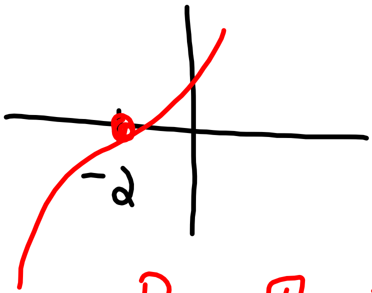
# Multiplicity

$y = (x+2)^5$

Zero: $-2$ mult. of $\underline{\underline{5}}$

Odd



Pass Thru

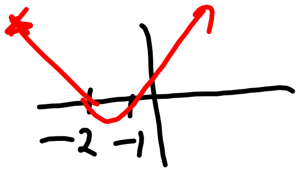
## Fundamental Theorem of Algebra

Polynomial of Degree "n"
Then there are exactly "n"
zeros & factors.

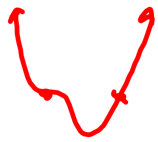
— They can be repeated or complex.

PASS PASS

$y = (x+1)^{1}(x+2)^{3}$




Poly of Degree: $\underline{4}$ Has



zeros: $-1, -2, -2, -2$

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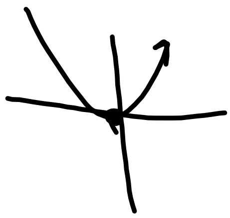
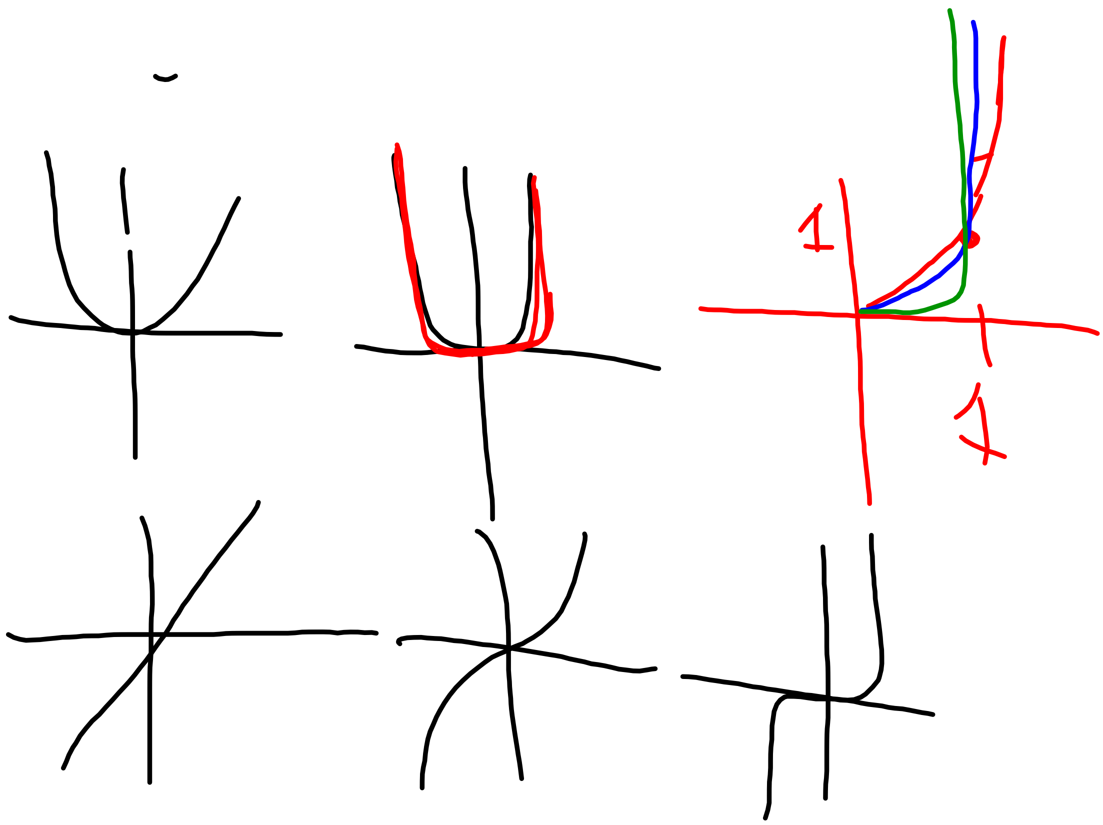
$y = x^2$

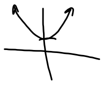


Poly of Degree: 2

zeros: 0, 0



Ex $y = x^2+4$

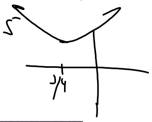


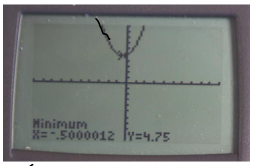
zeros: $2i$ $-2i$

$0 = x^2+4$ $\quad x^2 = -4$

$x = \pm\sqrt{-4} = \pm 2i$

Ex $y = x^2+x+5$






Complex Zeros

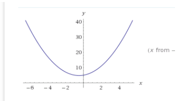
$0 = x^2+x+5$

$x = \dfrac{-1 \pm \sqrt{1-4(5)}}{2}$

$x = \dfrac{-1 \pm \sqrt{19}\,i}{2}$

$y = \left(x - \dfrac{-1+\sqrt{19}\,i}{2}\right)\left(x - \dfrac{-1-\sqrt{19}\,i}{2}\right)$ $\quad y = (x-2)(x+3)$

zeros: $2, -3$




Alternate form:

$-x^2 - x + y - 5 = 0$

Complex roots:

$x = -\frac{1}{2} i(\sqrt{19} - i)$ $\quad -\frac{1}{2}\sqrt{19}\,i \quad -\frac{1}{2}(-i^2) = \dfrac{-1-\sqrt{19}\,i}{2}$

$x = \frac{1}{2} i(\sqrt{19} + i)$

## Rational Functions

| n \ a | Lead + | Lead − |
|---|---|---|
| odd | Disco Right | Disco Left |
| even | Happy ◡ | Sad ⌒ |

$$r(x) = \frac{P(x)}{q(x)}$$

Polynomials

| Degree | Leading | Zeros |
|---|---|---|
| DN < Deg Num | LN = Numerator | ZN = Num |
| DD = Denom | LD = Denominator | ZD = Zeros of Denom. |

Ex $y = \dfrac{3(x+1)(x-3)^2}{7(x-4)^3(x+5)^5}$

DN = 3 $\quad$ LN: 3 $\quad$ ZN: $-1, 3, 3$

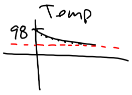
DD: 8 $\quad$ LD: 7 $\quad$ ZD: $4, -5$

## END Behavior:

Sales

Horizontal Asymptote
$y=0$

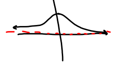
Temp

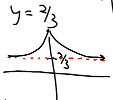




$y = 72°$

| DD > DN | DD = DN | DD < DN |
|---|---|---|
| HA: $y=0$ | HA: $y = \frac{LN}{LD}$ | HA: NONE; SLANT ASYMPTOTE |
| Ex $y = \frac{1}{x^2+1}$ | Ex $y = \frac{2x^2+4}{3x^2}$ | Ex $y = \frac{x^3}{1}$ |
| DN: 0, DD: 2, HA: $y=0$ | DN: 2, DD: 2, LN: 2, LD: 3, $y = 2/3$ | END Behavior Like Poly |
| $y_1 = 1/(x^2+1)$ | $y_1 = (2x^2+4)/(3x^2)$ | |







LN/LD

DN−DD

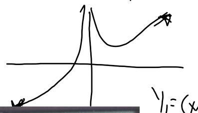
| DN−DD | Lead (+) | Lead (−) |
|---|---|---|
| odd | Disco Right | Disco Left |
| even | Happy ◡ | Sad ⌒ |

$DN - DD = 5 - 2 = 3$ (odd)

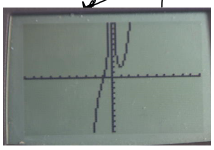
$\frac{x^5+1}{x^2}$   $x^2 \overline{)\,x^5}$   $x^3 + \ldots$

$LN/LD = 1/1 = 1 = (+)$

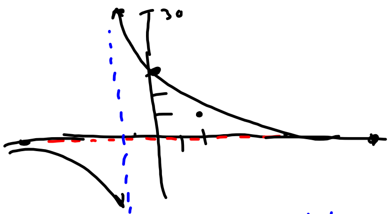
END Behavior $y = \frac{x^5+1}{x^2}$ is Disco Right



$Y_1 = (x \wedge 5 + 1)/x^2$



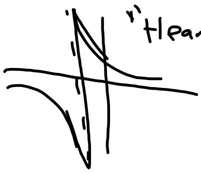
Ex $y = \frac{3}{x+1}$





At $x = -1$ ZD Vertical Asyptote.

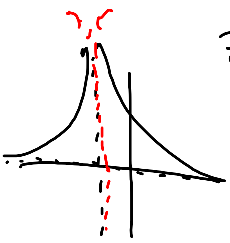
mostly All ZD X ⇒ Vertical Asymptotes



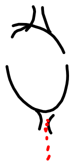
"Heartbeat"

ZD repeated odd number of times

$y = \frac{3}{(x+1)^2}$ "Volcano"



ZD even multiplicity



END Behavior (STARTS & END) $y=$

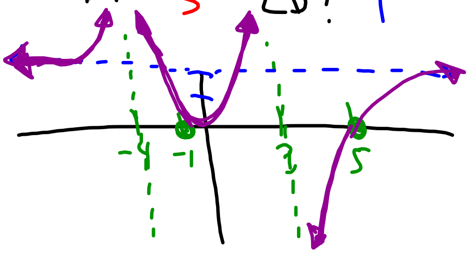
Zeros: ZN $x=$ ZN

Vertical Asymptote: $x=$ ZD

Ex $y=\dfrac{2(x+1)^2(x-5)^1}{1(x-3)^1(x+4)^2}$

DN: 3 LN: 2 ZN: $-1$ (Touch), 5 (Pass)

DD: 3 LD: 1 ZD: 3 (HB), $-4$ (Vok.)




VERT. ASYM

END Behavior?

DD = DN

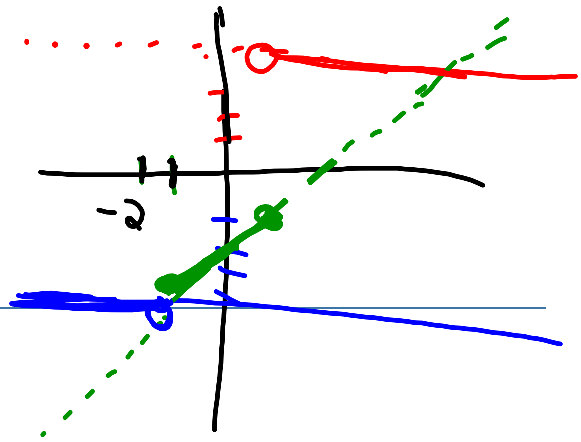
$y=$ LN/LD

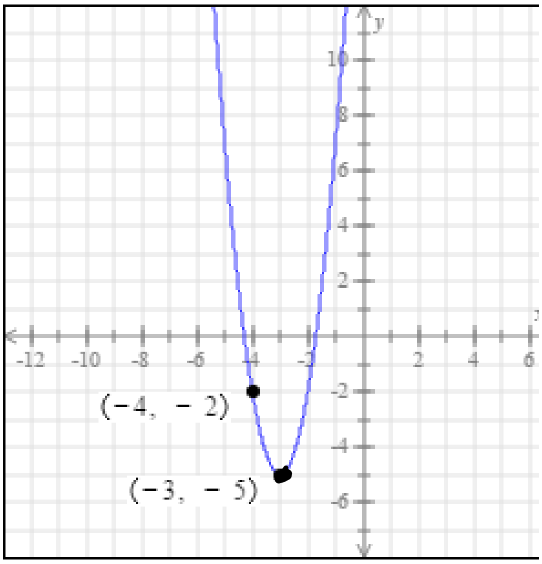
**6. Graphing a piecewise-defined function: Problem type 1**

Suppose that the function $g$ is defined, for all real numbers, as follows.

$$g(x)=\begin{cases} y=-4 & \text{if } x<-2 \\ y=x-2 & \text{if } -2\le x\le 1 \\ y=4 & \text{if } x>1 \end{cases}$$

Graph the function $g$.






$$y = a\underbrace{(x-h)}_{0}^2 + k$$

$x = -3 \quad y = -5$

$$y = a(x+3)^2 - 5$$

$$-2 = a(-4+3)^2 - 5$$

$$-2 = a - 5$$

$$a = 3$$