

# Related Rates

151 h.  
d12

Rates "per"  $\div$   
miles per hour  $\frac{\text{miles}}{\text{hrs.}}$

$\frac{dy}{dt}$

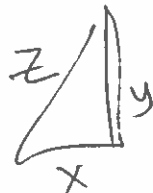
values miles, hrs., degrees

$$\frac{d}{dt} x^2 = 2x \cdot \frac{dx}{dt}$$

Ex:  $x^2 + y^2 = z^2$

$$\frac{d}{dt} x^2 + y^2$$

$$\frac{d}{dt} z^2$$

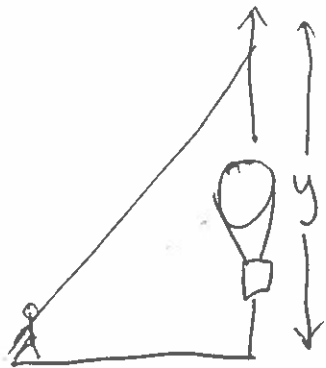


$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

3 rates  
 $\frac{dx}{dt}$   $\frac{dy}{dt}$   $\frac{dz}{dt}$

3 values  
 $x, y, z$

# Balloon Problems



Balloon rising at  $10'$  per sec =  $\frac{ft}{sec}$   
 $\frac{dy}{dt} = 10'$

$x = 25'$

$\frac{dx}{dt} = 0$

$x^2 + y^2 = z^2$

**STAY**

Walk Away  $2'/sec$

$\frac{dx}{dt} = 2$

~~Walk Away~~

$\frac{dz}{dt} = ?$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$

$= 0$

Not walking

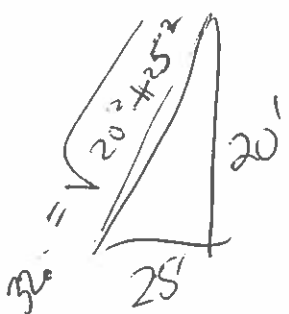
Walk Towards  $2'/sec$

$\frac{dx}{dt} = -2$

At  $t = 2 sec \rightarrow y = 20'$

At  $t = 20 sec \rightarrow y = 200'$

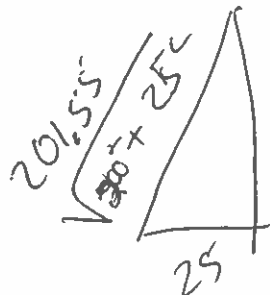
At  $2 sec$



$\frac{y}{z} \frac{dy}{dt} = \frac{20}{32.5} \cdot 10$

$\frac{dz}{dt} = 6.25' / sec$

At  $20 sec$



$\frac{y}{z} \frac{dy}{dt} = \frac{200}{201.55} \cdot 10$

$\frac{dz}{dt} = 9.92' / sec$



Infinitely large

$$\frac{dz}{dt} = \frac{1000}{25} \frac{dy}{dt}$$

Volume



$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 1000 \text{ cc per sec}$$

$$\frac{d}{dt} V = \frac{d}{dt} \left( \frac{4}{3} \pi r^3 \right)$$

At  $r = 7$

$$\frac{dr}{dt} = \frac{1000}{4\pi(7)^2}$$

$$= 1.62 \text{ cm/sec}$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

Rates

$$\frac{dV}{dt} = 1000 \text{ cc/sec}$$

Value

$$r = 7 \text{ cm}$$

$$\frac{dr}{dt} =$$

$$1000 = 4\pi(7)^2 \cdot \frac{dr}{dt}$$

At  $r = 100$

$$\frac{dr}{dt} = \frac{1000}{4\pi(100)^2}$$

$$= .0079 \text{ cm/sec}$$

# Selling Balcons

$$\text{Revenue} = P \times S(P)$$

$$R = P \cdot (1000 - P)$$

$$R = 1000P - P^2$$

Derivate  $d/dt$

$$\frac{dR}{dt} = 1000 \frac{dP}{dt} - 2P \left( \frac{dP}{dt} \right)$$

Now

$$P = \$1.$$

I want  
Raise Price  
by  $\$0.05/\text{wk.}$

$$\frac{dP}{dt} = +0.05$$

$$P = \$550$$

$$\frac{dR}{dt} = -\$5/\text{wk}$$

Rate

$$\frac{dR}{dt} = \text{Chang Revenue}$$

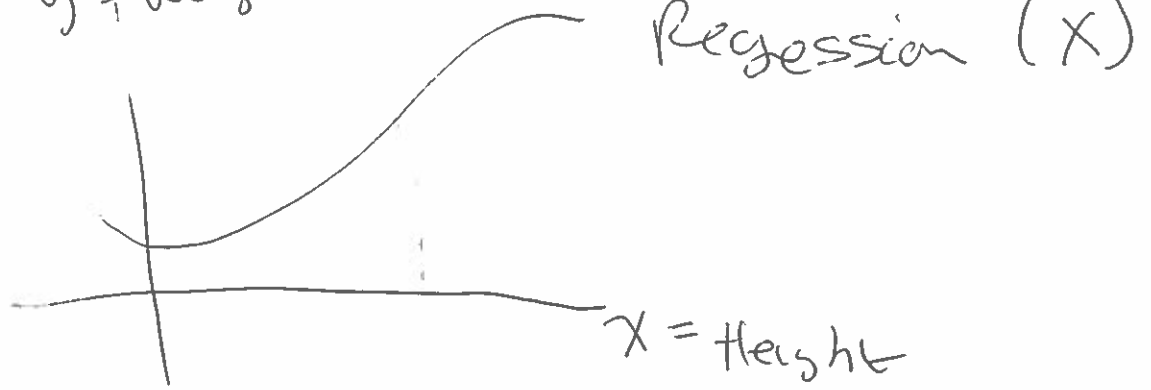
$$\frac{dP}{dt} = \text{Chang Price}$$

Value

$$P = \text{Price}$$

$$\begin{aligned} \frac{dR}{dt} &= (1000 - 2(1)) \cdot 0.05/\text{wk.} \\ &= \$49.90/\text{wk} \end{aligned}$$

$y = \text{weight}$



$$\frac{dx}{dy} = + .01''/\text{wts.}$$



$$\frac{dx}{dt} = \text{time}/\text{time?}$$

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$y_1 = \text{reg. eq.}$

$$\frac{dy}{dt} = (\text{reg. eq.})' \cdot \frac{dx}{dt}$$

\$/year \cdot \text{yrs}/\text{day}

$$y = (3x)^{8x}$$

$$\ln y = \ln 3x^{8x}$$

$$\ln y = 8x \cdot \ln 3x$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} 8x \cdot \ln(3x)$$

$$\frac{1}{y} \frac{dy}{dx} = 8x \cdot \frac{1}{3x} \cdot 3 + \ln(3x) \cdot 8$$

$$\frac{dy}{dx} = y [8 + 8 \ln(3x)]$$

$$= (3x)^{8x} [8 + 8 \ln(3x)]$$

GROUP NAME: <u>WHO PROBABILITIES</u> Date: <u>3/6</u>	Student Names (First and Last) Speaker/Presenter: <u>Jenna</u>
Independant Variable (x-axis): <u>years</u>	Writer/Prep: <u>Kathleen</u>
Dependant Variable (y-axis): <u>steroids in food in babies (ppm)</u>	Leader/Collaborator: <u>Cathryn</u>

Conclusion (in words):

The rate of change in food in babies is decreasing by 21.05 ppm per year

Supporting Work:

SIN REG

$$y_1 = a * \sin(bx + c) + d$$

$$a = 51.49869637$$

$$b = .4431990559$$

$$c = -2.666224053$$

$$d = 147.0192513$$

X	Y <sub>1</sub>	Y <sub>2</sub>
0	123.45	-20.29
10	197.54	-4.422
14	127.11	-21.05

$$74.09 \text{ ppm}$$

$$-15.868$$

$$y_2 = \text{hDeriv}(y_1, x, x)$$

L <sub>1</sub>	L <sub>2</sub>
.01	122
3	100
6	143
9	200
12	170

$$-21.05 \text{ ppm / yr}$$

GROUP NAME: International Avengers!!  
ANTI FLUFFY POINTS

Student Names (First and Last)  
 Speaker/Presenter: Ahmed & June

Date: 03/06/2014

Independent Variable (x-axis): Salary Per K.

Writer/Prep: Ahmed & June

Dependant Variable (y-axis): Crime Rate

Leader/Collaborator: Tyler.

Conclusion (in words):  
 At 70K Per Year if your salary is increasing by 2K Per Year your chance of committing a crime is increasing by 4.35% Per Year!

Supporting Work:

70	$y_1$	$y_2$
	.13575	.00435
100	<del>281198</del>	<del>-12229</del>

$$y = ax^3 + bx^2 + cx + d$$

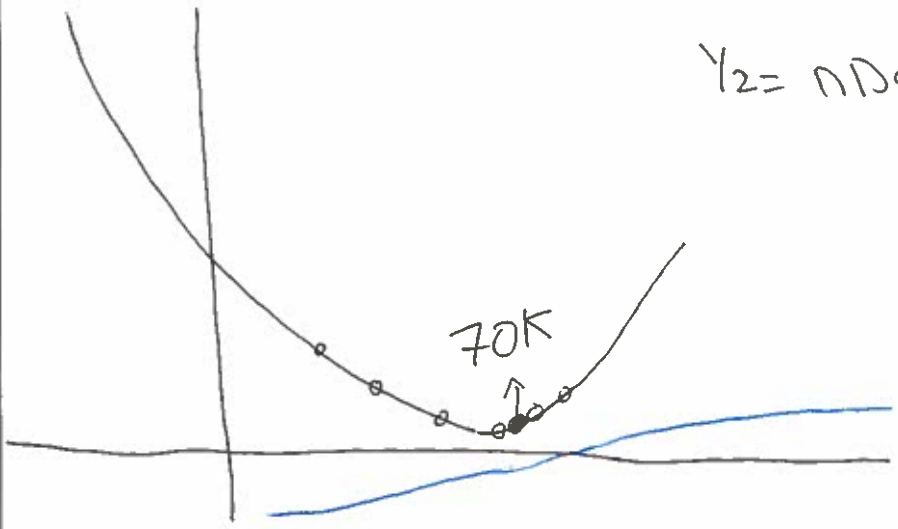
$$a = 4.1666667E^{-7}$$

$$b = 1.107 \dots E^{-4}$$

$$c = -.019 \dots$$

$$d = .812$$

$$y_2 = n \text{Deriv}(y_1, x, x) * 2$$





GROUP NAME: Fluffy Ponies

Date: 3/6/14

Student Names (First and Last)

Speaker/Presenter: Milton

Writer/Prep: Courtney

Leader/Collaborator: Tyler

Independent Variable (x-axis): Income

Dependant Variable (y-axis): crime rate

Conclusion (in words):

At \$40K / year, if your salary is increasing by \$2K per year then your chance of committing a crime ~~increasing~~ decreasing by ~~1.35%~~ 1.72% per year.

Supporting Work:

$$y_1 = 4.166 \cdot 10^{-7} x^3 + 1.107 \cdot 10^{-4} x^2 - .0195 \cdot x + .812$$

$$y_2 = \text{deriv}(y_1, x, x)$$

x	y <sub>1</sub>	y <sub>2</sub>
70	.13575	.00435
40	.23771	-.0172



GROUP NAME: Turkish Burvish?  
 Date: 3/6

Student Names (First and Last)  
 Speaker/Presenter: Jason

Independent Variable (x-axis): Years  
 Dependant Variable (y-axis): Tuition Prices

Writer/Prep: Dallen  
 Leader/Collaborator: \_\_\_\_\_

Conclusion (in words): In the year 2014 tuition is rising at \$25.452 dollars per semester

Supporting Work:

$$Y_1 = 2820.509... * 1.014...^x$$

$$Y_2 = nDeriv(Y_1, x, x)$$

x	Y <sub>1</sub>	Y <sub>2</sub>
14	3464.7	50.906

$$\frac{50.906}{2} = 25.452...$$

GROUP NAME: Porter's minions

Student Names (First and Last)

Date: mar. 16 2013

Speaker/Presenter: Kero

Independent Variable (x-axis): years

Writer/Prep: JANN

Dependant Variable (y-axis): \$ tuition

Leader/Collaborator: Daniella

Conclusion (in words):

In 2015 tuition is going down \$.02 a day

Supporting Work:

$$y_1 = -2.04 \dots x^4 + 97.91 \dots x^3 + -1733.95 \dots x^2 + 113477.08 \dots x + -35573.99 \dots$$

$$y_2 = nDeriv(y_1, x, x)$$

x	y <sub>1</sub>	y <sub>2</sub>
15	3551	-10.42

$$\frac{-10.42}{365} = \$ \text{MINOR} / \text{day}$$