Agenda:

Review of Derivative Rules

Review of Cumulative Past Work

Lecture: Postion Velocity Acceleration

Project: Do Transcendals and Higher orders

Lecture: Implicit Differentiation

Derivative Rules

Math = Language

Calculus = Change

2 Rates Ave & Instant

2 pts → 1 pt

Limits → 1 pt

Instant. = derivate

slope of Tangent Line

Velocity

Derivative

$\frac{d}{dx} c = 0$

$\frac{d}{dx} mx+b = m$

Power $\boxed{\frac{d}{dx} x^n = n x^{n-1}}$

Product $\frac{d}{dx} f \cdot g = f' \cdot g + g' f$

Quotient $\frac{d}{dx} \frac{f}{g} = \frac{g f' - f g'}{g^2}$

Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'$$

Transcendentals

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \ln x = 1/x$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} a^x = a^x \ln a \qquad \frac{d}{dx} \log_B x = \frac{1}{x \ln B}$$

Chain

$$\frac{d}{dx}(f \circ g)(x)$$

$$= \frac{d}{dx} f(g(x))$$

$$= f'(g(x)) \cdot g'(x)$$

Ex $\frac{d}{dx} \sin^{100} x$

$$\frac{d}{dx} \cdot \sin^{100} x$$

$$\frac{d}{dx} x^{100}$$

$$\frac{d}{dx} \left(\sin x \right)^{100}$$

$$100 x^{99}$$

$$100 (\sin x)^{99} \cdot \frac{d}{dx} \sin x$$

$$100 (\sin x)^{99} \cdot \cos x$$

$$100 \cdot \cos x \cdot \sin^{99} x$$

Ex

$$\frac{d}{dx}(\sin x)^{100}$$

$$100(\sin x)^{99}\cdot\cos x$$

$$100\cos x\cdot\sin^{99}x$$

Quiz Review

Sec. Ex. 3 - 2.1 Section Exercise 3

Your response:

Find an equation of the tangent line to $y = x^2 - 9x$ at $x = -8$.

$\boxed{y = -25x - 208}$ ✗

Sec. Ex. 3 - 2.1 Section Exercise 3

x y

POINT ON LINE $(-8, 136)$

Your response:

Find an equation of the tangent line to $y = x^2 - 9x$ at $x = -8$.

$\boxed{y = -25x - 208}$ ✗

$Y(-8) = 64 + 72 = 136$

SLOPE OF TANGENT LINE

$y' = 2x - 9$

$y'(-8) = 2(-8) - 9 = -16 - 9 = -25$

$y - 136 = -25(x + 8)$

$y = -25(x + 8) + 136$

Sec. Ex. 3 -

X	Y_1	Y_2
-8	136	-25

Your response:

Find an equation of the tangent line to $y = x^2 - 9x$ at $x = -8$.

$y = -25x - 208$ ✗

Point slope tan line

$y' = 2x - 9$ ← need $x = -8$

$$m_{tan} = y'(-8) = 2(-8) - 9 = -25$$

Point $(-8, y(-8)) = (-8, 64 + 72)$

$= (-8, 136)$

$$y - 136 = -25(x + 8)$$

$$y - 136 = -25x - 200$$

$$\underline{\quad +136 \qquad\qquad +136}$$

$$y = -25x - 64$$

Determine values of a and b that make the given function continuous.

$$f(x) = \begin{cases} \dfrac{33\sin x}{x} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ b\cos x & \text{if } x > 0 \end{cases}$$

a = [ ] and b = [ ]

Determine values of a and b that make the given function continuous.

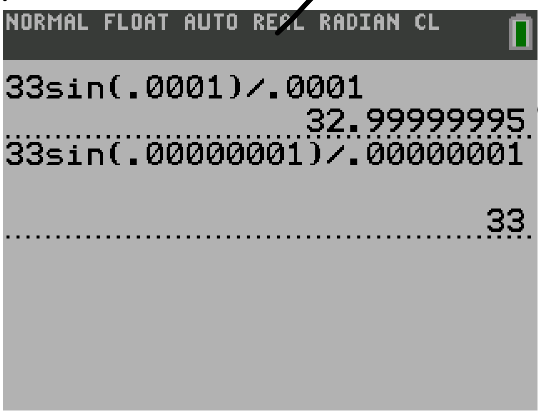
$$f(x) = \begin{cases} \dfrac{33\sin x}{x} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ b\cos x & \text{if } x > 0 \end{cases}$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$$

$$\lim_{x\to 0} \frac{33\sin x}{x} = \lim_{x\to 0} b\cos x$$

$$\| \quad 33 = a$$

$a = 33$ and $b = 33$



$$\lim_{x\to 0} b\cos x = 33$$

$$b = 33$$

Question

Suppose that the height of a falling object t seconds after being dropped from a height of 276 feet is given by $f(t) = 276 - 16t^2$ feet. Find the average velocity between times $t = 3$ and $t = 4$.

Your Answer: ______ ft/s

Question

Suppose that the height of a falling object t seconds after being dropped from a height of 276 feet is given by $f(t) = 276 - 16t^2$ feet. Find the average velocity between times $t = 3$ and $t = 4$.

Your Answer: -112 ft/s

t	f
0	276
1	260
2	212
3	132
4	20

2 Points

$\cdot f(3) = 276 - 16 \cdot 9 = 132 \leftarrow$ ft.

$\cdot f(4) = 276 - 16 \cdot 16 = 20 \leftarrow$

sec

$$\frac{\Delta y}{\Delta x} = \frac{132 - 20}{3 - 4} = \frac{ft}{sec}$$

-112

Question

Determine the limit (answer as appropriate, with a number, ∞, −∞ or does not exist).

$$\lim_{x \to -\infty} \frac{-9x}{\sqrt{16 + x^2}}$$

Question

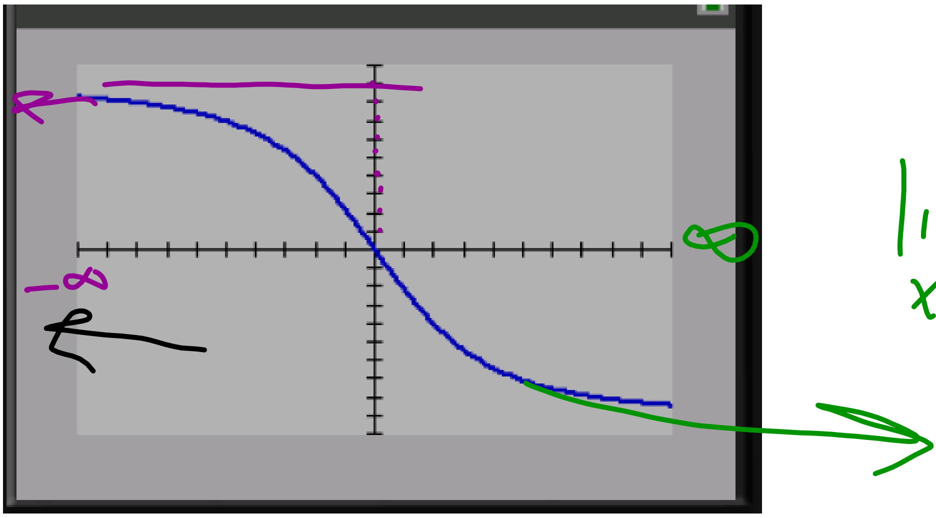
Determine the limit (answer as appropriate, with a number, ∞, −∞ or does not exist).

$$\lim_{x \to -\infty} \frac{-9x}{\sqrt{16 + x^2}} = \lim_{x \to -\infty} \frac{-9x / x}{\sqrt{\frac{16}{x^2} + 1}} = -9$$

$$\frac{16}{x^2} \to 0$$

$$\lim_{x \to -\infty} \frac{-9x}{|x|} = (-1)(-9)$$

$$= 9$$



$$\lim_{x \to \infty} f(x) = -9$$

Evaluate the indicated limit.

$$\lim_{x \to 289} \frac{x - 289}{\sqrt{x} - 17}$$

Evaluate the indicated limit.

$$\lim_{x \to 289} \frac{x - 289}{(\sqrt{x} - 17)} \cdot \frac{(\sqrt{x} + 17)}{(\sqrt{x} + 17)} =$$

$$\lim_{x \to 289} (\sqrt{x} + 17) \qquad \frac{\cancel{x - 289}}{\cancel{x - 289}}$$

$$= 17 + 17 = 34$$

Examples of chain, product,quotient rules

Find the derivative of $f(x) = -2x^2 + 3x + 2$ at $x = -5$.

$f'(-5) =$ $\boxed{}$.

Find the derivative of $f(x) = -2x^2 + 3x + 2$ at $x = -5$.

$f\,'(-5) =$ $\boxed{}$

$f'(x) = -4x + 3$

$f'(-5) = -4(-5) + 3$

$= 23$

Differentiate $y = (x^3 + x - 8)^9$.

$$\frac{dy}{dx} = \square(\square)^{\square}(\square)$$

Differentiate $y = (x^3 + x - 8)^9$.

Outside

$\frac{dy}{dx} = \boxed{9}\left(\boxed{x^3+x-8}\right)^{\boxed{8}}\left(\boxed{3x^2+1}\right)$

$\frac{d}{dx}(x^3+x)$

x^9

$\downarrow$

$9x^8$

Find $\frac{d}{dt}(\sqrt{113+14t})$.

Find $\frac{d}{dt}(\sqrt{113+14t})$.

$\frac{1}{2}$

$\frac{d}{dt}(113+14t)$

$\frac{1}{2}(113+14t)^{-1/2} \cdot \frac{d}{dt}(113+14t)$

$\frac{1}{2}(113+14t)^{-1/2} \cdot (14)$

$7/\sqrt{(113+14t)}$

Compute the derivative of $f(x) = x^3\sqrt{18x+7}$.

- A. $3x^2\sqrt{18x+7} + 9x^3(18x+7)^2$
- B. $3x^2\sqrt{18x+7} + 18x^3(18x+7)^{-1/2}$
- C. $3x^2\sqrt{18x+7} + 9x^3(18x+7)^{-1/2}$
- D. $12x^2\sqrt{18x+7}$

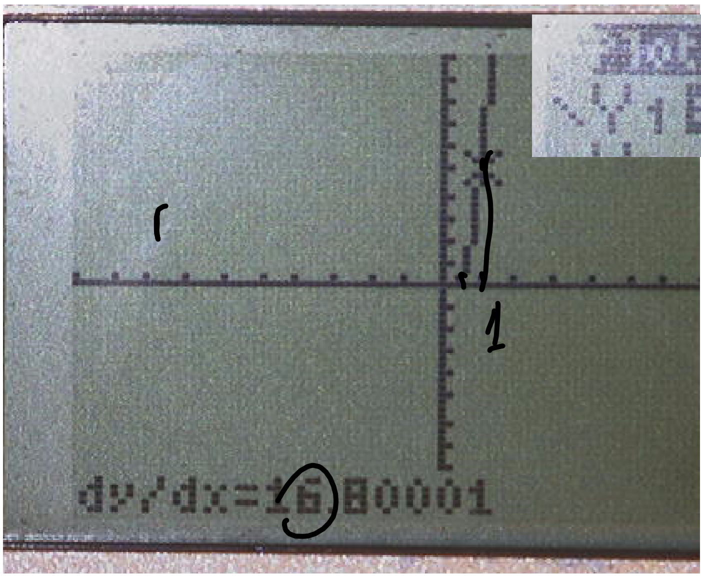
Compute the derivative of $f(x) = x^3\sqrt{18x + 7}$.

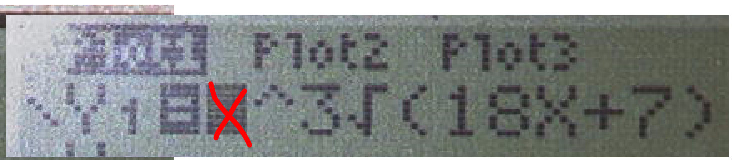
- A. $3x^2\sqrt{18x+7} + 9x^3(18x+7)^2$
- B. $3x^2\sqrt{18x+7} + 18x^3(18x+7)^{-1/2}$
- C. $3x^2\sqrt{18x+7} + 9x^3(18x+7)^{-1/2}$
- D. $12x^2\sqrt{18x+7}$

$$\sqrt{18x^7 + 7x^6}^{\,1/2} = \frac{1}{2}(18x^7 + 7x^6)^{-1/2} \cdot (126x^6 + 42x^5)$$

$$x^3 \cdot \frac{d}{dx}(18x+7)^{1/2} + (18x+7)^{1/2} \cdot \frac{d}{dx}x^3$$

$$x^3 \cdot \frac{1}{2}(18x+7)^{-1/2} \cdot 18 + 3x^2 \cdot \sqrt{18x+7}$$





zoom 6

calc: 6 dy/dx

Compute the derivative of $f(x)=\dfrac{7x}{(x^3+8)^2}$.

Compute the derivative of $f(x) = \dfrac{7x}{(x^3+8)^2}$.

Quotient Rule

$$\frac{(x^3+8)^2 \cdot 7 - 7x \cdot 6x^2(x^3+8)}{(x^3+8)^4}$$

$\frac{d}{dx}(x^3+8)^2 =$

$2(x^3+8)^1 \cdot 3x^2$

chain rule

Product Rule

$\frac{d}{dx}\, 7x \cdot (x^3+8)^{-2}$

$7x \cdot (-2)(x^3+8)^{-3} \cdot (3x^2) + (x^3+8)^{-2} \cdot 7$

$f \cdot g' + g \cdot f'$

Compute the derivative of $f(x) = \dfrac{7x}{(x^3+8)^2}$.

$(x^3+8)^2 = x^6 + 16x^3 + 64$

Product Rule

$\frac{d}{dx}\, 7x \cdot (x^3+8)^{-2}$

$7x \cdot (-2)(x^3+8)^{-3} \cdot (3x^2) + (x^3+8)^{-2} \cdot 7$

$f \cdot g' + g \cdot f'$

Lecture:Postion Velocity Acceleration

$s(x)$ $\quad$ $V(x)$ $\quad$ $a(x)$

$s(t)$ $\quad$ $v(t)$ $\quad$ $a(t)$

$s'(x)$ $\quad$ $s''(x)$

$s'(t)$ $\quad$ $s''(t)$

$v'(x)$

$v'(t)$

$0 \longrightarrow 60$ mph in 5 sec

5 sec =

~~30~~ $5/60$ min

$$\frac{60 \text{ mph}}{\frac{5}{3600} \text{ hr}}$$

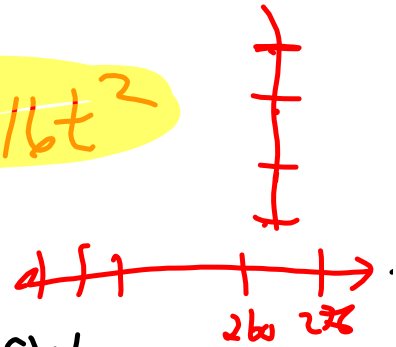
$= 43{,}200$ mph/hr $\quad \frac{5}{3600}$ hr.

m/hr^2

Postion Velocity Acceleration

$s(t) = 276 - 16t^2$

(ft)



speed + direction

$s'(t) = v(t)$ ft/sec

acceleration $= s''(t) = v'(t) = a(t)$

$\frac{ft/sec}{sec} = ft/sec^2$

Position $s(t)$

Velocity $s'(t) = V(t)$

Acceleration $s''(t) = v'(t) = a(t)$

Second derivative

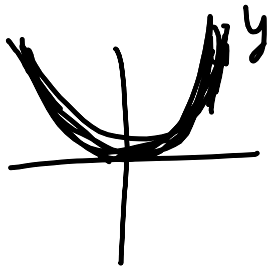
y'' $\qquad \frac{d^2}{dx^2} f$ $\qquad D_{xx} f$

$\frac{d}{dx}\frac{d}{dx} x^2$

$\frac{d}{dx} \cdot 2x$

$= 2$

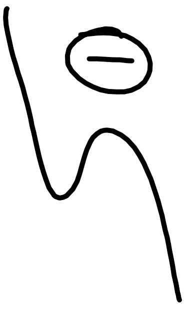
$\frac{d}{dx}\frac{d}{dx} f$



$y'' = 2$

Third derivative

y''' $\qquad \frac{d^3}{dx^3} f$ $\qquad D_{xxx} f$



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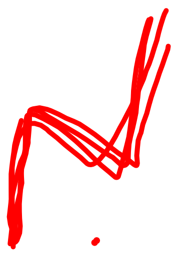
Jerk



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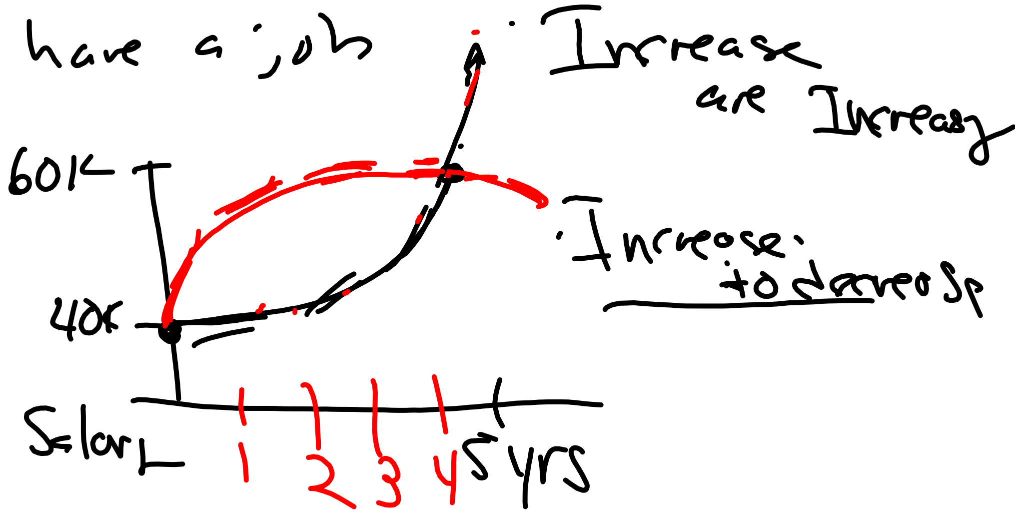
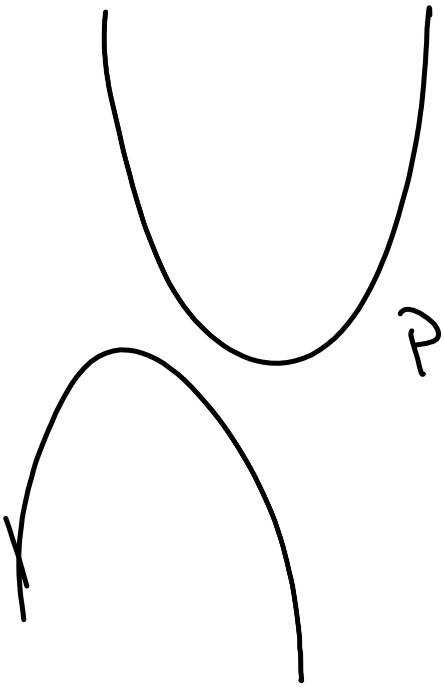
Question?

You have a job



Concave Up

Concave Down



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Examples

1 out of

Use the position function $s(t) = \dfrac{86t}{\sqrt{t^2+3}}$ to find the velocity at time $t = 2$.

(Assume units of meters and seconds.)

Ex

1 out of

Use the position function $s(t) = \frac{86t}{\sqrt{t^2+3}}$ to find the velocity at time $t = 2$.

(Assume units of meters and seconds.)

$$y = 86t \cdot (t^2+3)^{-1/2}$$

Product

$$86t \cdot -\frac{1}{2}(t^2+3)^{-3/2} \cdot \frac{d}{dt}(t^2+3) + (t^2+3)^{-1/2} \cdot \frac{d}{dt} 86t$$

$$86t \cdot -\frac{1}{2}(t^2+3)^{-3/2} \cdot (2t) + (t^2+3)^{-1/2} \cdot 86$$

1 out of

Use the position function $s(t) = \frac{86t}{\sqrt{t^2+3}}$ to find the velocity at time $t = 2$.

(Assume units of meters and seconds.)

chain

$$\frac{\sqrt{t^2+3}\cdot\frac{d}{dx}86t - 86t\frac{d}{dx}\sqrt{t^2+3}}{\sqrt{t^2+3}}$$

$S(t) =$ position

$V(t) = S'(t) =$ Velocity (Instant)

$V'(t) = S''(t) =$ acceleration

Second derivation

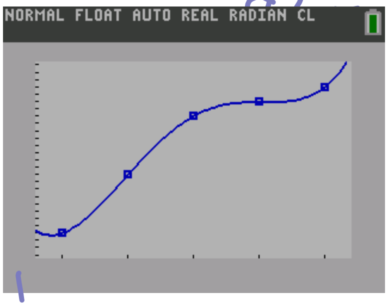
Groupwork

Higher order derivatives

y_1 = quartic regression

y_2 = derivative of y_1 ← third

y_3 = derivative of y_2 ←



NORMAL FLOAT AUTO REAL RADIAN CL

Plot1 Plot2 Plot3

\Y1=.49999999999871X^4+-5.9999999999845X^3+23.499999999936X^2+-27.999999999896X+49.999999999945

\Y2=4*.49999999999871X^3+3*-5.9999999999845X^2+2*23.499999999936X^1+-27.999999999896

\Y3=nDeriv(Y2,X,X)

NORMAL FLOAT AUTO REAL RADIAN CL

X	Y1	Y2	Y3
5	60	7	17

X=

$$\boxed{\begin{aligned} f(5) &= 60 \\ f'(5) &= 7 \\ f''(5) &= 17 \end{aligned}}$$

According to quartic regress, at 5th year salary is $60K and its increasing by $7K per year and its accelerating ~~by $17K per yr per yr.~~

Project:

Complete derivative of Transcendentals

$y = a\cancel{b}\, b^x \qquad y' = a \cdot b^x \ln b$

$y = a + b\ln x \qquad y' = b/x$

$y = a\sin(bx+c) + d \qquad y' = a\cos(bx+c)b$

$a \cos(bx+c) \cancel{+d}$

$*(1, 2, 3 \ldots$

$Y_1 = \text{regEq}$

$Y_2 = \text{nderiv}(Y_1, X, X)$

$Y_3 = \text{nder}(Y_2, X, X)$

or

$Y_3 = \text{nderiv}(\text{nderiv}(Y_1, X, X), X, X)$

Picture

X	Y_1	Y_2	Y_3	
20	103.2	1.5271	-2.047	

Math

$f(20) = 103.2$

$f'(20) = 1.52\ldots$

$f''(20) = -2.04\ldots$

Words

According to sine regression

In 2020,

Profits are increasing by 1.52 \$ per year

And decellerating

Implicit Differentiation

Implicit Differentiation

Implied (suggest)
needs will roll.

Explicit $\rightarrow$ cut off head

$f(x) = y$

Ex: $y^2 + 2y - 7x = 3$

$\frac{d}{dx}\left(y^2+2y-7x\right) = \frac{d}{dx}3$

$\frac{d}{dx}y^2 + 2\frac{dy}{dx} - 7 = 0$

$2y\cdot\frac{dy}{dx} + 2\frac{dy}{dx} - 7 = 0$

$\frac{dy}{dx}(2y+2) = 7$

$\frac{dy}{dx} = 7/(2y+2)$

$$\frac{d}{dx} x^2 = 2x$$

$$\frac{d}{dx} (\underline{x^4-7})^2 = 2(x^4\underline{-}7)\cdot \underline{\underline{4x^3}}$$

$$\frac{d}{dx} \underline{y}^3 = 3\underline{y}^2 \cdot \frac{dy}{dx}$$

Explicit $y = f(x) = x^2 + 7x$

Implicit $y = xy^2 + x^2$

differentiation (Take $\frac{d}{dx}$ of both sides)

$$\frac{d}{dx} y = \frac{d}{dx}(xy^2 + x^2)$$

$$\frac{dy}{dx} = \frac{d}{dx}(xy^2) + \frac{d}{dx} x^2$$

$$\frac{dy}{dx} = x \frac{d}{dx} y^2 + y^2 \cdot \frac{d}{dx} x + 2x$$

Chain rule

$$\frac{dy}{dx} = x \cdot 2y \cdot \frac{dy}{dx} + y^2 + 2x$$

Variables don't match

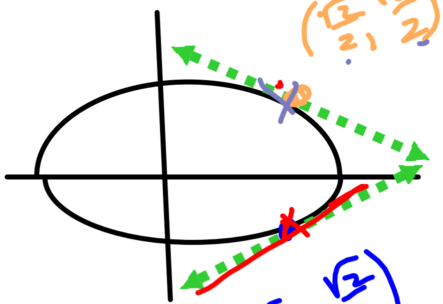
As a chain Rule

$$\frac{dy}{dx} - 2xy \frac{dy}{dx} = y^2 + 2x$$

$$\frac{dy}{dx}(1 - 2xy) = y^2 + 2x$$

$$\boxed{\frac{dy}{dx} = \frac{y^2 + 2x}{1 - 2xy}}$$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\quad m_{tan} = -1$



$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$m_{tan} = 1$

$$\boxed{x^2 + y^2 = 1}$$

$$\boxed{\frac{d}{dx}}(x^2 + y^2) = \boxed{\frac{d}{dx}} 1$$

$$2x + 2y\boxed{\frac{dy}{dx}} = 0$$

$$\boxed{\frac{dy}{dx}} = \frac{-2x}{2y} = \boxed{\frac{-x}{y}}$$

$$\frac{d}{dx} y^3 = 3y^2 \frac{dy}{dx}$$

chain

$$\frac{d}{dx} t^{30} = 30t^{29} \cdot \frac{dt}{dx}$$

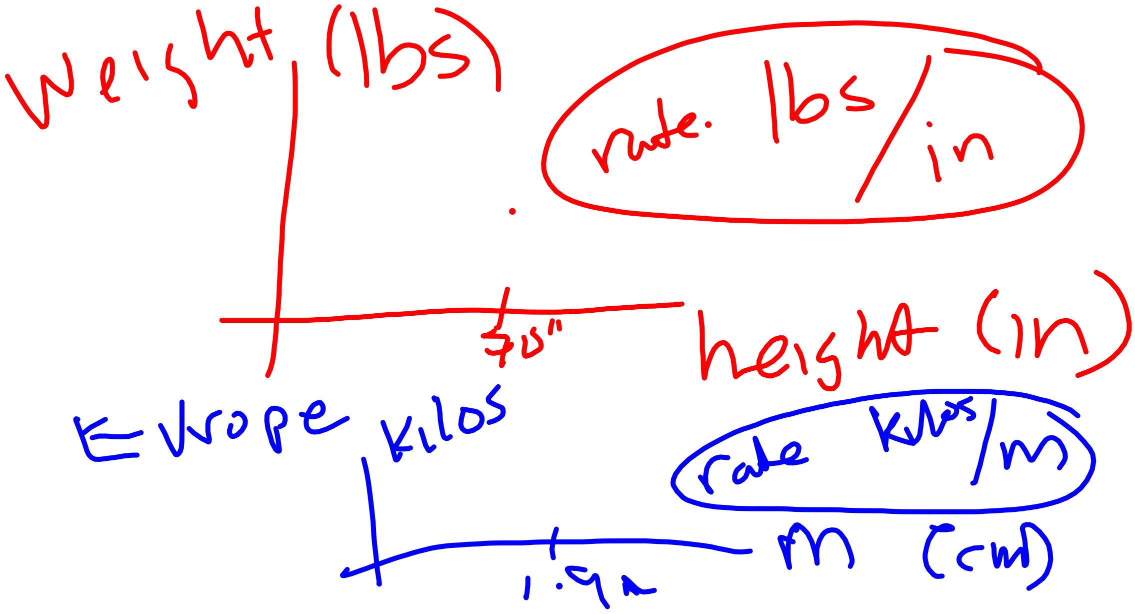
Meaning of Composite Functions (preview)

OPTIONAL

$(f \circ g)(x) = f(g(x))$

Assembly Line

$x \longrightarrow g(x) \longrightarrow f(g(x))$


Weight (lbs)
rate: lbs/in
70"
height (in)
Europe
kilos
rate kilos/m
1.9m
m (cm)

cm	Height	Weight	Kilo
$C(x) = x/2.5$	60"	120	2.2 lb = 1 K.
	68	148	
	71	156	$K(lb) = lb/2.2$

$$\overset{cm}{x} \longrightarrow C(x) \xrightarrow{in} r(x) \xrightarrow{lb} K(x) \quad \text{Kilos}$$

$$K(r(\underset{cm}{c(x)})) = \underset{\text{Kilos}}{\boxed{}}$$

$$\frac{d}{dx} K(r(c(x))) =$$

$$= K'(r(c(x))) \cdot r'(c(x)) \cdot c'(x)$$

$$\frac{d}{dx} f(r(g(x)))$$

$g(x) = x - 2$

$g'(x) = 1$

$$f'(r(g(x))) \cdot \frac{d}{dx} r(g(x))$$

$$f'(r(g(x))) \cdot r'(g(x))\, g'(x)$$

$f(x) = x \cdot 1\text{mil}$

$f'(x) = 1\text{mil}$

$1{,}000{,}000 \cdot r'(g(x))$

(14) mean 2014

$r' = 250$

$\rightarrow$ \$250 million/yr.