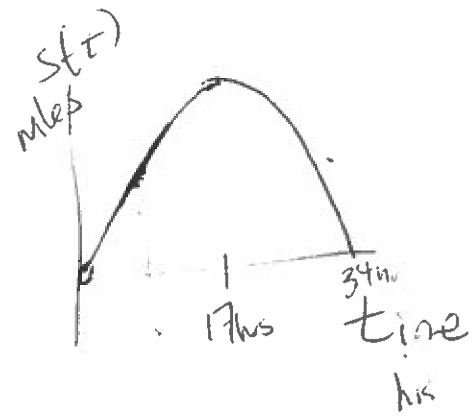
Three Topics

1. Initial Value Problems
2. Integration by substitution
3. Definite Integral w/ substitution

$s(t)$ — position



$v(t)$ — velocity

$= s'(t)$

$a(t)$ — acceleration

$= s''(t) = v'(t)$

Ex acceleration of gravity $-9.8\,m/sec^2$

throw a ball up at $6\,m/sec = v(0)$

How fast is ball moving after 7 sec.

$a(t) = -9.8$

$v(t) = \int a(t)\,dt = -\int 9.8\,dt = -9.8t + c$

$V(t) = -9.8t + C$

$V(0) = \cancel{-9.8 \cdot 0} + C = 6$

$C = 6$

$V(t) = -9.8t + 6$

After 7 sec

$V(7) = -9.8(7) + 6$

$= -68.6 + 6$

$= -62.6$

Ex

How high is the ball if I thru the ball from 1 meter high after 3 sec?

$V(t) = -9.8t + 6$ $\quad S(0) = 1$

$S(3) = ?$

$S(t) = \int v(t)\,dt = \int -9.8t + 6\,dt$

$= -\frac{9.8t^2}{2} + 6t + C = S(t)$

$$= \quad 0 + 0 + c = S(0) = 1$$
$$c = 1$$

$$S(t) = -4.9t^2 + 6t + 1 \Rightarrow$$

$$S(3) = (-4.9)(9) + 6(3) + 1 = -25.1$$
$$-44.1 \quad +19$$

$$S(t) = -\frac{1}{2}at^2 + V_0 t + S_0$$

$$S'(t) = -at + V_0$$

$$S''(t) = -a$$

Ex

$$V(t) = 3t^2 + \sin t$$

$$V(0) = 8 \qquad \text{Find } S(t)$$

$$S(t) = \int V(t)\,dt = \int 3t^2 + \sin t \, dt$$

$$= t^3 - \cos t + c$$

$$8 = S(0) = 0 - 1 + c \qquad c = 9$$

Ex $a(t) = 3 \quad V(0) = 7 \quad S(1) = 8$

Find $S(2) = ?$

$$V(t) = \int 3\,dt = 3t + C_1$$

$$V(0) = 0 + C_1 = 7$$

$$V(t) = 3t + 7$$

$$S(t) = \int 3t + 7\,dt = \frac{3t^2}{2} + 7t + C_2$$

$$S(1) = \frac{3}{2} + 7 + C = 8$$

$$C = -1/2$$

$$S(t) = \frac{3}{2}t^2 + 7t - \frac{1}{2}$$

$$S(2) = 6 + 14 - 1/2 = 19.5$$

$$\frac{d}{dx} e^{x^2} = e^{x^2} \cdot (2x)$$

$\rightarrow$ Ex $\quad \int e^{x^2} (2x)\,dx$

substitute $\quad u = x^2$

$du = 2x\,dx$

$$\int e^u \, du$$

$$e^u + C$$

$$= e^{x^2} + C_1$$

Ans

Ex

$$\int \sin(2x)\,dx$$

Substitution $u = 2x$

$$du = 2\,dx$$

$$\frac{du}{2} = dx$$

$$\int \sin(u) \cdot \frac{du}{2} = \frac{1}{2}\int \sin u\,du$$

$$= -\frac{1}{2}\cos u + C$$

Ans $-\frac{1}{2}\cos(2x) + C$

Ex

$$\int \frac{\sin x}{\cos x}\,dx$$

Substitution (look at denominators)

$u = \cos x$

$du = -\sin x\,dx$

$$\int \frac{1}{u} -du$$

$$-\int \frac{1}{u}\,du = -\ln|u| + C$$

$$-\ln|\cos x| + C$$

Substitution

$$\int x(x+1)^{1/2}\,dx$$

Sub $u = x+1 \rightarrow x = u-1$

$du = dx$

$$\int (u-1)u^{1/2}\,du$$

$$\int u^{3/2} - u^{1/2}\,du$$

$$\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C$$

$$\frac{2(x+1)^{5/2}}{5} - \frac{2(x+1)^{3/2}}{3} + C$$

$$\Delta x = \frac{8}{n}$$

$$\sum \left(41 + \frac{4}{n}i + 48.5\right)\frac{8}{n}$$

$$\sum_{i}^{n} \frac{89.5 \cdot 8}{n} + \frac{32}{n^2} i$$

$$89.5 \cdot 8 + \frac{32}{n^2} \cdot \sum i$$

$$\frac{32}{n^2} \cdot \frac{(n)(n+1)}{2}$$

$$89.5 \cdot 8 + 16\left(\frac{n^2+n}{n^2}\right)$$

$$\lim_{n \to \infty} 89.5 \cdot 8 + 16$$