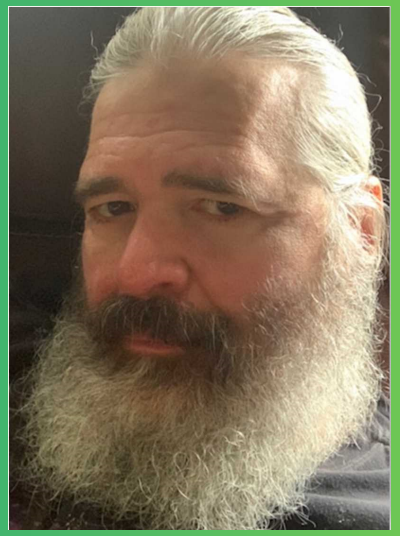
MAT 151 Calculus 1

Prof. Porter



Agenda

Homework questions

Lecture: Techniques of Differentiation

Group work

151d4

Homework Questions:

You can repeat homework as often as you like to the final due date, you should have them ready by the first due date if you want help with them. If you fail to open the assignment, by the deadline, it may never open.

Lecture : Techniques of Differentiation

What is Math?

What is Calculus?

What are the two rates of change?

How are two point slopes turned to one?

What is the derivative?

What is Math? A language

What is Calculus? Study of Change

What are the two rates of change?

Average and Instantaneous

How are two point slopes turned to one?

Limits

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

What is the derivative?

slope of the tangent line

is the derivative

is the instantaneous rate of change

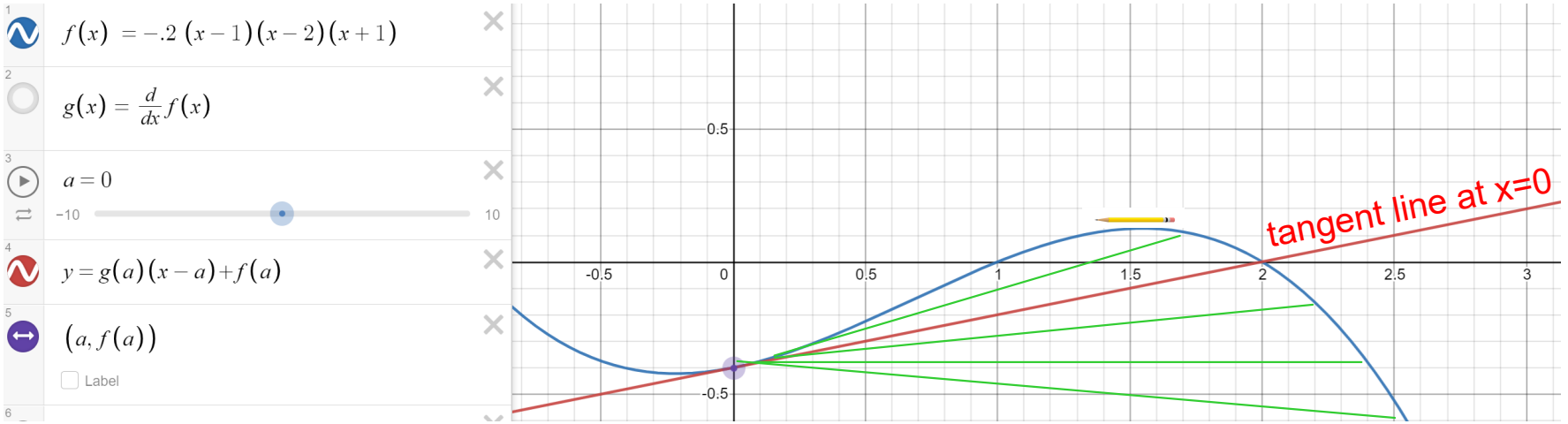
is the velocity

is dy/dx

Notation of Derivative of a function f(x) = y

'x' is a variable,'a' a value

Functions	Evaluated
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$	$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$
D_x	$D_x(a)$
y'	$y'\big\vert_{x=a}$
dy/dx	$dy/dx\vert_{x=a}$
f'	$f'(a)$
$\frac{dy}{dx}$	$\frac{dy}{dx}\big\vert_{x=a}$
$\frac{df}{dx}$	$\frac{df}{dx}\big\vert_{x=a}$
df/dx	$df/dx\vert_{x=a}$
+more	+more



secant lines between x=0 and other x values

2.5, 2.4, 2.2, 1.7

Tangent line is like holding a strait pencil to the curve

at x=1.5 the pencil rests flat....so the derivative is zero.

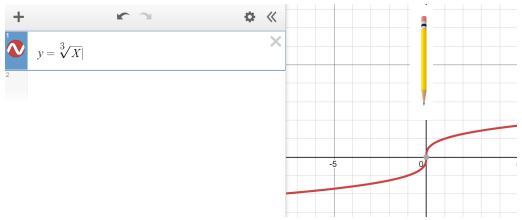
Differentiability:

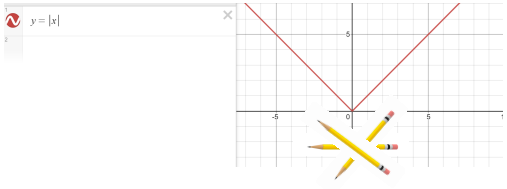
What do think it means?

Ability to differentiate/ Ability to take the derivative

What could go wrong with a pencil on a curve?

Cusp Corner Hole



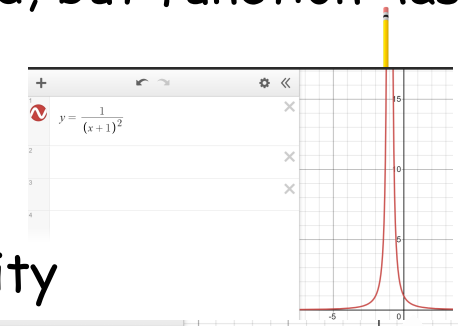


CUSP: Tangent pencil is strait up so slope is undefined

CORNER: Multiple tangent pencils so multiple slopes (none)

HOLE: Looks good, but function has to exist.

How about an asymptote?

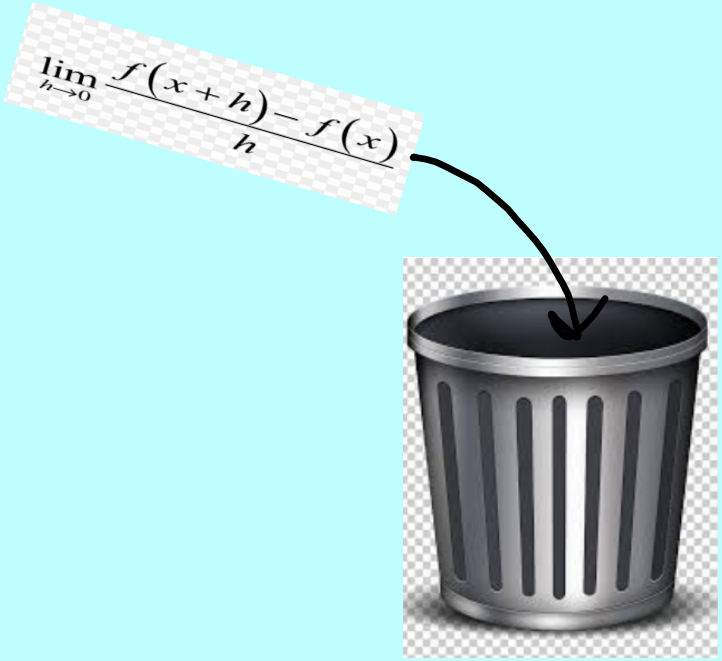


Nope.

How about any discontinuity

Nope.

Lecture: "Quick and Dirty" Derivatives



Rules for Finding Derivatives:

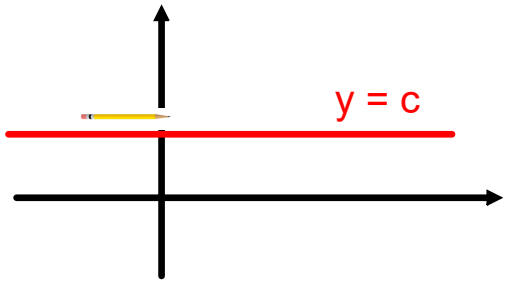
$\frac{d}{dx}$ means take the derivative with respect to 'x'

f'(t) means take the derivative with respect to the variable 't'

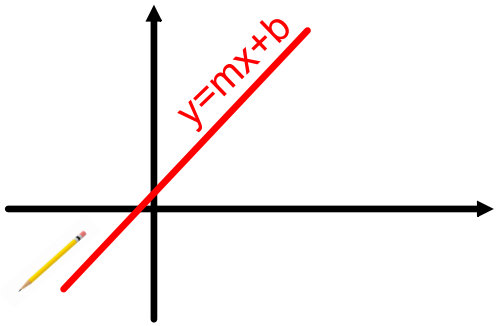
<u>Derivative of any constant 'c' is Zero</u>

$\frac{d}{dx} c = 0$

EX: $\frac{d}{dx} 100 = 0$



Derivative of a line is the slope of the line y=mx+b



$$\frac{d}{dx}x = 1$$

$$\frac{d}{dx}ax = a$$

$$\frac{d}{dx}ax+b = a$$

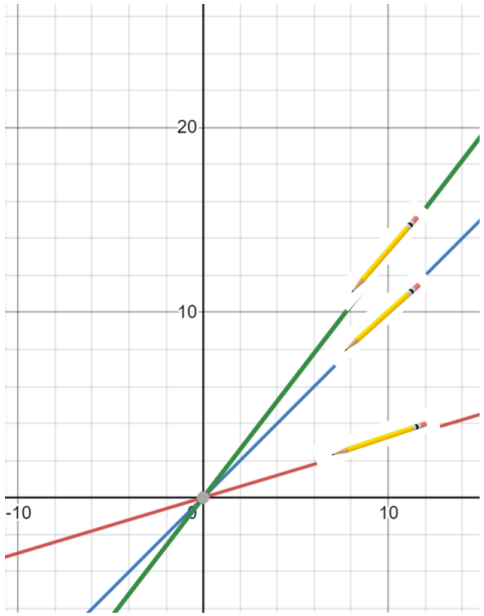
EX: $\frac{d}{dx}20x = 20$

The derivative of a sum is the sum of the derivatives

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

So: $$\frac{d}{dx}ax + b = \frac{d}{dx}ax + \frac{d}{dx}b = a + 0$$

EX: $\frac{d}{dx}$ 2x+5 = 2

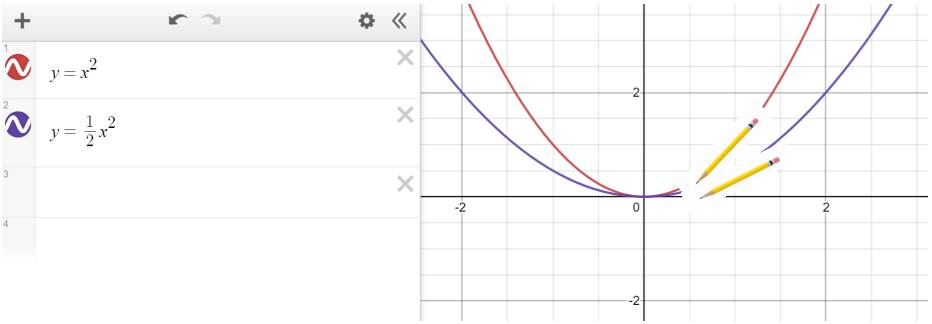


The derivative of a constant 'a' times a function f(x) is the product of the constant 'a' times the derivative of f(x)

$$\frac{d}{dx}[af(x)] = a\frac{d}{dx}f(x)$$

So: $\frac{d}{dx}ax = a\frac{d}{dx}x = a$

EX: $\frac{d}{dx}20f(x) = 20\,f'(x)$



a number in front of a function stretches (or shrinks) it by that much. So the slope of the pencil is changed by that amount

Power Rule: $\boxed{\frac{d}{dx}x^n = n\,x^{n-1}}$

EX: $\frac{d}{dx}x^2 = 2x$

$\frac{d}{dx}x^9 = 9x^8$

$\frac{d}{dt}t^7 = 7t^6$

$\frac{d}{dx}x^{1/3} = (1/3)x^{-2/3}$

to prove this, need to start with definition...

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

involves $(x+h)^n - x^n$

which will end up with nx^{n-1}

being the only term without 'h' going to zero.

Just memorize this rule...you will need it fast!

Combining Rules:

EX: $\frac{d}{dx}3x^2 - 3x + 7 =$

$\frac{d}{dx}3x^2 - \frac{d}{dx}3x + \frac{d}{dx}7 =$

$3\frac{d}{dx}x^2 - 3\frac{d}{dx}x + 0 =$

$3(2x) - 3 = \boxed{6x - 3}$

EX: $\frac{d}{dx}\left(\frac{\sqrt{x}}{2} - \frac{3}{x}\right) = \frac{d}{dx}\left(\frac{1}{2}x^{\frac{1}{2}} - 3x^{-1}\right)$

write as exponent

exponent minus 1

$= \frac{1}{4}x^{-\frac{1}{2}} + 3x^{-2}$

(1/2)(1/2) (-3)(-1)

It will get easier, except fractions are always messy.

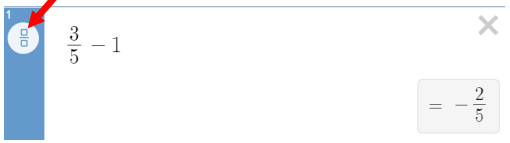
EX: $y = (1/2)\, x^{3/5}$

$y' = (1/2)(3/5)\, x^{3/5 - 1}$

Might have to calculate separately or...

$3/5 - 5/5 = -2/5$

Desmos click here for fraction answer



$$\boxed{y' = (3/10)\, x^{-2/5}}$$

Find an equation of the tangent line to $y = x^2 - 6x$ at $x = -7$.

$y =$ ☐

Point x = -7 y = (49)-6(-7) = 7 (-7,7)

Slope of tangent line: y' = 2x - 6 (from rules)

y'(-7) = 2(-7) - 6 = -20

Equation of Line: y - 7 = -20 (x - (-7))

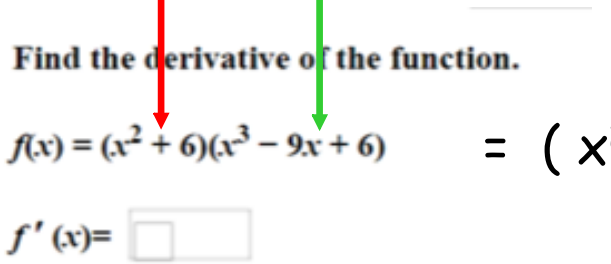
y = -20x -140 + 7

y = -20x -133

Product Rule

$$\frac{d}{dx}[f(x)*g(x)] = f(x)g'(x)+g(x)f'(x)$$

EX:



$= (x^2 + 6)(3x^2 - 9) + (x^3 - 9x + 6)(2x)$

Dashed lines represent derivatives being taken.

*represents MULTIPLICATION

Quotient Rule (fractions)

$$\frac{d}{dx}[f(x)/g(x)] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$$

EX: $\frac{d}{dx}\frac{x-1}{3x^5} = \frac{3x^5\,\frac{d}{dx}(x-1) - (x-1)\,\frac{d}{dx}3x^5}{(3x^5)^2}$

$$= \boxed{\frac{3x^5\,(1) - (\,x - 1)\,15x^4}{9x^{10}}}$$

Dashed lines represent derivatives being taken.

Note: Less is more. Don't oversimplify or you may make mistakes

Group Work

Derivatives

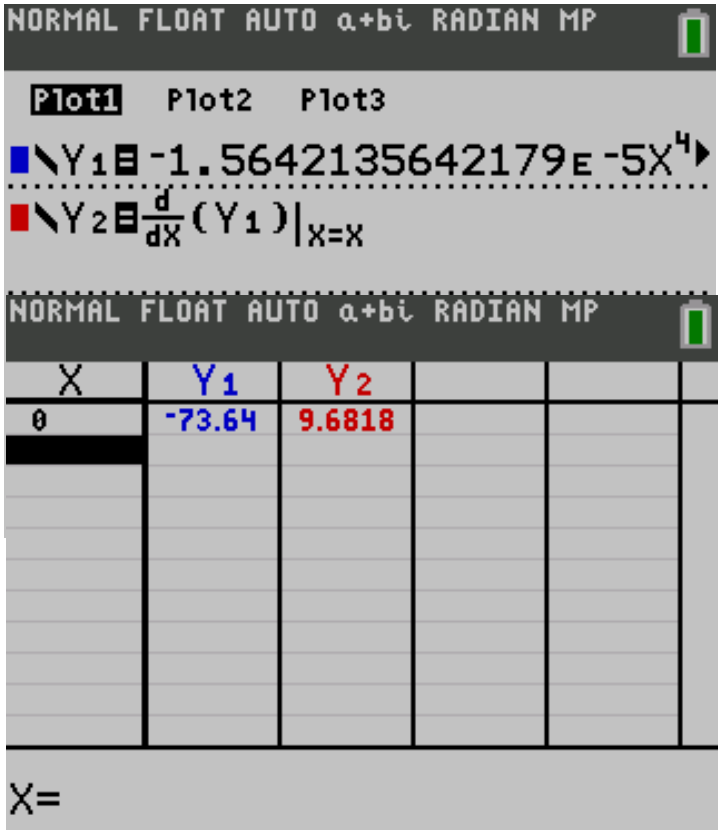
Calculator:

Y=

y1=quartic regression

y2=nderiv(y1,x,x)

Table



Words:

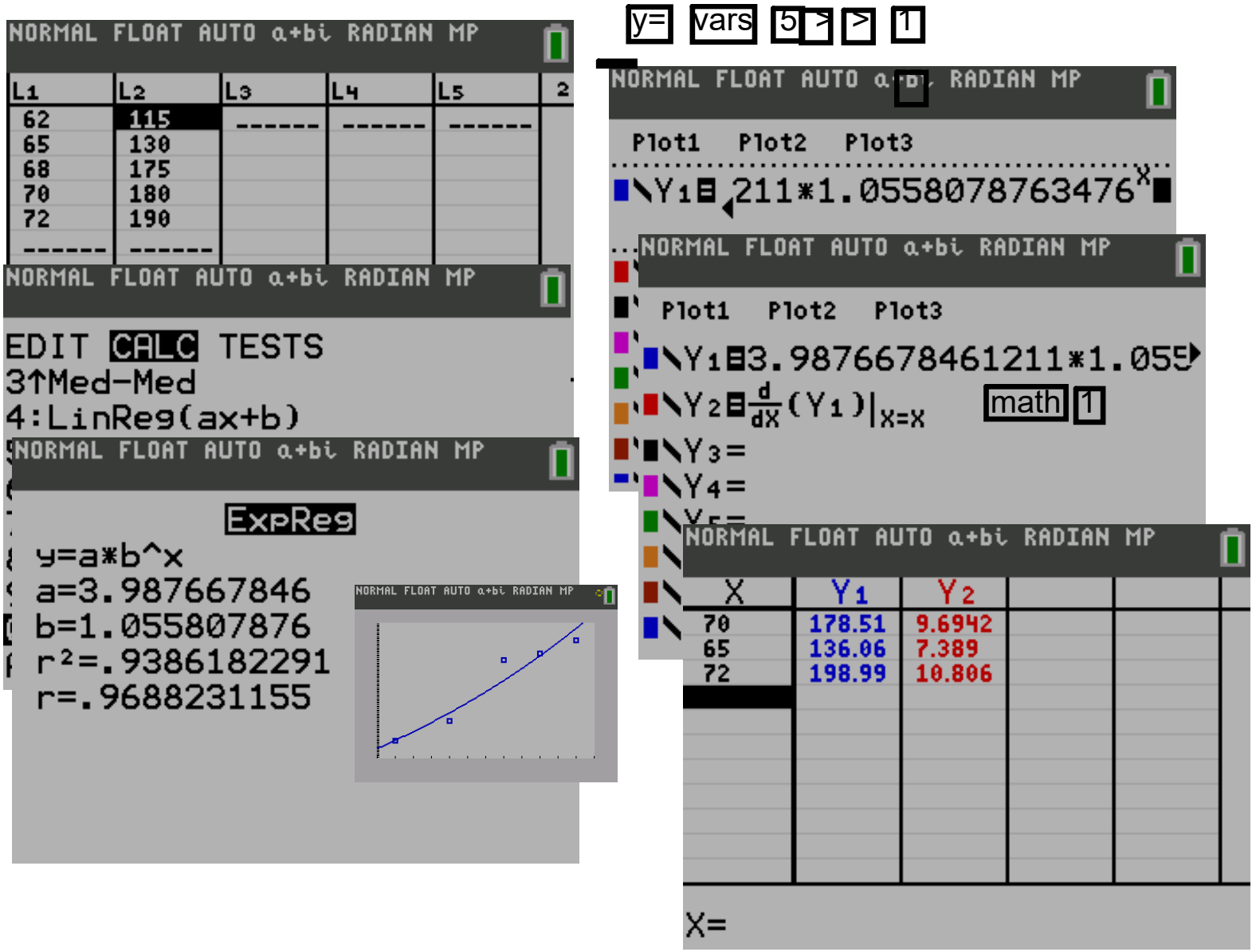
According to the cubic regression,

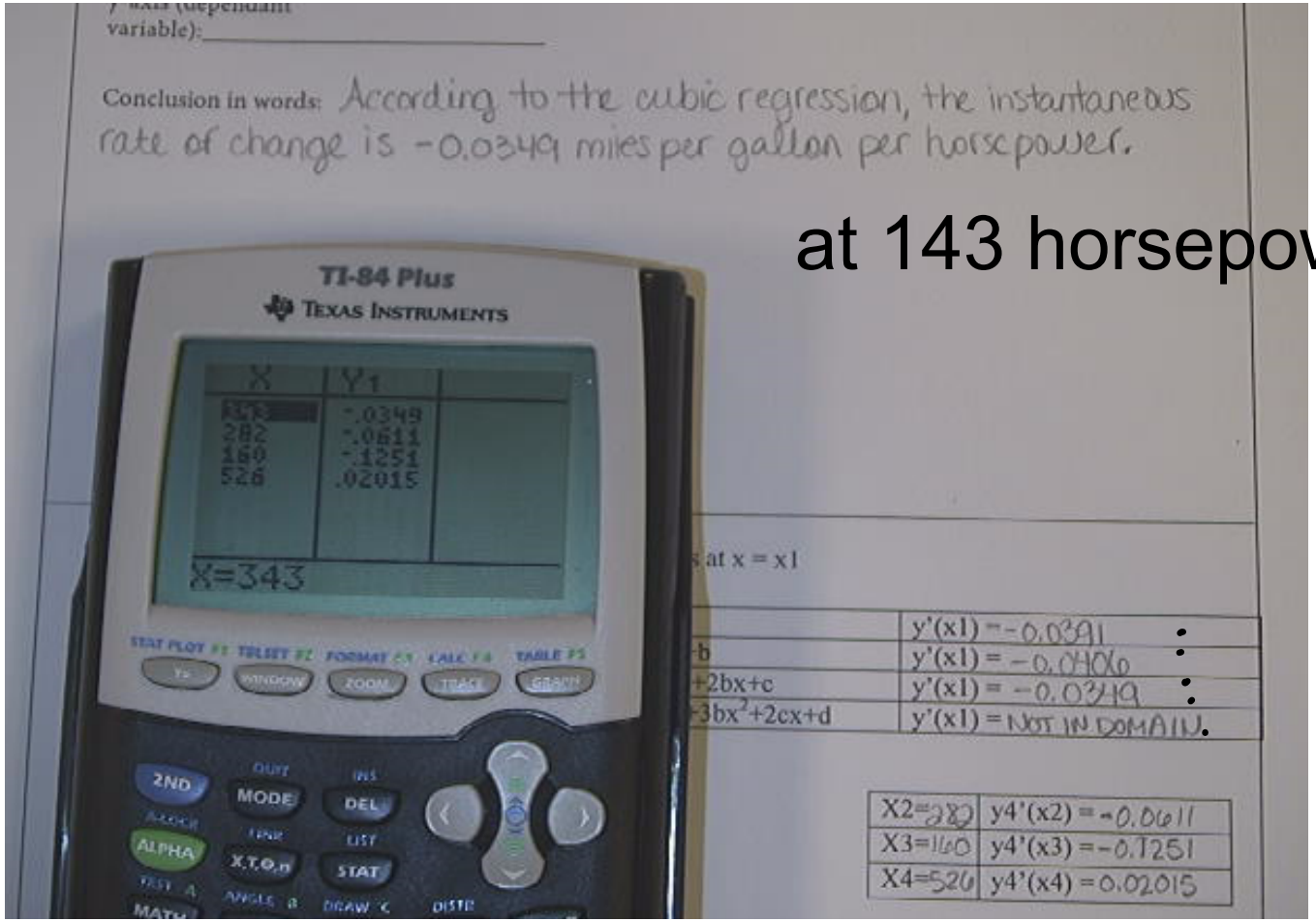
If I Charge (x=) $3, I will sell (y=) 5 items

with an instananeous rate of change of

(dy/dx=) -.5 items per dollar.

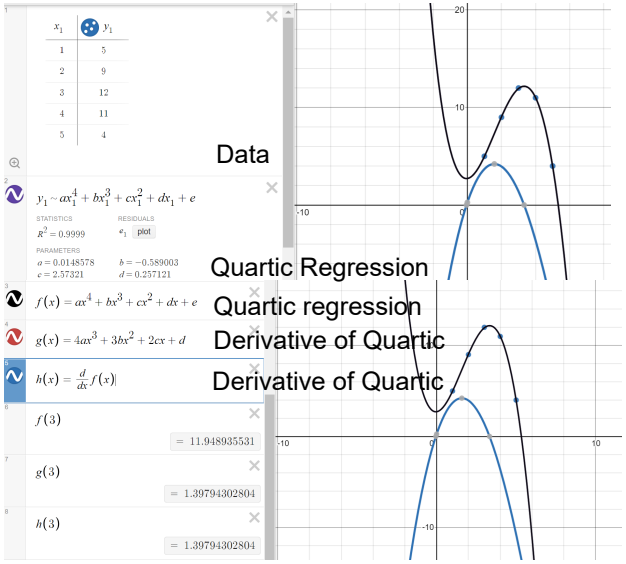
If I raise the price by one dollar, I'll sell 4.5 items.


y= vars 5 > > 1
NORMAL FLOAT AUTO a+bi RADIAN MP
L1 L2 L3 L4 L5 2
62 115 ------ ------ ------
65 130
68 175
70 180
72 190
------ ------
EDIT CALC TESTS
3↑Med-Med
4:LinReg(ax+b)
ExpReg
y=a*b^x
a=3.987667846
b=1.055807876
r²=.9386182291
r=.9688231155
Plot1 Plot2 Plot3
Y1=211*1.0558078763476^X
Y1=3.9876678461211*1.055
Y2=d/dX(Y1)|X=X
math 1
Y3=
Y4=
X Y1 Y2
70 178.51 9.6942
65 136.06 7.389
72 198.99 10.806
X=



at 143 horsepower

DESMOS



x=3	f(3) = 11.948..	f'(3) = 1.39...	

Reminders....

1. Go to Blackboard
2. Post your Selfie on Discussion Forum
3. Set up Connect by opening an assignment

You must complete a Connect assignment NOW