3}{3x - 9} \cdot \frac{x^2 - x - 6}{x - 2}
\]
Simplify your answer as much as possible.

62. Multiply.
\[
\frac{25x^2 - 49}{3x^2 + x - 14} \cdot \frac{4x - 8}{5x + 7}
\]
Simplify your answer as much as possible.

63. Divide.
\[
\frac{10b^2}{a^5b} ÷ \frac{4b}{3a^2}
\]
Simplify your answer as much as possible.
64. Divide.

\[
\frac{25x^2 - 49}{3x^2 + x - 14} \div \frac{5x + 7}{4x - 8}
\]

Simplify your answer as much as possible.

65. Divide.

\[
\frac{x^2 - 10xy + 16y^2}{x^2 - 25y^2} \div \frac{4x - 32y}{x + 5y}
\]

Simplify your answer as much as possible.

66. Fill in the blank to make equivalent rational expressions.

\[
\frac{3}{\nu^6} = \frac{\_\_}{\nu^3}
\]

67. Factor.

\[27\nu^3 - 54\]

68. Find the least common denominator of \(\frac{3}{2x - 8}\) and \(\frac{2x}{5x - 20}\).
69. Subtract.
\[
\frac{4c - 3d}{4c} - \frac{7c + 11d}{4c}
\]
Simplify your answer as much as possible.

70. Subtract.
\[
\frac{y^2 - y}{y^2 - 6y + 9} - \frac{6}{y^2 - 6y + 9}
\]
Simplify your answer as much as possible.

71. Subtract.
\[
\frac{-7}{10y^2} - \frac{5}{6y^3}
\]
Simplify your answer as much as possible.

72. Subtract.
\[
\frac{5x}{4x - 16} - \frac{x + 4}{3x - 12}
\]
Simplify your answer as much as possible.
73. Add.
\[ \frac{2}{3x^2 - 2x - 8} + \frac{1}{3x^2 + 13x + 12} \]
Simplify your answer as much as possible.

74. Divide.
\[ (5x^2 + 40x + 37) ÷ (x + 7) \]
Your answer should give the quotient and the remainder.
Quotient:
Remainder:

75. Divide.
\[ (20x^3 + 17x^2 + 5x + 3) ÷ (5x - 2) \]
Your answer should give the quotient and the remainder.
Quotient:
Remainder:

76. Solve for \( x \).
\[ \frac{-28}{x - 2} = \frac{-20}{x} \]
77. Solve for $v$.

\[
\frac{3}{5v} = \frac{1}{10} + \frac{7}{2v}
\]

78. Solve for $y$.

\[
\frac{3}{y-6} - \frac{5}{y-6} = 2
\]

79. Solve for $y$.

\[
\frac{1}{y-4} + \frac{3}{y+2} = \frac{10}{y^2 - 2y - 8}
\]

80. Solve for $x$.

\[
\frac{x}{5x + 12} = \frac{-2}{x}
\]

81. Solve for $y$.

\[
\frac{y-5}{y-1} + 1 = \frac{y-3}{y+2}
\]
82. Working together, it takes two different sized hoses 20 minutes to fill a small swimming pool. If it takes 25 minutes for the larger hose to fill the swimming pool by itself, how long will it take the smaller hose to fill the pool on its own?

Do not do any rounding.

83. Simplify.

\[ \sqrt[3]{20t^{10}u^3} \]

Assume that all variables represent positive real numbers.

84. Write the following expression in simplified radical form.

\[ \sqrt[5]{81y^2z^6} \]

Assume that all of the variables in the expression represent positive real numbers.

85. Write the following expression in simplified radical form.

\[ \sqrt[5]{192s^{10}t^{14}} \]

Assume that all of the variables in the expression represent positive real numbers.

86. Simplify.

\[ \frac{2}{16^{\frac{3}{2}}} \]
87. Simplify. Write your answers without exponents.

\[ \left( \frac{1}{25} \right)^{\frac{3}{2}} = \]

\[ 8 \cdot \frac{4}{3} = \]

88. Simplify.

\[ \frac{3}{u^5} \cdot \frac{2}{u^2} \]

Assume that the variable represents a positive real number.

89. Simplify the expression.

\[ \frac{1}{y^{\frac{3}{2}}} \]

\[ \frac{1}{y^4} \cdot y^{-\frac{1}{2}} \]

Write your answer using only positive exponents. Assume that all variables are positive real numbers.

90. Simplify.

\[ \sqrt{63} + 3\sqrt{28} \]

91. Simplify.

\[ 4 \sqrt{5} + 5 \sqrt{45} - \sqrt{20} \]
92. Simplify.
\[ \sqrt{75} w - \sqrt{12} w \]
Assume that the variable represents a positive real number.

93. Simplify.
\[ 6 x^2 \sqrt{72} x - \sqrt{18} x^5 \]
Assume that the variable represents a positive real number.

94. Simplify as much as possible.
\[ x \sqrt{75 u^5} - 2 u^2 \sqrt{3 u x^2} \]
Assume that all variables represent positive real numbers.

95. Simplify.
\[ -\sqrt[3]{54} - 4 \sqrt[3]{16} \]

96. Simplify.
\[ -\sqrt[3]{54} x^{14} - 3 \sqrt[3]{16} x^{14} \]
Assume that the variable represents a positive real number.
97. Simplify.
\[
\sqrt{7}z \cdot \sqrt{3}z
\]
Assume that the variable represents a positive real number.

98. Simplify.
\[
\sqrt{3}c^3 \sqrt{6}c^2
\]
Assume that the variable represents a positive real number.

99. Simplify.
\[
\sqrt{15}y^6 \sqrt{x^2} \sqrt{5}y^3 x^3
\]
Assume that all variables represent positive real numbers.

100. Simplify.
\[
\sqrt[4]{4} \cdot \sqrt[4]{20}
\]

101. Multiply.
\[
(8\sqrt{6} + 2)(5 + 4\sqrt{10})
\]
Simplify your answer as much as possible.
102. Multiply and simplify.

\[
(\sqrt{x} + 2\sqrt{3})^2 = \square \\
(\sqrt{x} + \sqrt{3})(\sqrt{x} - \sqrt{3}) = \square
\]

103. Simplify.

\[
\frac{\sqrt{15}}{\sqrt{60}}
\]

104. Simplify.

\[
\frac{\sqrt{28} - 4}{12}
\]

105. Rationalize the denominator and simplify.

\[
\frac{\sqrt{5}}{\sqrt{6}}
\]

106. Rationalize the denominator and simplify.

\[
\frac{\sqrt{3} + \sqrt{5}}{3\sqrt{3} - \sqrt{5}}
\]
107. Rationalize the denominator and simplify.

\[
\frac{3}{\sqrt{16}}
\]

108. Write in simplified radical form with at most one radical.

\[
\frac{3}{\sqrt{y}} \cdot \frac{\sqrt{y^2}}{\sqrt{y^2}}
\]

Assume that the variable represents a positive real number.

109. Solve for \( y \), where \( y \) is a real number.

\[
\sqrt{y} + 6 = 8
\]

110. Solve for \( y \), where \( y \) is a real number.

\[
3 + \sqrt{y} = \sqrt{y + 39}
\]

111. Solve for \( y \), where \( y \) is a real number.

\[
y = \sqrt{2y^2 - 8y + 12}
\]
112. Solve the following equation for $B$.

$$I = \sqrt{\frac{B}{7}}$$

113. Solve for $\nu$, where $\nu$ is a real number.

$$\frac{1}{\nu^4} = 2$$

114. Solve for $u$, where $u$ is a real number.

$$\left(3u + 4\right)^{\frac{1}{4}} + 6 = 8$$

115. Simplify the expressions below as much as possible.

Leave no negative numbers under radicals and no radicals in denominators.

$$\frac{\sqrt{-120}}{\sqrt{-10}} = \square$$

$$\sqrt{-25} \cdot \sqrt{-100} = \square$$

116. Multiply.

$$\left(-2 + 6i\right)\left(-4 + 5i\right)$$

Write your answer as a complex number in standard form.
117. Divide.
\[
\frac{-3 + 4i}{2 - 5i}
\]
Write your answer as a complex number in standard form.

118. \(\xi^3 = 3\) where \(\xi\) is a real number.
Simplify your answer as much as possible.

119. Solve \((z + 4)^\frac{3}{2} = 5\) where \(z\) is a real number.
Simplify your answer as much as possible.

120. Solve \(5y - \frac{1}{5} = 10\) where \(y\) is a real number.
Simplify your answer as much as possible.

121. Write the quadratic equation whose roots are \(-1\) and \(5\), and whose leading coefficient is 4.

122. Solve \(\nu^2 = 25\), where \(\nu\) is a real number.
Simplify your answer as much as possible.

123. Solve \((\gamma - 6)^2 - 50 = 0\), where \(\gamma\) is a real number.
Simplify your answer as much as possible.

124. Fill in the blank to make the expression a perfect square.
\[x^2 - 14x + \square\]
125. Solve the quadratic equation by completing the square.

\[ x^2 - 16x + 57 = 0 \]

First, choose the appropriate form and fill in the blanks with the correct numbers. Then, solve the equation. If there is more than one solution, separate them with commas.

<table>
<thead>
<tr>
<th>Form:</th>
<th>Solution:</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x + \square)^2 = \square)</td>
<td>(x = \square)</td>
</tr>
<tr>
<td>((x - \square)^2 = \square)</td>
<td></td>
</tr>
</tbody>
</table>

126. Use the quadratic formula to solve for \(x\).

\[ 2x^2 + 7x - 6 = 0 \]

127. Use the quadratic formula to solve for \(x\).

\[ 5x^2 - 3x = 3 \]

Round your answer to the nearest hundredth.

128. Find all complex solutions of \(x^2 + 3x + 4 = 0\).
129. Compute the value of the discriminant and give the number of real solutions of the quadratic equation.

\[ 7x^2 - 3x + 2 = 0 \]

Discriminant:
Number of real solutions:

130. A model rocket is launched with an initial upward velocity of 201 \( \text{ft/s} \). The rocket's height \( h \) (in feet) after \( t \) seconds is given by the following.

\[ h = 201t - 16t^2 \]

Find all values of \( t \) for which the rocket's height is 93 feet.

Round your answer(s) to the nearest hundredth.
(If there is more than one solution, use the "or" button.)

131. Solve.

\[ y^4 - 13y^2 = -36 \]

If there is more than one solution, separate them with commas.
132. Find all real number solutions.

\[ x - 3x^2 - 4 = 0 \]

133. Simplify.

\[ \frac{4}{\nu + 6} \]

\[ = \frac{12\nu}{\nu^2 + 12\nu + 36} \]

134. Simplify.

\[ \frac{3 - \frac{5}{2\nu}}{7 - \frac{5}{2\nu}} \]

135. Simplify.

\[ \frac{1 - \frac{3}{x + 6}}{\frac{9}{x + 6} + x} \]
136. Simplify.

\[ \frac{1}{u^2 + v^{-1}} \]

Write your answer using only positive exponents.

137. Graph the solution to the following inequality on the number line.

\[ (x + 3)(x - 7) > 0 \]


138. Graph the solution to the following inequality on the number line.

\[ x^2 - 6x < -8 \]


139. Solve the following inequality.

\[ \frac{x - 1}{x + 5} \leq 0 \]

Write your answer using interval notation.

140. Solve the following inequality.

\[ \frac{x + 7}{3 - x} \geq 1 \]

Write your answer as an interval or union of intervals.