Solving a distance, rate, time problem using a rational equation

A plane has a cruising speed of 200 miles per hour when there is no wind. At this speed, the plane flew 500 miles with the wind in the same amount of time it flew 300 miles against the wind. Find the speed of the wind.

\[
\frac{d}{\text{hr}} = \begin{cases} 
200 & \text{mile} \\
\text{hr} 
\end{cases}
\]

\[
\frac{\text{d}}{\text{t}_1} = \frac{500}{200 + w}
\]

\[
\frac{\text{d}}{\text{t}_2} = \frac{300}{100 - w}
\]

\[
\frac{500}{200 + w} = \frac{300}{100 - w}
\]

\[
500(200-w) = 300(200+w)
\]

\[
100000-500w=60000+300w
\]

\[
-60000+500w -60000+500w
\]

\[
40000 = 800w
\]

\[
400/8 = 50 = w
\]

Simplify each radical expression as much as possible.

Assume that the variables represent any real numbers. Use the absolute value button only when necessary.

(a) \(\sqrt{z^2} = \boxed{z}\)

(b) \(\sqrt[6]{4y + 3} = \boxed{|4y + 3|}\)

\[\sqrt[3]{-1} = -1\], \[\sqrt[4]{-1} = \text{complex}\]

\[\sqrt{-41}, \sqrt{-1}, \sqrt{4}, 2i\]
\[-\frac{13 + 10x^3 + 3x^4 - 19x}{-x^2 - x + 3} : \]

\[-x^2 - x + 3 \quad \frac{-3x^2 - 7x - 2}{\quad \frac{7}{x^2 + 3} \quad}

\[-3x^4 + 10x^3 + 0x^2 - 19x - 13

- (3x^4 + 3x^3 - 9x^2)

\[= \frac{7x^3 + 9x^2}{(7x^3 + 7x^2 - 21x)} \quad \]

\[-(7x^3 + 7x^2 - 21x)\]

\[\frac{2x^2 - 7x - 13}{(2x^2 + 2x - 6)}\]

\[-7\]

\[x^2 + 11 = 13^2\]

\[x^2 = 169 - 121\]

\[x^2 = 48\]

\[x = \sqrt{48}\]

\[= 4\sqrt{3}\]

\[= 6.92\ldots\]
Factoring to solve an equation

Ex: \( x + 2 = 0 \) \( x = -2 \)
Ex: \( 4x = 0 \) \( x = 0 \)
Ex. \((x + 1)(x - 2) = 0 \) \( x = -1,2 \)
Ex: \( x^2 + 5x + 6 = 0 \)
factor first
\((x + 3)(x + 2) = 0 \) \( x = -2,-3 \)
Ex: \((x - 1)^2 + 4 = 5 \)
\[ \sqrt{(x - 1)^2} = \pm 1 \]

\[ x - 1 = \pm 1 \]

\[ |x - 1| = 1 \]
\[ x = 1 \pm 1 \]

or
\[ x^2 - 2x + 1 + 4 = 5 \]
\[ x^2 - 2x = 0 \]
\[ x(x - 2) = 0 \] \( x = 0,2 \)

Completing the Square

\((x - 1)^2 \)
\[ x^2 - 2x + 1 \]

\((x - 5)^2 \)
\[ x^2 - 10x + 25 \]
\[ \frac{25}{2} \]
\[ (x - 5)^2 \]

ex: \( x^2 + 20x + 100 = 144 + 100 \)
\[ \frac{20}{2} \]
\[ (x + 10)^2 = 244 \]
\[ x + 10 = \pm \sqrt{244} = \pm 2\sqrt{61} \]
\[ x = -10 \pm 2\sqrt{61} \]
QUADRATIC FORMULA

Ax^2 + Bx + C = 0

Note: we used completing the square to find the formula

\[ x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \]

\[ x^2 + 20x = 144 \]
\[ x^2 + 20x - 144 = 0 \]

\[ x = \frac{-20 \pm \sqrt{20^2 - 4(1)(-144)}}{2(1)} \]
\[ A = 1 \]
\[ B = 20 \]
\[ C = -144 \]

Finding the roots of a quadratic equation with leading coefficient greater than 1

Solve for \( w \).

\[ 4w^2 - 6 = -5w \]

\[ -2, \frac{3}{4} \]

If there is more than one solution, separate them with commas. If there is no solution, click on "No solution."

We first rewrite the quadratic equation as follows.

\[ 4w^2 + 5w - 6 = 0 \]

Then, we factor the left-hand side.

\[ (4w - 3)(w + 2) = 0 \]

\[ 4w^2 + 8w - 3w - 6 \]
\[ 4w(w + 2) - 3(w + 2) \]
\[ (w + 2)(4w - 3) = 0 \]
Solving an equation of the form \( x^2 = a \) using the square root property

Solve \( \sqrt{v^2} = 49 \), where \( v \) is a real number. Simplify your answer as much as possible.

If there is more than one solution, separate them with commas. If there is no solution, click on "No solution".

\[
\sqrt{v^2} = \pm 49
\]

\[
\sqrt{v} = \pm 7
\]

\[
(v - 7)(v + 7) = 0
\]

\[
v = 7, -7
\]

\[
a = 2
\]
\[
b = 1
\]
\[
c = 6
\]

We use these values in the quadratic formula.

\[
x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2}
\]

\[
= \frac{-1 \pm \sqrt{-47}}{4}
\]

Note that the discriminant, \( b^2 - 4ac = -47 \), is negative. So, our solutions are not real numbers. We get the following.

\[
x = \frac{-1 \pm \sqrt{47}i}{4}
\]

Using the fact that \( \sqrt{-1} = i \)

\[
x = \frac{-1 + \sqrt{47}}{4} - i, \frac{-1 - \sqrt{47}}{4} - i
\]

Here is the answer.
Solving a quadratic equation by completing the square: Exact answers

Solve the quadratic equation by completing the square.

\[ x^2 + 4x - 6 = 0 \]

First, choose the appropriate form and fill in the blanks with the correct numbers.

Then, solve the equation. If there is more than one solution, separate them with commas.

\[ \frac{4}{1} = \frac{10}{1} \]

\[ (x + 2)^2 = 10 \]

\[ x + 2 = \pm \sqrt{10} \]

Discriminant of a quadratic equation

Compute the value of the discriminant and give the number of real solutions of the quadratic equation.

\[ -8x^2 - x - 2 = 0 \]

Background:

The solutions of the quadratic equation \[ ax^2 + bx + c = 0 \] are given by the quadratic formula.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

The expression under the square root, \[ b^2 - 4ac \], is the discriminant of the quadratic equation.

The value of the discriminant determines the number of real solutions of \[ ax^2 + bx + c = 0 \].

- If \[ b^2 - 4ac > 0 \], there are 2 real solutions to \[ ax^2 + bx + c = 0 \].
- If \[ b^2 - 4ac = 0 \], there is 1 real solution to \[ ax^2 + bx + c = 0 \].
- If \[ b^2 - 4ac < 0 \], there are no real solutions to \[ ax^2 + bx + c = 0 \].

\[ x^2 - 2x + 1 = 0 \]

\[ D = 4 - 4(1)(1) = 0 \]

\[ (x - 1)(x - 1) = 0 \]
Happy List
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Solving an equation that can be written in quadratic form: Problem type 1

Solve,

\((y + 2)^2 - 10(y + 2) = -9\)

If there is more than one solution, separate them with commas.

\[u^2 - 10u = -9\]

\[u^2 - 10u + 9 = 0\]

\[(u - 9)(u - 1) = 0\]

\((u - 9)(u - 1) = 0\)

\[u = 9, 1\]

\[y + 2 = 9 \quad y + 2 = 1\]

\[y = 7 \quad y = -1\]