Agenda

Homework

Lecture: Intermediate Value Theorem

Lecture: Quick and Dirty Derivatives

Group Work

Homework

Quiz

Find an equation of the tangent line to $y = x^2 - 6x$ at $x = -7$.

$y =$ ▯

Quiz

Find an equation of the tangent line to $y = x^2 - 6x$ **at** $x = -7$.

$y =$ ☐

slope

1. Definition
2. Calclator
3. Quick + Dirty

point

$(-7, 91)$

$y(-7) = 49 + 42$

POINT $(-7, 91)$

Find an equation of the tangent line to $y = x^2 - 6x$ at $x = -7$.

$y(-7) = (-7)^2 - 6(-7) = 91$

$y =$ ☐

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{[(x+h)^2 - 6(x+h)] - [x^2 - 6x]}{h}$$

$$y' = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{6x} - 6h - \cancel{x^2} + \cancel{6x}}{h}$$

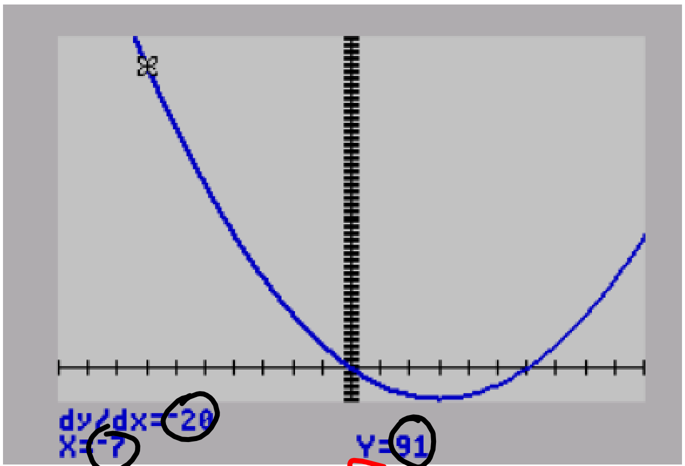
$$\lim_{h \to 0} \frac{\cancel{h}(2x + \cancel{h}^{\to 0} - 6)}{\cancel{h}} = 2x - 6$$

$$m = y'(-7) = 2(-7) - 6 = -20$$

slope $= -20$

Point $(-7, 91)$ $m = 20$

$$\boxed{y = 20(x+7) + 91}$$



$(-7, 91)$

$m = -20$

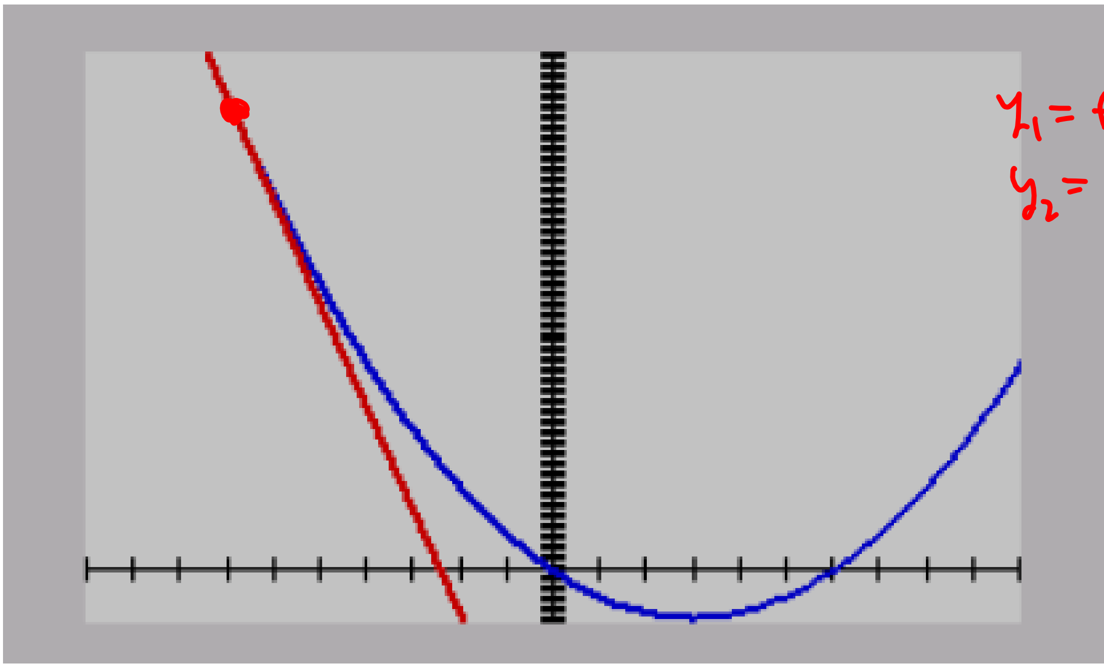
$y - 91 = -20(x+7)$

$y = 91 - 20(x+7)$

TI 83

Calc 6: dy/dx

$x = -7$



y_1 = function
y_2 = tangent line

Lecture Intermediate Value Theorem

Lecture

What is calculus?

What are the rates of change?

What is a limit?

What can we do with limits?

What theroems do we have?

Continuity

1.) limit exists

2.) funtion must exist

3.) limit = functio

Theorems:

Squeeze Theoerm (limits)

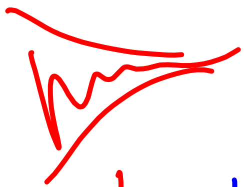
Intermediate Value Theorem (Continuity)

Mean Value Theorem (Continuity)

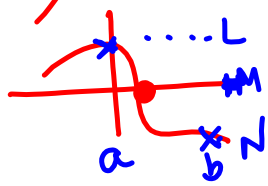
Fundamental Theorem (later)

Theorems:

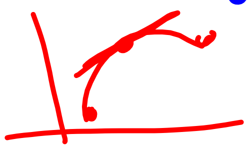
Squeeze Theorem



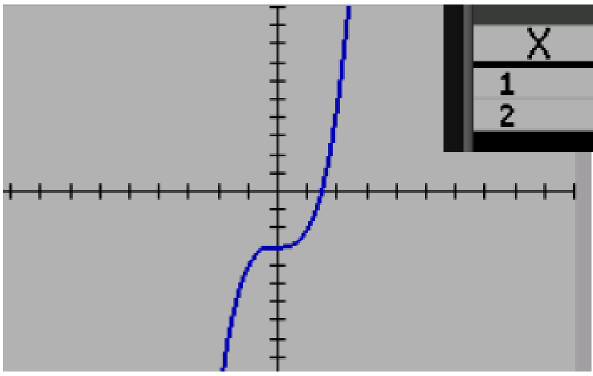
Intermediate Value



Mean Value



Where used:



X	Y_1
1	-2
2	5

1) y_1 is continuous

2) $f(1) = \ominus$

$f(2) = \oplus$

$\ominus <$ zero $< \oplus$

3) By I.V.T. There must be a point c

$1 < c < 2$

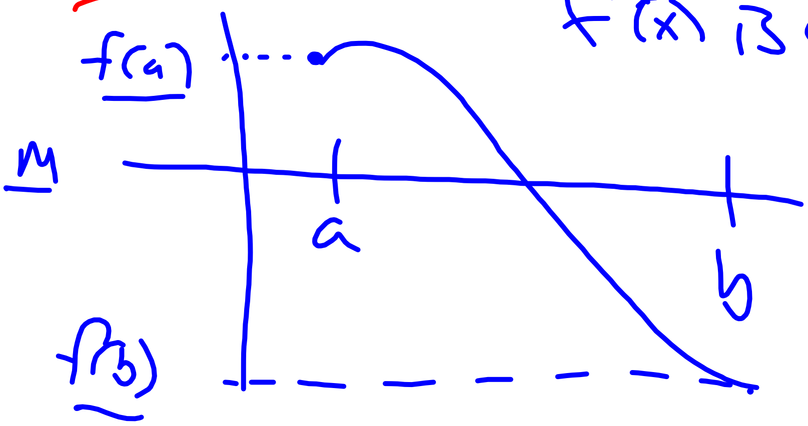
so that $f(c) = 0$

Lecture: IVT

Continuity

Intermediate Value Theorem

given $f(x)$ is cont on $[a,b]$ ~~(a,b)~~



and $f(a) > M > f(b)$

and $f(x)$ is cont on (a,b)

Then, there is a $c \in [a,b)$

so that $f(c) = M$

Lecture Quick and Dirty Derivatives

Calculus is:

- Limits

[L'Hopital's Rule]

- Derivatives

- Integrals

Derivative

||

Slope of Tangent line

||

Instantaneous Rate

f' $\quad$ $f'(x)$ $\quad$ $\frac{d}{dx}f$ $\quad$ $\frac{df}{dx}$ $\quad$ $\frac{dy}{dx}$ $\quad$ $D_x f$

$f(x)=y$

calculators

nderiv(Y_1, X, X)

$$\frac{d}{d\boxed{X}}\boxed{Y_1}\Bigg|_{\boxed{X}=\boxed{X}}$$

Evaluate at a point.

$f'(2)$ $\qquad$ $\frac{d}{dx}f\Big|_{x=2}$

nderiv$(Y_1, X, 2)$

Old way

~~Find~~ the equation of the tangent line to $y = \dfrac{5}{x+2}$ at $x = 2$.

Point $(2, 5/4)$

$m = -5/16$

An equation of the tangent line is y = ▢

Use definition of derivative

$$\lim_{h\to 0} \frac{\frac{5}{x+h+2} - \frac{5}{x+2}}{h} = \lim_{h\to 0} \frac{5(x+2)-5(x+h+2)}{h(x+2)(x+h+2)}$$

$$= \frac{-5}{(x+2)(x+2)} = \frac{-5}{(x+2)^2}$$

$$y'(2) = \frac{-5}{16} \checkmark$$

Find the equation of the tangent line to $y = \dfrac{5}{x+2}$ at $x = 2$.

$y = \dfrac{5}{x+2}$ $\qquad f'(2)$

$f(x) = \dfrac{5}{x+2}$

$f(x+h) = \dfrac{5}{x+h+2}$

$$\boxed{\text{definition} \quad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$\lim_{h \to 0} \frac{\dfrac{5}{x+h+2} - \dfrac{5}{x+2}}{h}$$

$$\lim_{h \to 0} \frac{5(x+2) - 5(x+h+2)}{(x+h+2)(x+2)} \cdot \frac{1}{h}$$

$$\lim_{h \to 0} \frac{\cancel{5x} + \cancel{10} - \cancel{5x} - 5h - \cancel{10}}{h(x+h+2)(x+2)}$$

$$\lim_{h \to 0} \frac{-5\cancel{h}}{\cancel{h}(x+h+2)(x+2)}$$

$$\lim_{h \to 0} \frac{-5}{(x+\cancel{h}+2)(x+2)}$$

$$f'(x) = \lim_{h \to 0} \frac{-5}{(x+\cancel{h}+2)(x+2)} = \frac{-5}{(x+2)^2}$$

$x+2$

$$f'(2) = \frac{-5}{4^2} = \frac{-5}{16} = m_{tan}$$

EQN OF TANGENT LINE

$x = 2 \quad \overset{f(2)}{y} = \dfrac{5}{2+2} = \dfrac{5}{4}$ $\qquad$ Point - Slope

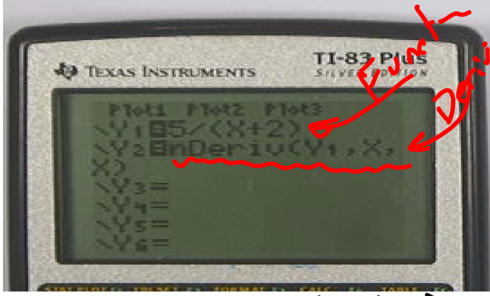
$(2, 5/4) \quad m = -5/16$

$$y - 5/4 = -5/16(x-2)$$

$$y = -5/16(x-2) + 5/4$$

$(2, 1.25)$ Point

$-.3125 = m$



Function

Deriv

$Y_1 = 5/(x+2)$

$Y_2 = \text{nderiv}(Y_1, x, x)$

Table $\quad 5/4$

X	Y1	Y2
2	1.25	-.3125

$(2, 5/4)$

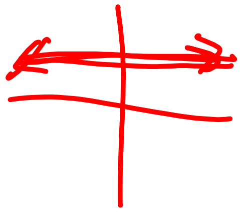
$m = -3.125$

Find the equation of the tangent line to $y = \dfrac{5}{x+2}$ at $x = 2$.

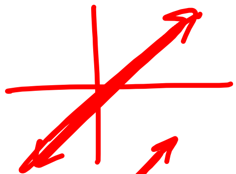
An equation of the tangent line is $y = -\dfrac{5}{16}(x-2) + \dfrac{5}{4}$

New
Quick + Dirty

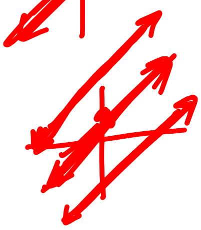
$$\frac{d}{dx}(\text{Constant}) = 0$$



$$\frac{d}{dx}\,x = 1$$



$$\frac{d}{dx}\,mx+b = m$$



Sum Rule: $\frac{d}{dx}(f+g) = \frac{d}{dx}f + \frac{d}{dx}g$

Rule: $\frac{d}{dx}\,mf = m\,\frac{df}{dx}$ $m = \#$

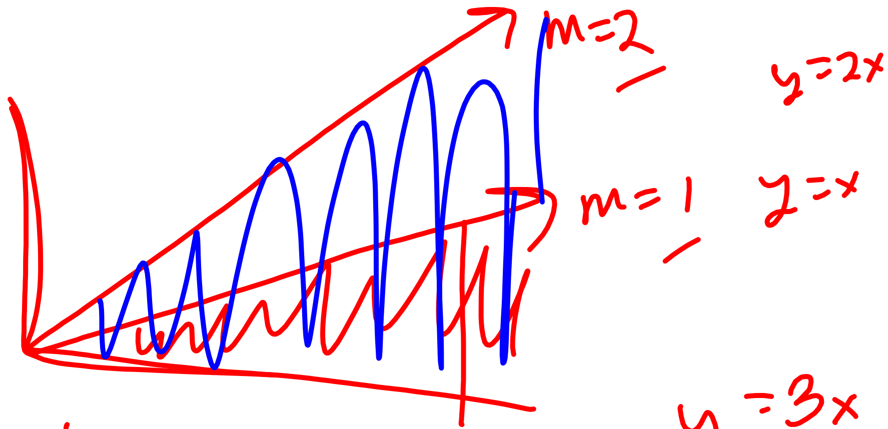
Power Rule: $\frac{d}{dx}\,x^{n} = n\,x^{n-1}$ $n = \#$

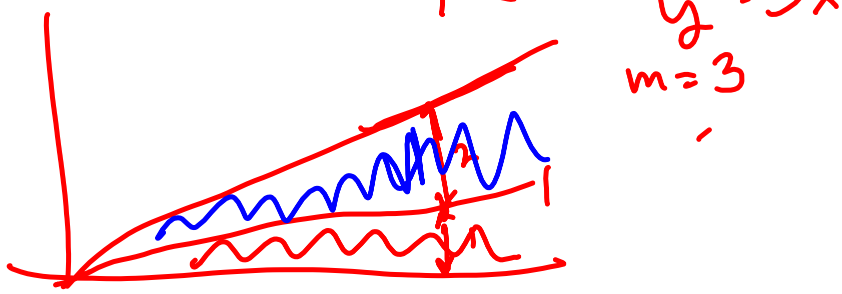
Ex $y = x^5$ $y' = 5x^4$

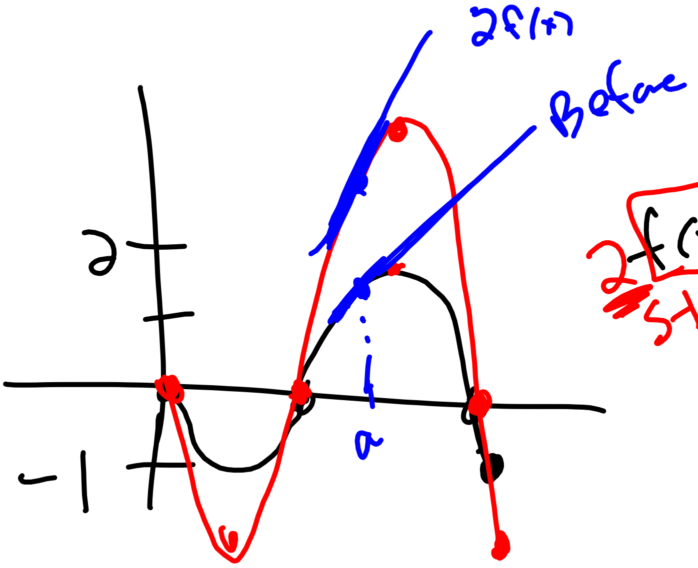
Ex $y = 3x^4$ $\longrightarrow$ $y' = 12x^3$

$4x^3$

Ex $y = 2x^3 - 5x^1 = 6x^2 - 5$


m=2
y=2x
m=1
y=x


y=3x
m=3



$\boxed{2f(x)}$ stretches the function by $\underline{2}$

Derivative equally stretched

Common Functions	Function	Derivative
Power Rule	x^n	nx^{n-1}

$$y = \frac{1}{x} = x^{-1}$$

$$y' = (-1)\,x^{-2} = -\frac{1}{x^2}$$

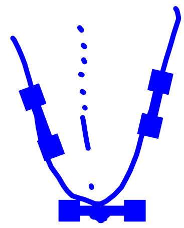
$$\frac{5}{x+2}$$

$$-\frac{5}{(x+2)^2}$$

Ex $\quad y = \frac{1}{2} x^{3/5}$

$$y' = \frac{1}{2} \cdot \frac{3}{5} \, x^{3/5 - 1}$$

$$= \frac{3}{10} x^{-2/5}$$



f $f(x) = 2x^2 + 4x - 4$ at $x = -1$.

$f'(x) = 4x' + 4$

$f'(-1) = 4(-1) + 4 = 0$

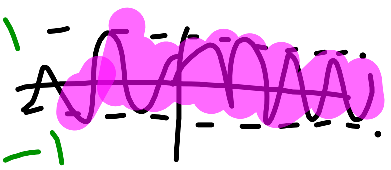
$2f(x)$

stretch

$f(2x)$

Smushes

$\sin(2x)$



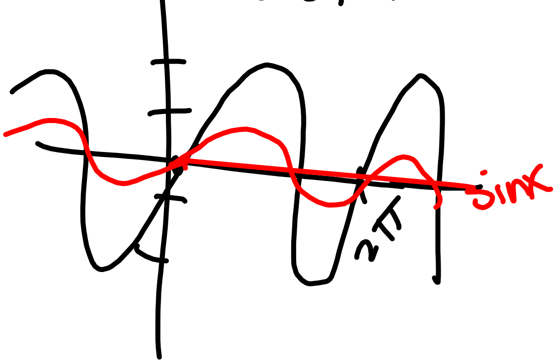
$2\sin x$

$f(x) = x^2$

$f(2x) = 4x^2$

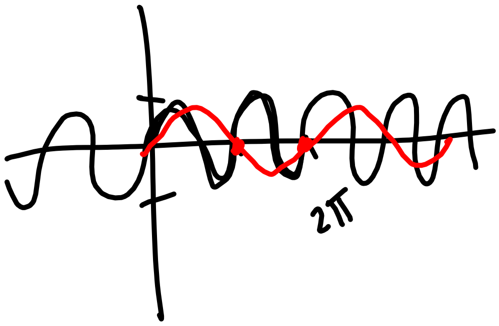
$f(x) = \sin x$

$2 \sin x$



"buster"

$\sin(2x)$



Find an equation of the tangent line to $y = x^2 - 7x$ at $x = -2$.

$y(-2) =$
$= 4 + 14$
$= 18$

$(-2, 18)$

$m = y' = 2x - 7$

$y'(-2) = 2(-2) - 7 = -11$

m=-11

$m = -11$

$x = -2 \quad y = 4 + 14 = 18$

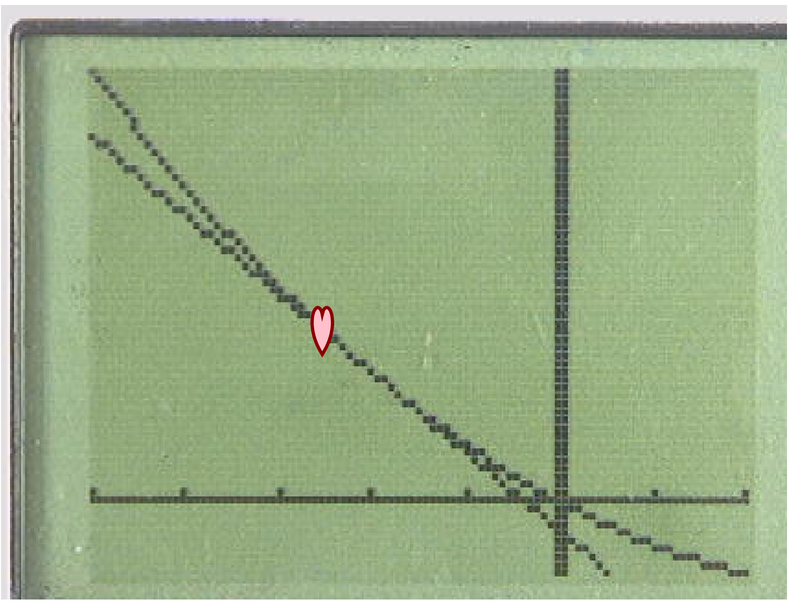
(-2 , 18)

$y - 18 = -11(x+2)$

y - y1 = m (x - x1)

y - 18 = -11 (x +2)

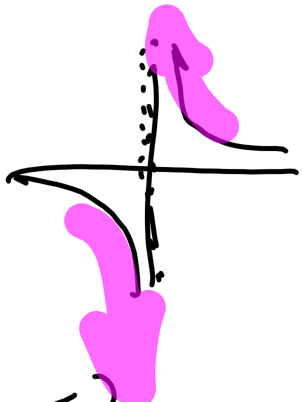
y = -11(x + 2) + 18

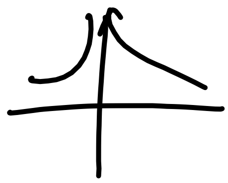


If $f(x) = \frac{5}{x}$ $(x \neq 0)$, find $f'(x)$.

$f(x) = 5x^{-1}$

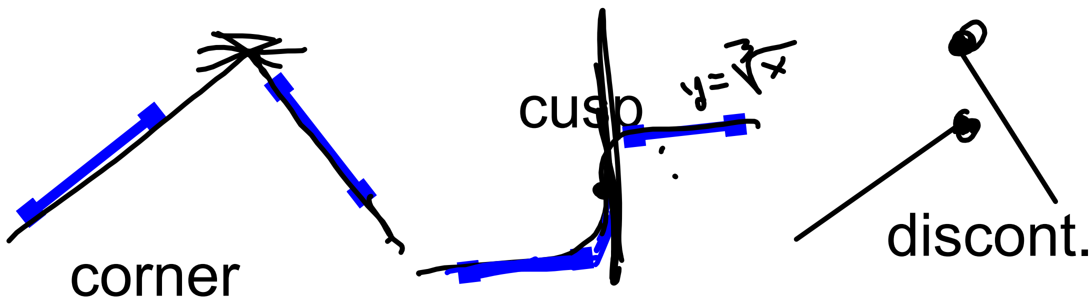
$f'(x) = -5x^{-1-1} = -5x^{-2}$





Differentiability Review

Differentiable



$y = |x|$

$y = \sqrt[3]{x}$

$y = x^{1/3} \; -1$

$y' = \frac{1}{3} x^{-2/3}$

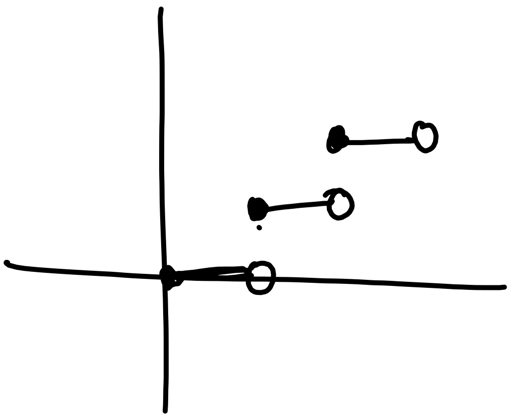
$y'(0) = \frac{1}{3} 0^{-2/3} = \frac{1}{3 \cdot 0} = \frac{1}{0}$

$y = [\![x]\!]$

$1.\overline{9} = 2$

$y = [\![x]\!]$

step function

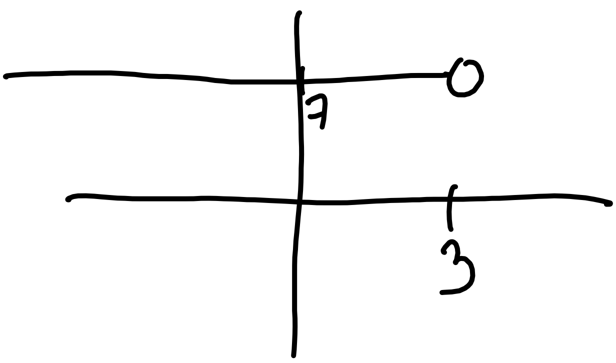


True or false?

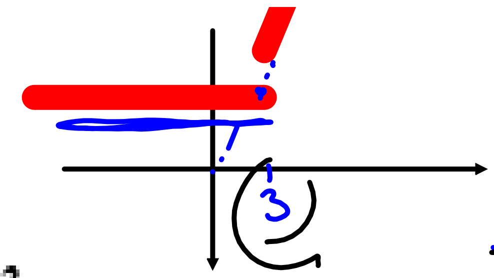
$$f(x) = \begin{cases} 7 & \text{if } x < 3 \\ 7x & \text{if } x \geq 3 \end{cases}$$ **is differentiable at** $x = 3$**.**

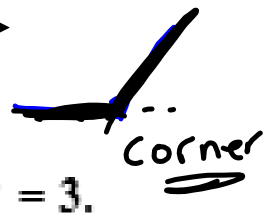
True or false?

$$f(x) \overset{y}{=} \begin{cases} 7 & \text{if } x < 3 \\ 7x & \text{if } x \geq 3 \end{cases}$$ is differentiable at $x = 3$.



Discontinues at $x = 3$





True or false?

$$f(x) = \begin{cases} 7 & \text{if } x < 3 \\ 7x & \text{if } x \geq 3 \end{cases}$$ is differentiable at $x = 3$.

left sided derivative is 0

right sided derivative is 7

differentiable= able to take the derivative

continuous, not a corner, defined

Classwork

Project

According to the cubic regression,

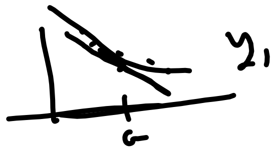
If I Charge (x=) \$3, I will sell (y=) 5 items

with an instananeous rate of change of

(dy/dx=) -.5 items per dollar.

If I raise the price by one dollar, I'll sell 4.5 items.

Calculator

Choose a regression 

Y_1 = VARS 5: > > 1:

math 8:

Y_2 = nderiv(Y_1 , X , X)

Pick a point of interest a

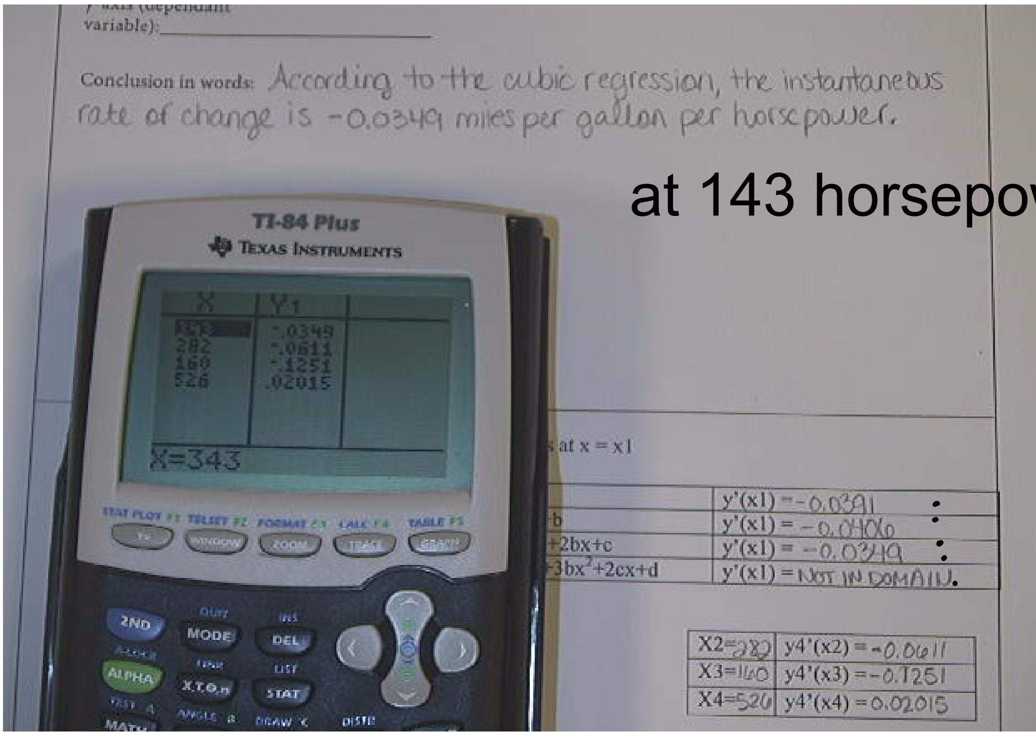
Table

X	Y_1	Y_2
→ x=a	y= $\boxed{}$	y'= $\boxed{}$

$y'(a)=$

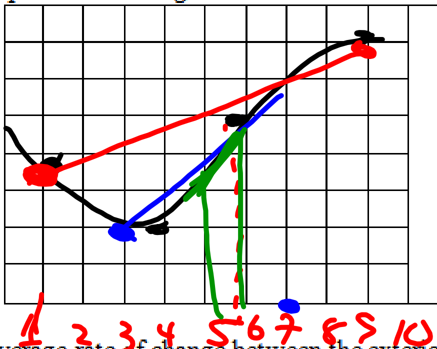
According to the cubic regression, ~~if~~ if I charge $3, the instan rate is -.54 sales per dollar.

~~If~~ If I raise the price by $1, I'll lose 1/2 sale.



at 143 horsepower

7. Roughly plot data and regression. Draw the secant and tangent lines at x = a Label Axis.



Pick x values in order

X1=	1
X2=	3
X3=	5
a=	5
X4=	6
X5=	7
X6=	9

4.9
4.99
4.999
5.001
5.01
5.1

Find the average rate of change between the exterior x-values around x = a using regression

{Y(x1) - Y(x6)}/{x1 – x6}= m_{sec}	Average Rate of Change	1.2203

Find the average rate of change between an interior x-values around x = a using regression

{Y(x2) - Y(x5)}/{x2 – x5}= m_{sec}	Average Rate of Change	1.2202

Find the average rate of change between the more interior x-values around x = a using regression

{Y(x3) - Y(x4)}/{x3 – x4}= m_{sec}	Average Rate of Change	1.2201

Find the instnataneous rate of change at x = a

nderiv(y1,x,a)	Instant Rate of Change	1.22

eaning:

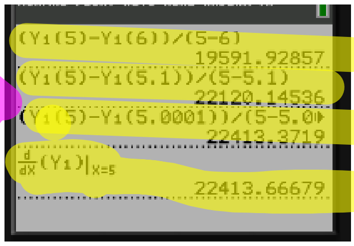
At $x = 5$ years

According to cubic regression.

The sales are increase

by \$1.22 per yrs.

$(y_1(5) - y_1(5))$



Instn

4
4.9
4.999
5
5.001
5.1
6

at ___ lbs per inch

is increasing by 18.3125 pounds per inch

, 73

x	y
72	18.3065
72.45	18.31276
72.4995	18.312517
72.5	18.3125175

223.7

At 72.5 inches tall, weight is increasing by 18.3125 pounds per inch.

according to the cubic data

y1(400) - y1(600) / (400 - 600) = ~~[illegible]~~ -.0048

y1(490) - y1(530) / (490 - 530) = 3.24 -.0074

y1(499) - y1(501) / (499 - 501) = 4.33 -.0077

nderiv(y1, x, a)

nderiv(y1, x, 500) = -.0077

At ~~[illegible]~~ $500,000, According to the cubic regression, the price of homes are decreasing by $ -.0077 per house, annually.

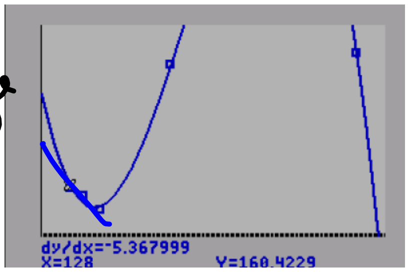
houses per dollar

Math $\dfrac{1.32 \text{ lb lifted}}{\text{weighted}}$

```
(Y1(290)-Y1(128))/(290-12▸
                 1.324646239
(Y1(128.1)-Y1(128))/(128.▸
                -5.346506361
```

$y'(128) = 5$

Picture



For a person weighing 128lbs, for each pound gained, they will be able to bench press 5.3 pounds less, according to the cubic regression.

Speech