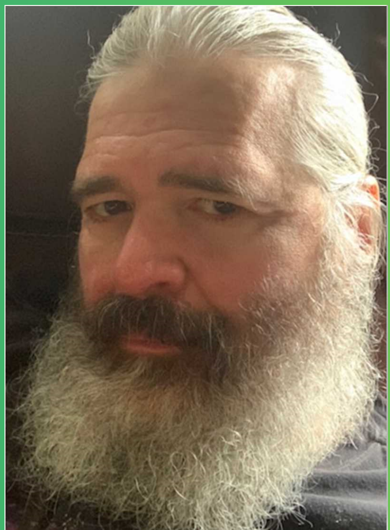
MAT 151 Calculus 1

Prof. Porter



Agenda

Review Homework

Lecture:

- Linearization
- Related Rates

Review Test

151d9

Homework 4* Implicit & Newtons

11 / 11 questions assigned | 10.0 po

Enter student instructions (optional)

add questions | org

question	questio
Sec. Ex. 1 - Tangent Lines	Multi
Sec. Ex. 7 - 2.8 Section Exercise 7	Multi
Sec. Ex. 9 - 2.8 Section Exercise 9	Multi
Example 2a - 2.8 Example 2a	Multi
Sec. Ex. 17 - Newton's Method	
Sec. Ex. 27 - Newton's Method	
Sec. Ex. 13a - 3.1 Section Exercise 13a	
Sec. Ex. 26a - 2.3 Section Exercise 26a	
Sec. Ex. 23b - Derivatives	
Sec. Ex. 19 - 2.3 Section Exercise 19	
Example 1 - 2.9 Example 1	

Question 1 (of 11) — Sec. Ex. 1 - Tangent Li

This is an algorithmic question.

Sec. Ex. 1 - Tangent Lines

3 attempts lef

Find the slope of the tangent line at the point (1, 3) for the ellipse $3x^2 + 8y^2 = 75$.

$y'(1) =$ ☐

Question 2 (of 11) — Sec. Ex. 7 - 2.8 Section Exercise 7

This is an algorithmic question. what's this? see another version

Sec. Ex. 7 - 2.8 Section Exercise 7

3 att

Find the derivative $y'(x)$ implicitly.

$\sqrt{xy} - 7y^2 = 46$

$y'(x) =$ ☐

Question 3 (of 11) — Sec. Ex. 9 - 2.8 Section Exercise 9

This is an algorithmic question. what's this? see another versio

Sec. Ex. 9 - 2.8 Section Exercise 9

3 attempts left | Check my work

Find the derivative $y'(x)$ implicitly for the equation $\frac{3x+7}{y} = 4x + y^3$.

$y'(x)=$ ☐

Question 4 (of 11) — Example 2a - 2.8 Example 2a

This is an algorithmic question. what's this? see another versio

Example 2a - 2.8 Example 2

Find the derivative $y'(x)$ imp

$x^2y^2 + 3y = 2x$

$y'(x)=$ ☐

Question 5 (of 11) — Sec. Ex. 17 - Newton's Method

This is an algorithmic question. what's this?

Sec. Ex. 17 - Newton's Method

3 attempts left | Check my

Use Newton's method to find an approximate root (accurate to six decimal places).

$$x^3 + 5x^2 - 2x + 1 = 0$$

$x =$ ☐

Question 6 (of 11) — Sec. Ex. 27 - Newton's Method

This is an algorithmic question. what's this? see another v

Sec. Ex. 27 - Newton's Method

3 attempts left | Check my work

Round your final answer to nine decimal places, if necessary.

Use Newton's method to approximate $\sqrt[3]{12}$.

$\sqrt[3]{12} \approx$ ☐

Question 7 (of 11) — Sec. Ex. 13a - 3.1 Section Exercise 13a

This is an algorithmic question. what's this? see another version

Sec. Ex. 13a - 3.1 Section Exercise 13a

3 attempts left | Check my work

Give your final answers as reduced improper fractions.

Use Newton's method with the given x_0 to compute x_1 and x_2 by hand.

$x^3 + 3x^2 - 5 = 0, x_0 = 1$

$x_1 =$ ☐ and $x_2 =$ ☐

Question 8 (of 11) — Sec. Ex. 26a - 2.3 Section Exercise 26a

This is an algorithmic question. what's this? see another version

Sec. Ex. 26a - 2.3 Section Exercise 26a

3 attempts left | Check my work

The given function represents the height of an object.

$h(t) = 10t^2 - 28t, t_0 = 9$

Compute the velocity and acceleration at time $t = t_0$.

At $t_0 = 9$, the velocity is ☐; the acceleration is ☐.

Question 9 (of 11) — Sec. Ex. 23b - Derivatives

This is an algorithmic question. what's this? see another versi

Sec. Ex. 23b - Derivatives

3 attempts left | Check my work

Use the given position function to find the acceleration function.

$s(t) = 9\sqrt{t} + 38t^2$

$a(t)=$ ☐

Question 10 (of 11) — Sec. Ex. 19 - 2.3 Section Exercise 19

This is an algorithmic question. what's this? see another versio

Sec. Ex. 19 - 2.3 Section Exercise 19

3 attempts left | Check my work

If $f(x) = 8x^4 + 7x^2 - 5$, compute $f^4(x)$.

$f^4(x)=$ ☐

Question 11 (of 11) — Example 1 - 2.9 Example 1

This is an algorithmic question. what's this? see another version

Example 1 - 2.9 Example 1

3 attempts left | Check my work

Compute the derivative of $f(x) = \sinh^2(4x)$.

$f'(x)=$ ☐

Find the derivative $y\,'(x)$ implicitly.

$$\sqrt{xy} - 6y^2 = 87$$

Find the derivative $y\,'(x)$ implicitly.

$\sqrt{xy} - 6y^2 = 87$

$$\frac{d}{dx}\left[(xy)^{1/2} - 6y^2 = 87\right]$$

$$\frac{1}{2}(xy)^{-1/2} \cdot \frac{d}{dx}(xy) - 12yy' = 0$$

$$2\sqrt{xy}\left[\frac{1}{2\sqrt{xy}} \cdot (xy' + y) - 12yy' = 0\right]$$

$$xy' + y - 24y\sqrt{xy}\,y' = 0$$

$$(x - 24y\sqrt{xy})\,y' = -y$$

$$y' = \frac{-y}{x - 24y\sqrt{xy}}$$

Lecture: Local Linear Approximations

What is Math?

What is Precalculus?

What is Calculus?

What are the two rates of change?

How do we get from two points to one?

What is the meaning of velocity?

What applications have done with derivative?

Velocity is

the Derivative is

the instantaneous rate of change is

the slope of the tangent line

Applications: Newtons Method for solving equations

and Local Linear Approximations

Local Linear Approximation

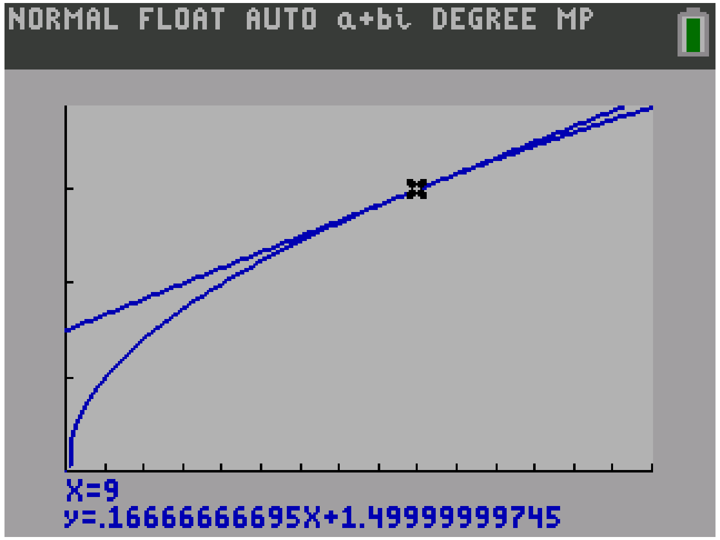
In mathematics, linearization is finding the linear approximation to a function at a given point.

Find the equation of the tangent line at a center point near where you will be working that can be evaluated in the function by hand.

Suppose you want the sqaure root of 12. You can approximate it by finding the tangent line of square root at 9 or 16 because they are perfect squares. 9 is closer to 12.

Calculator could just find square root of twelve,

so must do this by hand.



Equation of tangent line near 9.

$f(x) = x^{1/2}$ $f'(x) = (1/2)x^{-1/2}$

$f(9) = 3$ $f'(9) = (1/2)(1/3) = (1/6)$

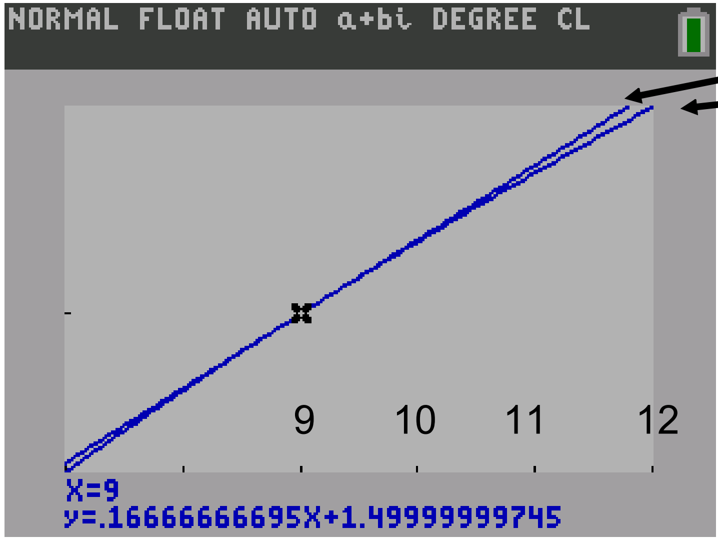
Point (9, 3) Slope is 1/6

$y - 3 = (1/6)(x - 9)$

Linearization: $y = (1/6)(x - 9) + 3$

(its the equation of a line)

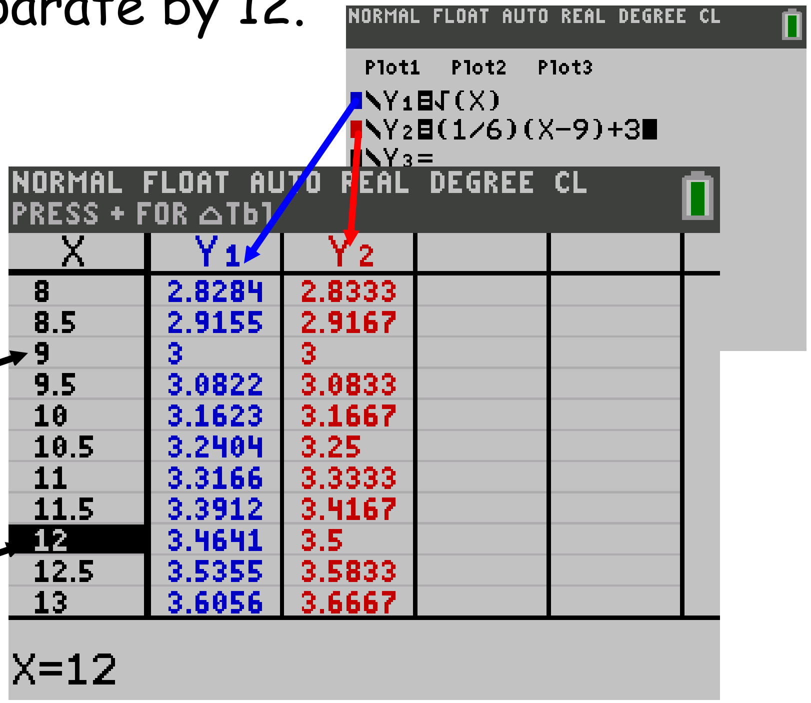
$y(12) = (1/6)(12-9) + 3 = 3.5$



Lineariation

Square root function

They are close near 9, but start to separate by 12.



X	Y1	Y2
8	2.8284	2.8333
8.5	2.9155	2.9167
9	3	3
9.5	3.0822	3.0833
10	3.1623	3.1667
10.5	3.2404	3.25
11	3.3166	3.3333
11.5	3.3912	3.4167
12	3.4641	3.5
12.5	3.5355	3.5833
13	3.6056	3.6667

Linearization Identical at x=9

Still pretty close at x = 12

For $\sqrt{(45)}$ use a center like a=49

Point (49,7) Slope = 1/14

Line: y = (1/14)(x-49)+7

For $\sqrt{(92)}$ use a center like a=100

Point (100,10) Slope = 1/20

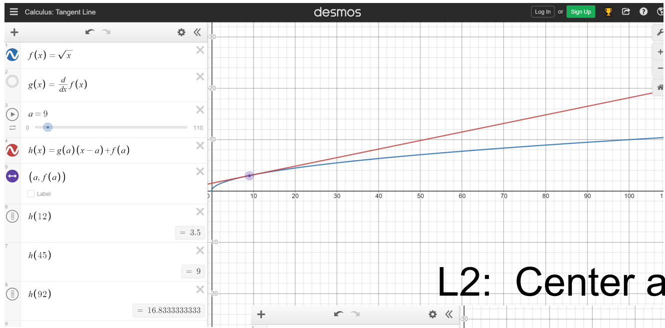
Line: y = (1/20)(x-100)+10

Note: For the (20) use a center of 27 because 27 is a perfect cube.

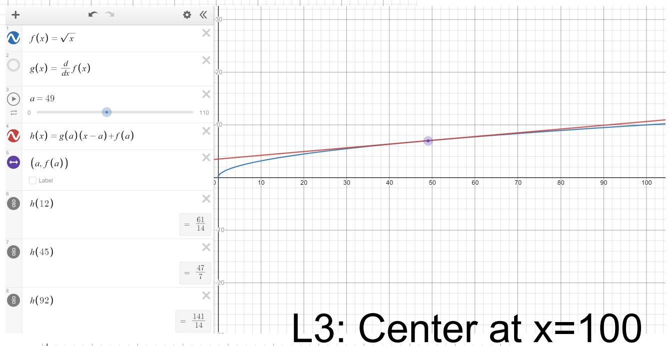
x=	f(x)	L1	L2	L3
12	3.46	3.5	4.36	5.6
45	6.71	9	6.71	7.25
92	9.59	16.9	10.1	9.6

When the center is close, the approximation is close. Otherwise....garbage

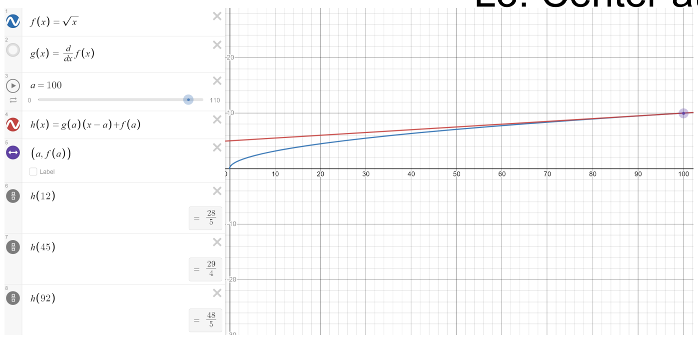
L1: Center at x=9



L2: Center at x=49



L3: Center at x=100



Where is this used?

In chemistry there is a point where you must find sin[x],

but [x] is always very small. So what is a good linear approximation for sin[x] when x is approximately zero?

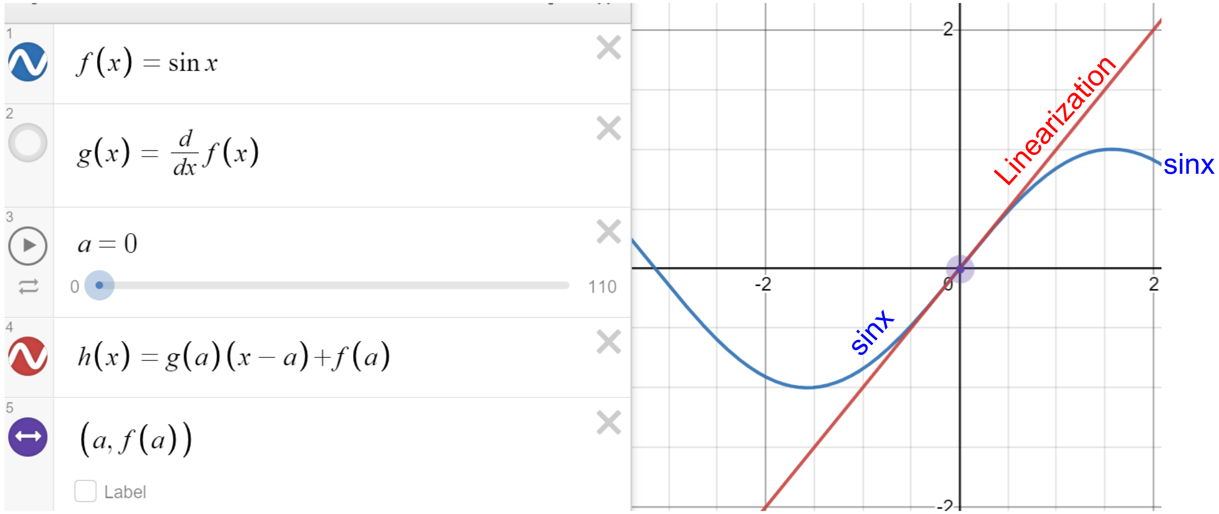
Center: $a = 0$ Point $(0, \sin 0) = (0,0)$

Slope of Tangent Line: $y' = \cos x$ $y'(0)=1$

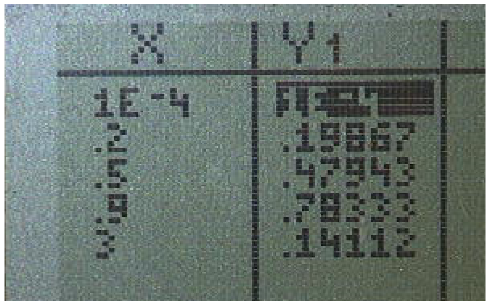
Linearization: $y - 0 = 1\,(x-0)$ or $y=x$

So they replace sin[x] with just [x]. Much easier to use!

EX: To find the sin(.001) just use .001.



$f(x) = \sin x$
$g(x) = \frac{d}{dx} f(x)$
$a = 0$
0
110
$h(x) = g(a)(x - a) + f(a)$
$(a, f(a))$
Label
2
-2
0
2
-2
Linearization
sinx
sinx

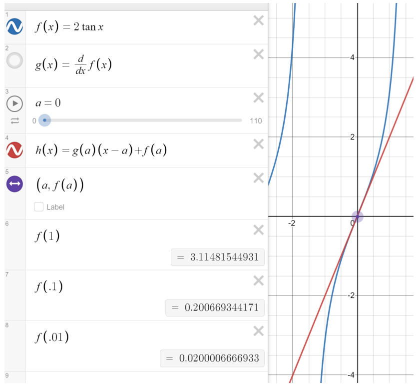


X
Y1
1E-4
.19867
.47943
.78333
.14112

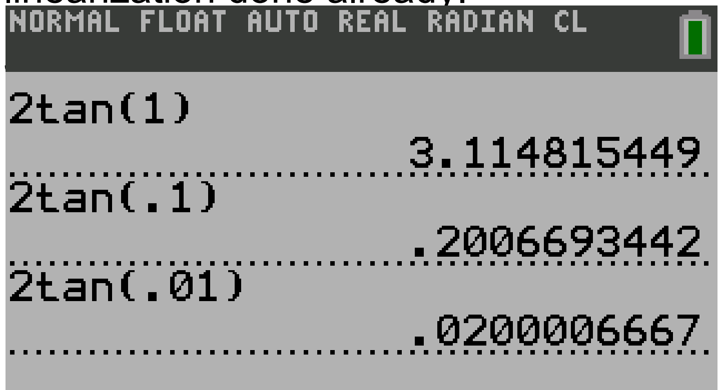
Find the linear approximation at $x = 0$ to show that the following commonly used approximations are valid for "small" x. Compare the approximate and exact values for $x = 0.01$, $x = 0.1$, and $x = 1$. Round your calculations to seven decimal places if needed.

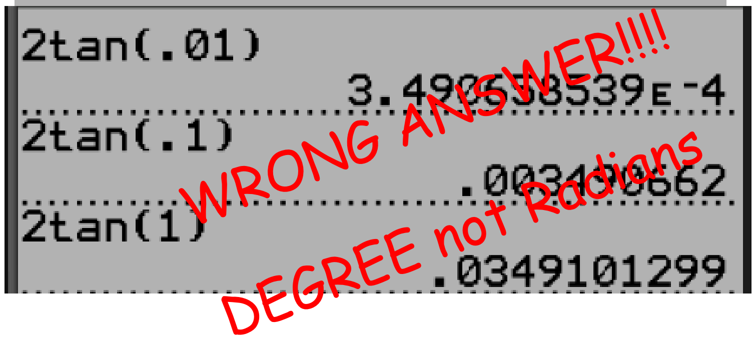
$2\tan(x) \approx 2x$

	$L(x)$	$f(x)$
$x = 0.01$	.02	
$x = 0.1$	.2	
$x = 1$	2	



linearization done already.





Round your final answer to four decimal places.

Use linear approximation to estimate the quantity $\sin\left(\frac{7}{3}\right)$.

$\sin\left(\frac{7}{3}\right) =$

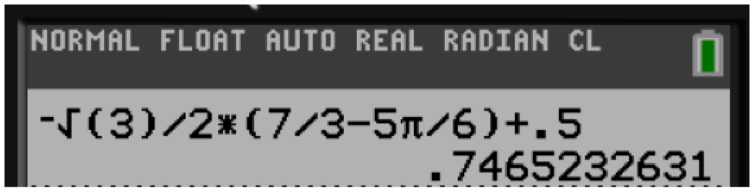
What is near 7/3 that can be evaluated exactly with sine function? sin(?) = 1/2

7/3 =2.33 near 5π/6 = 2.62

a= 5π/6 point: (5π/6, 1/2) slope: y'= cos(5π/6) = -√(3)/2

$y - 1/2 = -\sqrt{3}/2(x - 5\pi/6)$

$y(7/3) = (-\sqrt{3}/2)(7/3 - 5\pi/6) + 1/2$



NOTE: not a good exam questions. Need a calculator to answer, could just get sin(7/3)

RELATED RATES

In differential calculus, related rates problems involve finding a rate at which a quantity changes by relating that quantity to other quantities whose rates of change are known. The rate of change is usually with respect to time.

Process:

1. Find the main idea as a formula.

2. Differentiate with respect to time.

3. Identify all values and rates.

4. Solve for the unknown

The problem will contain rates that say "something per time" like: "miles per hour" or "feet per second" or "$ per day" Listen for the word "per"

These rates will be express as derivatives with respect to time like: dy/dt, df/dt, dR/dt

where t will be the time variable. Make sure units are same.

The problem will also contain values that say " time is", "distance is", "radius is", etc. These will be represnted with regular variables t, D, R, etc.

The problem will contain a main idea. It can be expressed as a regular function like "f(x) =..." or "R(p) = ..." or it can be one of the many formulas you already know like "Area=LxW", or the pyhagorean formula "$x^2 + y^2 = z^2$."

To solve thes problems, you are going to have to differentiate the entire formula with respect to time d/dt. That means likely none of your varaibles will match, and you'll have to use the chain rule.

EX: $\frac{d}{dt}(y^2) = 2y\frac{dy}{dt}$ OR $\frac{d}{dt}\text{Area} = d\,(L \times W) = L\frac{dW}{dt} + W\frac{dL}{dt}$

EX: **Suppose that the average yearly cost per item for producing x items of a business product is**

$$\overline{C}(x) = 14 + \frac{243}{x}. = 14 + 243x^{-1}$$

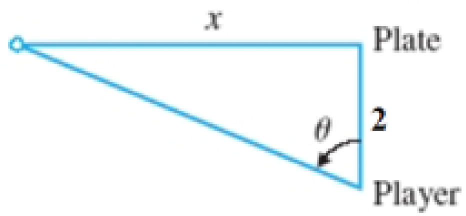
If the current production is $x = 9$ and production is increasing at a rate of 3 items per year, find the rate of change of the average cost.

Main idea given as a function, so $dC/dt = -243x^{-2}\,(dx/dt)$

$x = 9$ is given, and $dx/dt = 3$ is given too.

$dc/dt = -243\,(1/81)\,(3) = 9$ cost per year

EX: **A baseball player stands 2 meters from home plate and watches a pitch fly by. In the diagram, x is the distance from the ball to home plate and θ is the angle indicating the direction of the player's gaze. Find the rate θ' at which his eyes must move to watch a fastball with $x'(t) = -46$ m/s as it crosses home plate at $x = 0$.**



$\theta' =$ [ ] rad/s.

Main idea is from SOHCAHTOA

x is opposite, 2 is adjacent, TOA is needed

$\tan(\theta) = x/2$

$\frac{d}{dt}\tan(\theta) = \frac{d}{dt}x/2$

$\sec^2(\theta)\, d\theta/dt = (1/2)dx/dt$

Three unknown, a value and two rates

$dx/dt = -46$ When $x=0$, so does θ

$\sec^2(\theta)\, d\theta/dt = (1/2)dx/dt$

$\sec^2(0)\, d\theta/dt = (1/2)(-46)$

$\sec(0)=1$, so $d\theta/dt = -23$m/s

EX: **The frequency at which a guitar string vibrates (which determines the pitch of the note we hear) is related to the tension T to which the string is tightened, the density ρ of the string, and the effective length L of the string by the equation $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$. By running a finger along a string, a guitarist can change the distance between the bridge and their finger. Suppose that $L = 1/2$ m and $\sqrt{\frac{T}{\rho}} = 70$ m/s so that the units of f are Hertz (cycles per second).**

If the guitarist's hand slides so that $L'(t) = -1$, find $f'(t)$. At this rate, how long will it take to raise the pitch one octave (that is, double f)?

The length of time it takes to raise the pitch one octave is ☐ seconds.

Main Idea: $f(t) = 1/(2L)*70$

or $f = 35L^{-1}$

$df/dt = -35L^{-2}\, dL/dt$

$= -35(1/2)^{-2}(-1)$

Don't let this problem overwhelm you with reading.

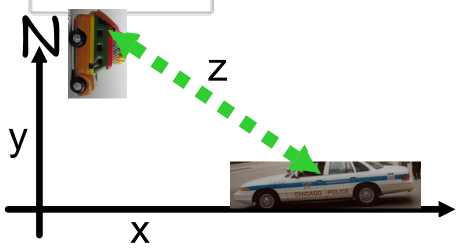
EX: A car is traveling at 65 km/h due south at a point $\frac{4}{5}$ kilometer north of an intersection. A police car is traveling at 49 km/h due west at a point $\frac{2}{5}$ kilometer due east of the same intersection. At that instant, the radar in the police car measures the rate at which the distance between the two cars is changing. What does the radar gun register?

Main Idea: $x^2 + y^2 = z^2$

$\frac{d}{dt}x^2 + \frac{d}{dt}y^2 = \frac{d}{dt}z^2$

$$\boxed{2x\,(dx/dt) + 2y\,(dy/dt) = 2z\,(dz/dt)}$$

[ ] km/h



65km/h is the rate dy/dt (negative)

4/5 km is the 'y' value

49km/h is the rate dx/dt (negative)

2/5 km is the 'x' value

$2x\,(dx/dt) + 2y\,(dy/dt) = 2z\,(dz/dt)$

5/6 variables; can find dz/dt

Use the main formula to find z:

$z = \sqrt{(2/5)^2 + (4/5)^2}$

$$z' = \frac{x\,x' + y\,y'}{z} = \frac{(-49)(2/5)+(-65)(4/5)}{\sqrt{(20/25)}} = -80.0512$$

NOTE: Give only positive answer

TEST Taking Advice: just intended to help- not required

1. Take the test in a similar environment to how you study.

2. Take the test at the same time you study

3. Practice the test at the same time and for the same time

4. Do the easy problems first, come back to the harder ones

5. Practice concentraing for two hours, its the length of final

Don't eat unusual or artificial foods before the test.

- No artificial sweeteners (sugar is OK for test)

- No artificial Salts (MSG) It is in almost everything. Avoid the ingrediaent "natural flavors." It's Sometimes MSG.

-MSG in most mexican, chinese dishes. Italian, Indian are fine, but not for the first time.

(because it's in there naturally)

Don't have to urinate during the exam.

-You may be losing mental resources without knowing it.

-You may not even know you have to "go" and give up too fast

-So don't eat melon, slurpee, soda, coffee, tea before exam

STOP

Derivatives Test NOW