Agenda

Quiz Review

Lecture: More Derivative Rules

Lecture: Derivative of Composite Functions

Group Work: Calculator Derivatives

Quiz #3 Review

Quiz #3

Find an equation of the tangent line to $y = x^2 - 3x$ at $x = -2$.

$y =$ [ ]

$y' = 2x - 3$

slope: $m = 2(-2) - 3 = -7$

POINT: $(-2, 10)$

$y(-2) = (-2)^2 - 3(-2) = 4 + 6 = 10$

$y - y_1 = m(x - x_1)$

$y - 10 = -7(x+2) + 10$

$+10$

Find an equation of the tangent line to $y = x^2 - 3x$ at $x = -2$.

$y =$ ☐

$y' = 2x' - 3$

$y'(-2) = 2(-2) - 3$

$= -7 = m$

$y(-2) = (-2)^2 - 3(-2)$

$= 4 + 6 = 10$

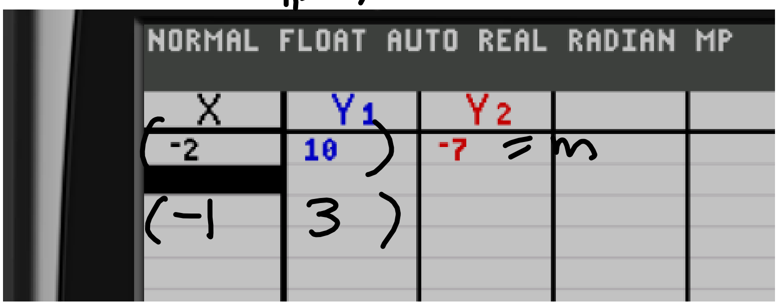
$(-2, 10)$ POINT

$\boxed{Y=}$ $y - 10 = -7(x+2)$ Point/slope.

$Y_1 = x^2 - 3x \leftarrow$

$Y_2 = \text{nder}(Y_1, X, x)$

$Y_1 = x^2 - 3x$



$(-2 \quad 10) \quad -7 = m$

$(-1 \quad 3)$

Find an equation of the tangent line to $y = x^2 - 3x$ at $x = -2$.

pt + slope

$y =$ ☐

$y(-2) = 4 + 6 = 10 \quad (-2, 10) \quad -7$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 3(x+h) - x^2 + 3x}{h}$$

$= 2x - 3$

$f'(-2) = -4 - 3 = -7 \quad$ Point $(-2, 10)$

$y = -7(x+2) + 10$

$$m = \frac{\Delta y}{\Delta x} = \boxed{\frac{y - y_1}{x - x_1} = m}$$

$$\boxed{y - y_1 = m(x - x_1)}$$

POINT $(-2, 10) \quad m = -7$

$y - 10 = -7(x+2)$

$y = -7(x+2) + 10$

Question

Suppose a state's income tax code states that tax liability is 15% on the first $18200 of taxable earning and 11% on the remainder. Find constants a and b for the tax function

$$T(x) = \begin{cases} a + 0.15x & x \leq 18200 \\ b + 0.11(x - 18200) & x > 18200 \end{cases}$$

such that $\lim_{x \to 0^+} T(x) = 0$ and $\lim_{x \to 18200} T(x)$ exists.

a = ☐, b = ☐

Question

Suppose a state's income tax code states that tax liability is 15% on the first $18200 of taxable earning and 11% on the remainder. Find constants a and b for the tax function

$$T(x) = \begin{cases} a + 0.15x & x \le 18200 \\ b + 0.11(x - 18200) & x > 18200 \end{cases}$$

such that $\lim_{x \to 0^+} T(x) = 0$ and $\lim_{x \to 18200} T(x)$ exists.

a = 0 , b = 2730

$$\lim_{x \to 0} T(x) = a + .15 \cdot 0$$

$$= a = 0$$

$$\lim_{x \to 18200^-} .15x = 2730$$

$$= \lim_{x \to 18200^+} b + .11(x - 18200)$$

$$= b$$

Lecture Rules & Transcendentals

What is Math?

What is Calculus?

What is the instantaneous rate of change?

What is Velocity?

Velocity is

the Derivative is

the instantaneous rate of change is

the slope of the tangent line

The "Quick and Dirty" Rules

1. $\frac{d}{dx}$ constant $= 0$
2. $\frac{d}{dx}$ $mx+b = m$
3. $\frac{d}{dx}$ $x^n = n x^{n-1}$ ← Power Rule

Ex $\frac{d}{dx} \; x^4 - 3x^2 + 7$

$4x^3 - 6x$

Ex $\frac{d}{dx} \; \frac{x^7 - 3x^2 + 7}{x^4} = \frac{d}{dx}\left(\frac{x^7}{x^4} - \frac{3x^2}{x^4} + \frac{7}{x^4}\right)$

$\frac{d}{dx}\left(x^3 - 3x^{-2} + 7x^{-4}\right)$

$3x^2 + 6x^{-2-1} - 28x^{-4-1}$

$3x^2 + 6x^{-3} - 28x^{-5}$

Ex $\frac{d}{dx}\sqrt{x}$

$\frac{d}{dx} x^{1/2} = \frac{1}{2} x^{1/2 - 1} = \frac{1}{2} x^{-1/2}$ or $\frac{1}{2\sqrt{x}}$

Examples

Ex $\dfrac{d}{dx}\; x^4 - 3x^2 + 7$

$4x^{4-1} - 3\cdot 2\, x^{2-1} + 0$

Ex $\dfrac{d}{dx}\; \dfrac{x^7 - 3x^2 + 7x^0}{x^4} = \dfrac{d}{dx}\; x^3 - 3x^{-2} + 7x^{-4}$

$= 3x^2 + 6x^{-2-1} + -28x^{-4-1}$

Ex $\dfrac{d}{dx}\sqrt{x} = \dfrac{d}{dx}\, x^{1/2} = \dfrac{1}{2}\, x^{1/2-1}$

$= \dfrac{1}{2}\, x^{-1/2} = \dfrac{1}{2\sqrt{x}}$

More Rules!

Product

Quotient

Chain

The Product Rule

$$\frac{d}{dx}(f \cdot g) = f \cdot \frac{d}{dx}g + g \cdot \frac{d}{dx}f$$

$$= fg' + gf'$$

The Quotient Rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}$$

Ex $\frac{d}{dx}\left(\underbrace{x^{50}+2x^{25}+7}_{f}\right)\left(\underbrace{x^{23}-2x+15}_{g}\right)$

$\underline{Ex}$ $\frac{d}{dx}(x^{50}+2x^{25}+7)(x^{23}-2x+15)$

$$(x^{50}+2x^{25}+7)\frac{d}{dx}(x^{23}-2x+15)+(x^{23}-2x+15)\frac{d}{dx}(x^{50}+2x^{25}+7)$$

$$(x^{50}+2x^{25}+7)\left[23x^{22}-2\right]+(x^{23}-2x+15)\left[50x^{49}+50x^{24}\right]$$

$$\frac{d}{dx}\ \frac{x^7-3x^2+7}{x^4}$$

$$\frac{x^4\cdot\frac{d}{dx}(x^7-3x^2+7)-(x^7-3x^2+7)\cdot\frac{d}{dx}x^4}{(x^4)^2}$$

$$\frac{x^4\cdot(7x^6-6x)-(x^7-3x^2+7)(4x^3)}{x^8}$$

$$\frac{7x^{10}-6x^5-4x^{10}+12x^5-28x^3}{x^8}$$

$$\frac{3x^{10}+6x^5-28x^3}{x^8}$$

$$3x^2+\frac{6}{x^3}-\frac{28}{x^5}$$

Ex

$$\frac{d}{dx}\left(\frac{1}{x-5}\right)\begin{matrix}=f\\=g\end{matrix}$$

$$\frac{(x-5)\cdot\underbrace{\frac{d}{dx}(1)}_{=0}-1\cdot\frac{d}{dx}\underbrace{(x-5)}_{=1}}{(x-5)^2}$$

$$=\frac{-1}{(x-5)^2}$$

Ex

$$y = \frac{1}{(x+5)^2} = \frac{1}{x^2+10x+25}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2} = \frac{-\frac{d}{dx}(x^2+10x+25)}{(x^2+10x+25)^2}$$

$$= \frac{-2x-10}{(x^2+10x+25)^2}$$

Ex

$$y = \frac{1}{(x+5)^2} = \frac{1}{x^2+10x+25}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2} = \frac{\cancel{(x^2+10x+25)\frac{d}{dx}}(1) - (2x+10)}{(x+5)^4}$$

$$= \frac{-2(x+5)}{(x+5)^4} = \frac{-2}{(x+5)^3}$$

$$y'(2) = \frac{-2}{3^3}$$

Chain Rule

Chain Rule

[Composite function $(f \circ g)(x)$

$$f(g(x))$$

Outside Inside

$$\frac{d}{dx} \underbrace{f(g(x))}_{(f \circ g)(x)} = f'(g(x)) \cdot g'(x)$$

derivative of outside
leave inside alone

$*$

derivative of inside.

$$\frac{d}{dx}\left(x^2+3x+8\right)^{100} = 100\left(x^2+3x+9\right)^{99} \cdot \frac{d}{dx}\left(x^2+3x+9\right)$$

$$(2x+3)$$

Power Rule.

$$\frac{d}{dx}\left(x\right)^{100} = \underline{100}\, x^{99} \cdot g'(x)$$

$g(x)$ $g(x)$

$$= 100 \cdot (2x+3)\left(x^2+3x+8\right)^{99}$$

Ex

$$\frac{d}{dx}(x)^{100} = 100(x)^{99}$$

$$\frac{d}{dx}(x^2+3x+8)^{100} = 100(x^2+3x+8)^{99} \cdot \frac{d}{dx}(x^2+3x+8)$$

$$\boxed{= 100(x^2+3x+8)^{99} \cdot (2x+3)}$$

DONE

Ex $\frac{d}{dx}\boxed{y}^{100} = 100(\boxed{y})^{99}\frac{dy}{dx}$

Ex $\frac{d}{dx}\pi^3 = 3\pi^2 \cancel{\frac{d\pi}{dx}}^{\to 0}$

$= 0$

$$\frac{d}{dx}\frac{5}{(x+2)} = \frac{d}{dx}5(x+2)^{-1}$$

$$= -5(x+2)^{-2}\cdot\frac{d}{dx}(x+2)$$

Ex $\dfrac{d}{dx}\left(\dfrac{1}{(x+5)^2}\right)$

$\dfrac{d}{dx}(x+5)^{-2}$

$= -2(x+5)^{-3}\cdot\dfrac{d}{dx}(x+5)$

$= \dfrac{-2}{(x+5)^3}$

Transcendentals

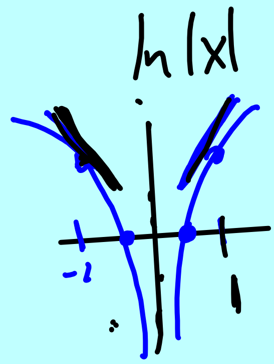
$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} a^x = a^x \ln a.$$

$$\frac{d}{dx} \ln x = \frac{1}{x} \quad (x>0)$$

$\underline{(x>0)}$

$$\frac{d}{dx} \ln|x| = \frac{1}{x}$$



$$\frac{d}{dx} \sin(x) = \cos(x)$$

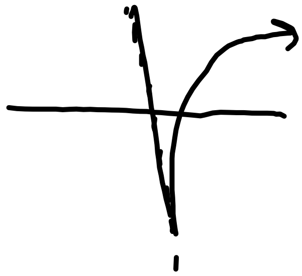
$$\frac{d}{dx} \cos(x) = -\sin(x)$$

More Specials

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} a^x = a^x \ln a \quad (a > 0)$

$\frac{d}{dx} \sin x = \cos x$



$\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \ln x = \frac{1}{x}$

$\frac{d}{dx} \log_B x = \frac{1}{x \ln B}$

$\frac{d}{dx} \sec x = \sec x \tan x$

$$\frac{d}{dx} e^{(\text{crap})} = e^{(\text{crap})} \cdot \frac{d}{dx}(\text{crap})$$

$$\frac{d}{dx} \ln(\text{crap}) = \frac{1}{(\text{crap})} \cdot \frac{d}{dx}(\text{crap})$$

$$\frac{d}{dx} \sin(\text{crap}) = \cos(\text{crap}) \cdot \frac{d}{dx}(\text{crap})$$

Proof

$$\frac{d}{dx}e^x = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x\left(e^h - 1\right)}{h}$$

$$e^x \lim_{h\to 0} \frac{e^h - 1}{h} \qquad = e^x$$

$$e^{x+h} = e^x \cdot e^h$$

Examples

$$\frac{d}{dx} e^{x^2} = e^{x^2} \cdot \frac{d}{dx} x^2$$

$$\frac{d}{dx} \ln(x+1) = \frac{1}{x+1} \cdot \frac{d}{dx}(x+1)$$

$$\frac{d}{dx} \sin^2(x) =$$

$$\frac{d}{dx}(\sin x)^2 = 2(\sin x)^1 \underbrace{\cos x}_{\frac{d}{dx}\sin x}$$

$$\frac{d}{dx} e^{x^2}$$

$\exp(x^2)$

$$\frac{d}{dx} \ln(x+1)$$

$$\frac{d}{dx} \sin^2(x)$$

$$\frac{d}{dx} \cos(x^2) =$$

$$\frac{d}{dx} e^{x^2} = e^{x^2} \cdot \frac{d}{dx} x^2 \quad \text{Chain Rule}$$

$\underline{exp(x^2)}$ $\quad = e^{x^2} \cdot 2x \overset{or}{=} 2xe^{x^2}$

$$\frac{d}{dx} \ln(x+1) = \frac{1}{(x+1)} \cdot \frac{d}{dx}(x+1) = \frac{1}{x+1}$$

$$\frac{d}{dx} \sin^2(x) = \frac{d}{dx}(\sin x)^2 = 2(\sin x)^1 \frac{d}{dx}(\sin x)$$

$$= 2 \sin x \cos x$$

$$\frac{d}{dx} \cos(x^2) = - \sin(x^2) \cdot \frac{d}{dx} x^2$$

$$= - \sin(x^2) \cdot 2x$$

tricky

y= sin (pi/3) $\longrightarrow$ $y' = 0$

y= f*sin (pi/3) $\longrightarrow$ $y' = f' \cdot \sin(\pi/3)$

y= x^.5*sin (pi/3) $\longrightarrow$ $y' = \frac{1}{2} x^{-1/2} \cdot \sin(\pi/3)$

$\frac{1}{2} x^{1/2 - 1}$

tricky

y= sin (pi/3) = .68... so y'=0

y= fsin (pi/3) = .68...*f so y'=.68...*f '

y= x^.5*sin (pi/3) = .68*x^.5 so

power rule y'=.68*.5*x^(-.5)

$y = \sin\left(\frac{\pi}{3}\right) \quad y' = 0$

$\frac{d}{dx} \sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$

$$\frac{d}{dx}\,\frac{\sin x}{\cos x} = \frac{\cos x \cdot \cos x - \sin x\,(-\sin x)}{\cos^2 x}$$

Quotient Rule $\left(\frac{f}{g}\right)' = \frac{g f' - f g'}{g^2}$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx} \sec x = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$= \frac{-1 \cdot -\sin x}{\cos^2 x} = \boxed{\tan x \sec x}$$

Ex $\quad \frac{d}{dx} \sec(x^2) = \sec(x^2)\tan(x^2) \cdot 2x$

$$\frac{d}{dx} \sec(x^2) = \sec(x^2)\tan(x^2) \cdot \frac{d}{dx}(x^2)$$

Groupwork

Before

1) Put data In.

2) Find Quartic regression

3) Put it in $Y_1 = \boxed{\text{VARS}}$ 5: >> 1:

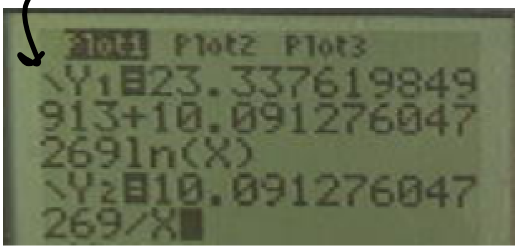
4) Put $Y_2 = \text{nderiv}(Y_1, X, x)$
or

$$Y_2 = \frac{d}{d\boxed{X}}(Y_1)\Big|_{x=\boxed{t}}$$

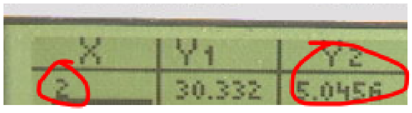
5) Table.

X	Y_1	Y_1'
Point of Int		

Ln reg.

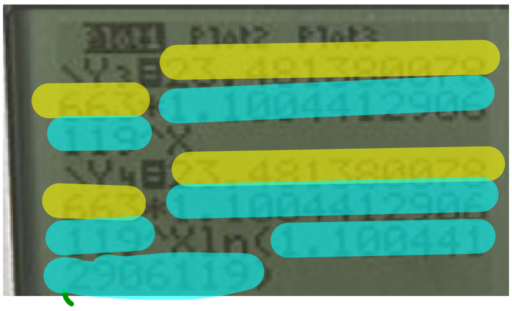


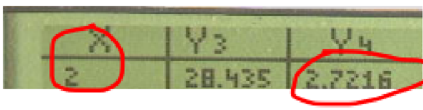
$$y = a + b \ln x$$
$$y' = 0 + b \cdot 1/x$$



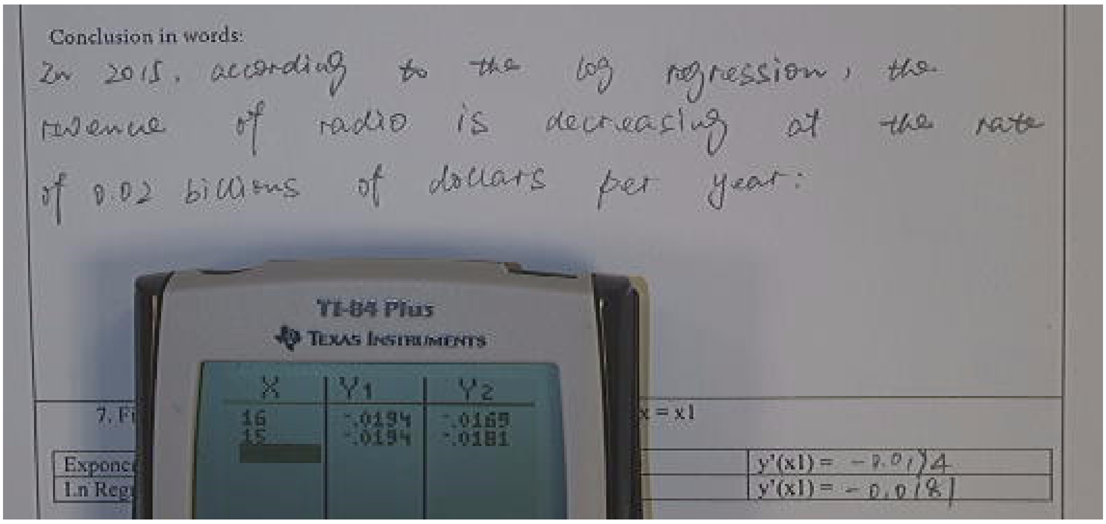
in 2nd year, according to the ln regression, her revenue is growing at 5.04 billion $ per year.

$$y = a * b^x$$
$$y' = a * b^x \cdot \ln b$$





in 2nd year, according to the exponential regression, her revenue is growing at 2.72 billion $ per year.



Conclusion in words:

In 2015, according to the log regression, the revenue of radio is decreasing at the rate of 0.02 billions of dollars per year.

7. Fi... x = x1

Exponen...	y'(x1) = −0.0194
Ln Reg...	y'(x1) = −0.0181

According to the exponential regression, at $4 the rate of sale lost is -.2838 people per dollar

I lose about 1/4 a person sales for every dollar I raise the price

For Posting

Pictr

TABLE

x	y_1	y_1'	y_1''
2020	A	B	C
2021			
2022			

Math

$y_1(2020) = A$

$y_1'(2020) = B$

$y_1''(2020) = C$

Words

According to The Quadr regression

at x = 2020 The function is ___A___

and changing by ___B___ units y/per unit x

and is accelerating/decellerating

[C(+)] [C(−)]

More..Class Project

```
Plot1  Plot2  Plot3
\Y1=
\Y2=-300.07142857143X²+73
08.8142857143X+-37054.7428
57143
\Y3=
```

$$y = ax^2 + bx + c$$

$$y' = 2ax + b$$

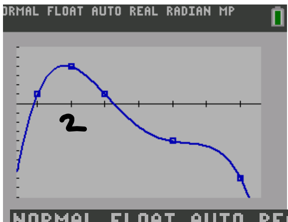
```
\Y3=126.08333333334X^3+-4
839.0714285716X^2+61348.13
0952384X+-249782.54285715
```

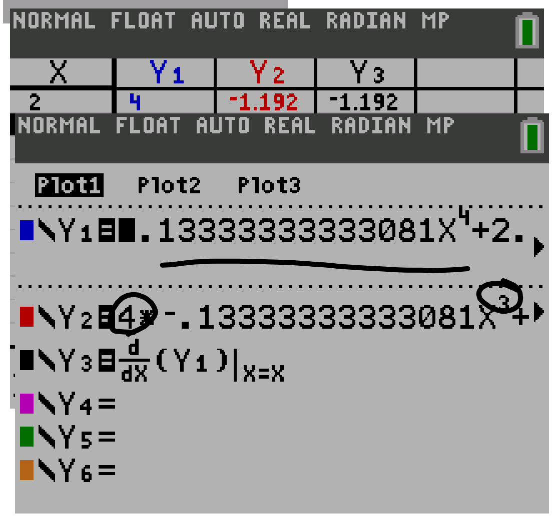
$$y = ax^3 + bx^2 + cx + d$$

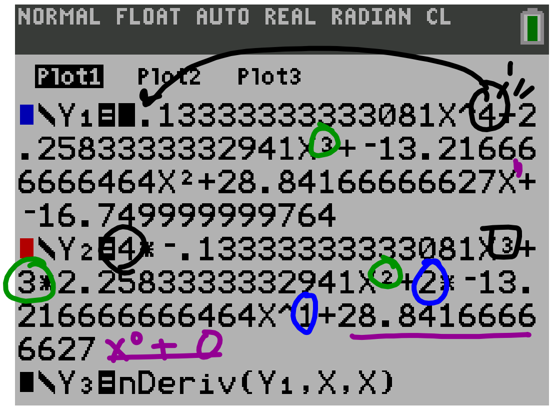
$$y' = 3ax^2 + 2bx + c$$

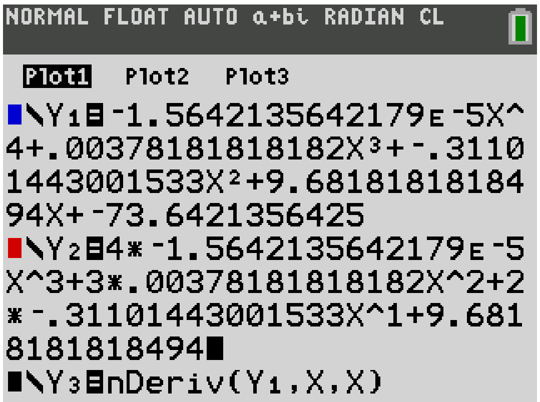
$y = ax^4 + bx^3 + cx^2 + dx + e$

$y' = 4ax^3 + 3bx^2 + 2cx + d$








NORMAL FLOAT AUTO a+bi RADIAN CL
Plot1 Plot2 Plot3
Y1=-1.5642135642179E-5X^4+.0037818181818182X3+-.31101443001533X2+9.6818181818494X+-73.6421356425
Y2=4*-1.5642135642179E-5X^3+3*.0037818181818182X^2+2*-.31101443001533X^1+9.6818181818494
Y3=nDeriv(Y1,X,X)

4 $y = a + b\ln x$

$y' = b/x$

5 $y = a\sin(bx+c)+d$

$y' = a\cos(bx+c) \cdot b$