

## Agenda

Homework Review

Lecture: Continuity

Classwork

## Homework:

### Limits at infinity and rational functions

Find the following limits.

If necessary, select the most informative answer from  $\infty$ ,  $-\infty$ , and "Does Not Exist."

$$(a) \lim_{x \rightarrow -\infty} \frac{4x^3 - 7}{-2 - 3x^2} =$$

$$(b) \lim_{x \rightarrow \infty} \frac{7x + 1}{4 - 5x} =$$

X	Y1
100	-133.3
-100	133.32
-9999	13332
-1E10	1.3E10

### Limits at infinity and rational functions

Find the following limits.

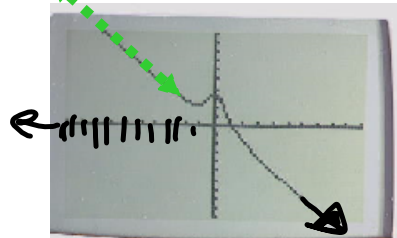
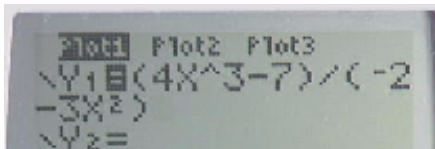
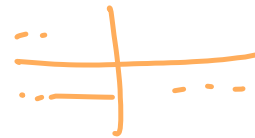
If necessary, select the most informative answer from  $\infty$ ,  $-\infty$ , and "Does Not Exist."

LEFT  
END

$$(a) \lim_{x \rightarrow -\infty} \frac{4x^3 - 7}{-2 - 3x^2} = \infty$$

RIGHT  
END

$$(b) \lim_{x \rightarrow \infty} \frac{7x + 1}{4 - 5x} = -\frac{7}{5}$$



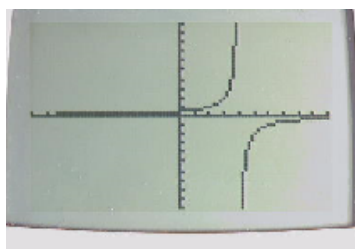
X	Y1
100	-133.3
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-9999	13332
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### Infinite limits and rational functions

Find the following limits for  $f(x) = \frac{2}{4-x}$  and  $g(x) = \frac{-4x}{(x-2)^2}$ .

If necessary, select the most informative answer from  $\infty$ ,  $-\infty$ , and "Does Not Exist."

$\lim_{x \rightarrow 4^-} f(x) = \square$	$\lim_{x \rightarrow 2^-} g(x) = \square$
$\lim_{x \rightarrow 4^+} f(x) = \square$	$\lim_{x \rightarrow 2^+} g(x) = \square$
$\lim_{x \rightarrow 4} f(x) = \square$	$\lim_{x \rightarrow 2} g(x) = \square$



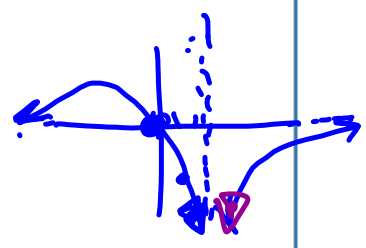
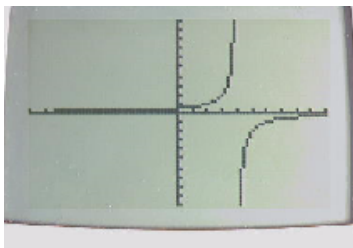
### Infinite limits and rational functions

Find the following limits for  $f(x) = \frac{2}{4-x}$  and  $g(x) = \frac{-4x}{(x-2)^2}$

If necessary, select the most informative answer from  $\infty$ ,  $-\infty$ , and "Does Not Exist."

Left  
Right  
Both

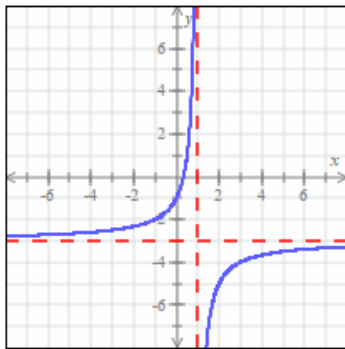
$\lim_{x \rightarrow 4^-} f(x) = \infty$	$\lim_{x \rightarrow 2^-} g(x) = -\infty$
$\lim_{x \rightarrow 4^+} f(x) = -\infty$	$\lim_{x \rightarrow 2^+} g(x) = -\infty$
$\lim_{x \rightarrow 4} f(x) = \text{DNE}$	$\lim_{x \rightarrow 2} g(x) = -\infty$



### Limits at infinity and graphs

Use the graph of  $f(x) = \frac{1 - 3x}{x - 1}$  and its asymptotes to find the following limits.

If necessary, choose the most informative answer from  $\infty$ ,  $-\infty$ , and "Does Not Exist."



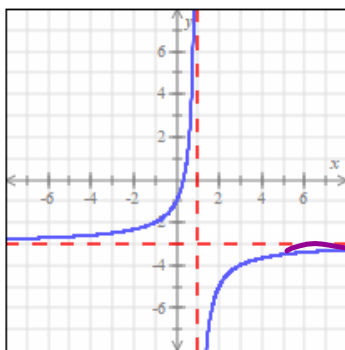
(a)  $\lim_{x \rightarrow \infty} f(x) = \square$

(b)  $\lim_{x \rightarrow -\infty} f(x) = \square$

### Limits at infinity and graphs

Use the graph of  $f(x) = \frac{1-3x}{x-1}$  and its asymptotes to find the following limits.

If necessary, choose the most informative answer from  $\infty$ ,  $-\infty$ , and "Does Not Exist."



$$\lim_{x \rightarrow \infty^+} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow -\infty^-} g(x) = \text{DNE}$$

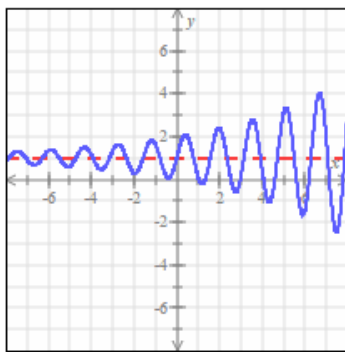
Right  
End

$$(a) \lim_{x \rightarrow \infty} f(x) = \boxed{-3}$$

$$(b) \lim_{x \rightarrow -\infty} f(x) = \boxed{-3}$$

Use the graph of  $h(x) = \sin(4x)e^{-0.25x} + 1$  and its asymptote to find the following limits.

If necessary, choose the most informative answer from  $\infty$ ,  $-\infty$ , and "Does Not Exist."



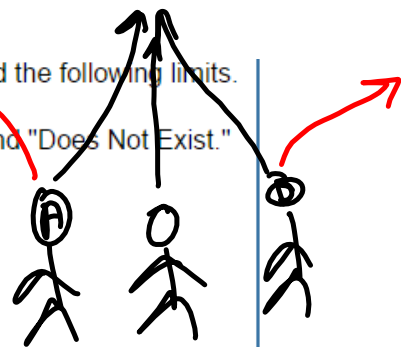
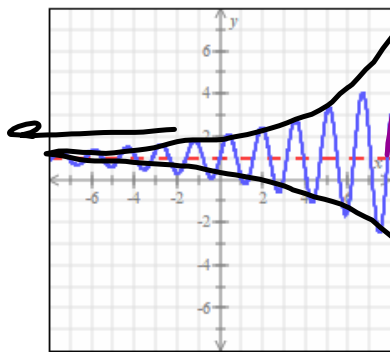
(a)  $\lim_{x \rightarrow \infty} h(x) = \square$

(b)  $\lim_{x \rightarrow -\infty} h(x) = \square$



Use the graph of  $h(x) = \sin(4x)e^{-x} + 1$ , and its asymptote to find the following limits.

If necessary, choose the most informative answer from  $\infty$ ,  $-\infty$ , and "Does Not Exist."



## SQUEEZE Theorem

(a)  $\lim_{x \rightarrow \infty} h(x) = \boxed{\text{DNE}}$

(b)  $\lim_{x \rightarrow -\infty} h(x) = \boxed{1}$

$$-e^{-x} \leq \sin(4x)e^{-x} \leq e^{-x}$$

$\lim_{x \rightarrow -\infty} -e^{-x} = \infty$        $\lim_{x \rightarrow -\infty} e^{-x} = \infty$   
 $\Rightarrow$  by  $\infty/\infty$  squeeze.

# Lecture: Continuity

Overview:

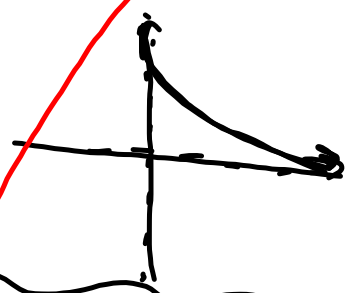
What is Calculus?

What is zero times infinity?

Zero \* Infinity } = could be anything?

$$\lim_{x \rightarrow \infty} x = \infty$$

$$\lim_{x \rightarrow \infty} \frac{B}{x} = 0$$



$$\lim_{x \rightarrow \infty} \cancel{x} \cdot \frac{B}{\cancel{x}} = B$$

$$\lim_{x \rightarrow \infty} x^2 \cdot \frac{B}{x^3} = 0$$

$$\lim_{x \rightarrow \infty} x^4 \cdot \frac{B}{x^2} = \infty$$

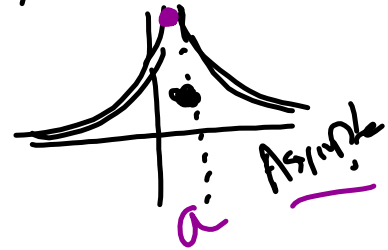
## CONTINUITY

1. LIMIT EXISTS

2. FUNCTION EXISTS

3. LIMIT = FUNCTION

Examples



CONTINUITY

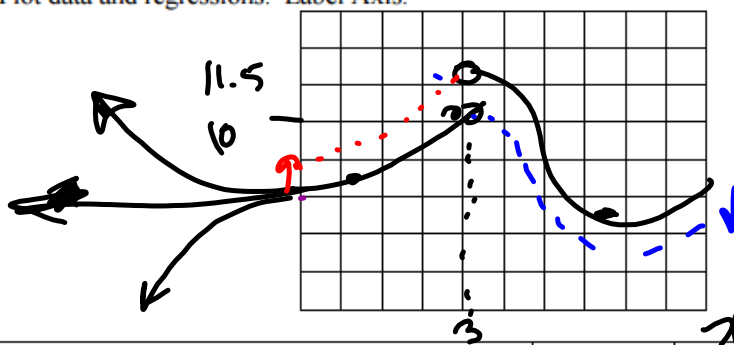
1. LIMIT EXISTS

2. FUNCTION EXISTS

3. LIMIT = FUNCTION

## Inner Limit

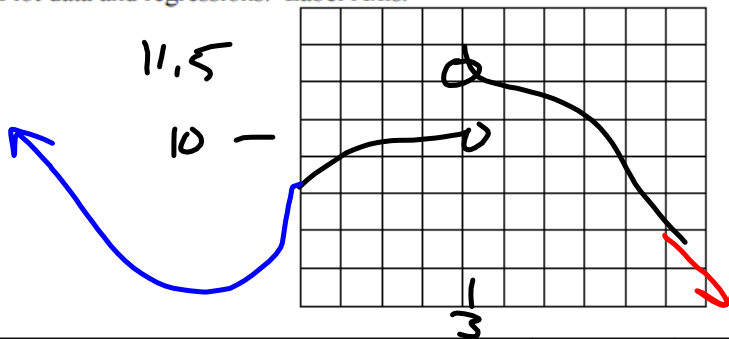
2. Roughly split the graph into two regions and perform different regressions on each side.  
Plot data and regressions. Label Axis.



left regression split at a $Y1 = \text{vars } 5: >> 1: \text{RegEq } /(x \leq a)$	Left Regression used:	Cubic $y = -3.$
right regression $Y2 = \text{vars } 5: >> 1: \text{RegEq } /(x \geq a)$	Right Regression used:	Quartic $y = -2.$
	Location of split (a)	3
Find $Y1(a)$ $Y2(a)$	$\lim_{x \rightarrow a^-} r(x)$	10
$\begin{array}{c c c} & Y_1 & Y_2 \\ \hline 3 & 10 & 11.5 \end{array}$	$\lim_{x \rightarrow a^+} r(x)$	11.5

# Outer Limit

3. Roughly split the graph into two regions and perform different regressions on each side.  
Plot data and regressions. Label Axis.



left regression split at a Y1=vars 5: >> 1: RegEq /(x≤a)	Left Regression used:	Cubic $y = \ominus$
right regression Y2=vars 5: >> 1: RegEq /(x≥a)	Right Regression used:	Quartic $y = \ominus$
Find Y1(-9999)	$\lim_{x \rightarrow -\infty} r(x)$	$+\infty$
Y2(9999)	$\lim_{x \rightarrow +\infty} r(x)$	$-\infty$

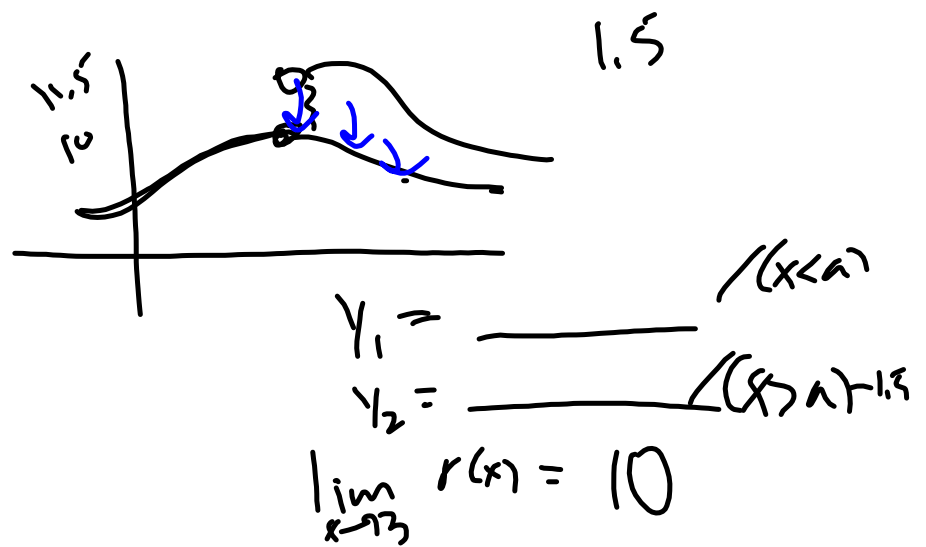
	$Y_1$	$Y_2$
9999	par	-19299
-9999	-1...	Error

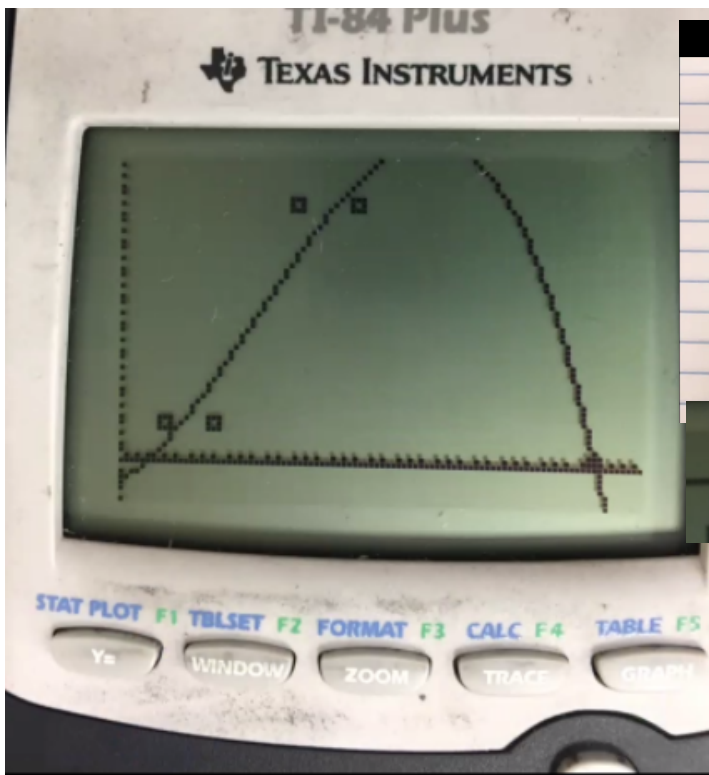
$\nearrow$  D<sub>50</sub>  
 $\nwarrow$  Left  
 $\nearrow$  Right



Classwork:

Make the function continuous





To make the function (c) continuous, add 1.532 to  $y_1$

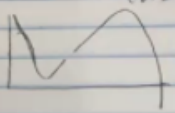
$$y_1 = 0.00172 \dots / (x \leq 22) + 1.532$$

$$y_2 = -0.009972 \dots (x > 22)$$

X	Y1	Y2
22	13.576	13.576

$y_1 = \text{quartic} \quad / \quad (x \leq 22)$   
 $y_2 = \text{cubic} \quad / \quad (x \geq 22)$

$y_1 = 0.0017x^4 - 0.22x^3 + 9.15x^2 - 165.92x + 1033.56 \quad / \quad (x \leq 22)$   
 $y_2 = -0.009972x^3 \dots \quad / \quad (x \geq 22)$

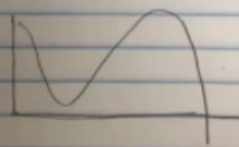
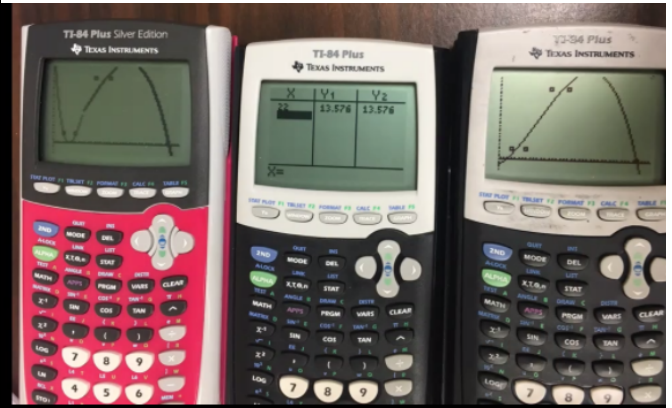


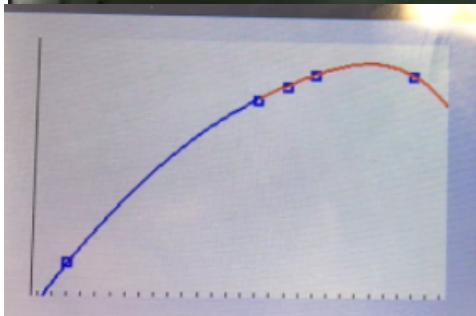
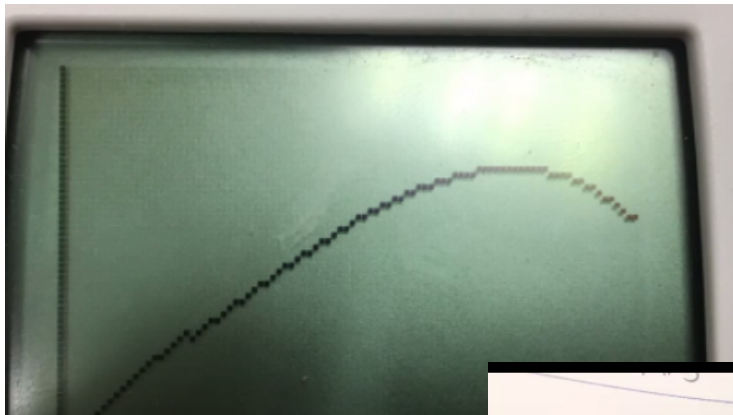
x	y <sub>1</sub>	y <sub>2</sub>
22	12.089	13.576

$\rightarrow 13.576 - 12.089 = 1.532$

To make the function (c) continuous, add 1.532 to y<sub>1</sub>

$y_1 = 0.0017x^4 \dots / (x \leq 22) + 1.532$   
 $y_2 = -0.009972x^3 \dots (x \geq 22)$



y value.

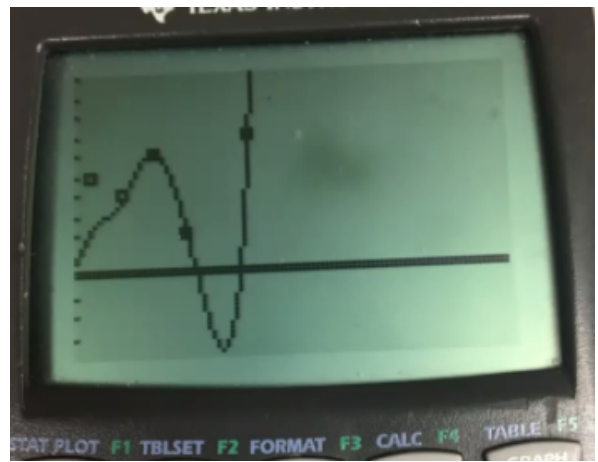
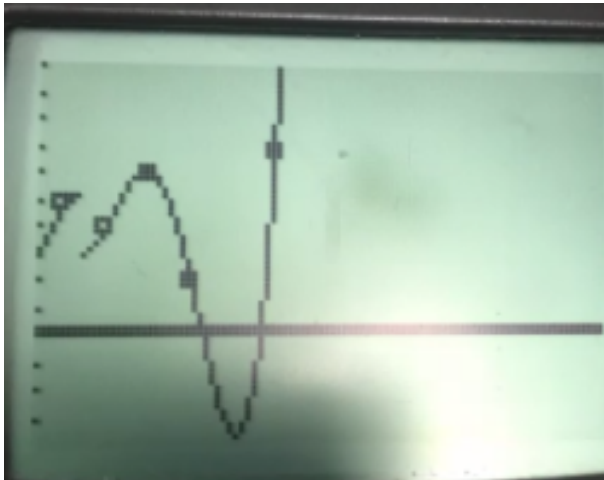
$$Y' = -3 \dots$$

Dwad

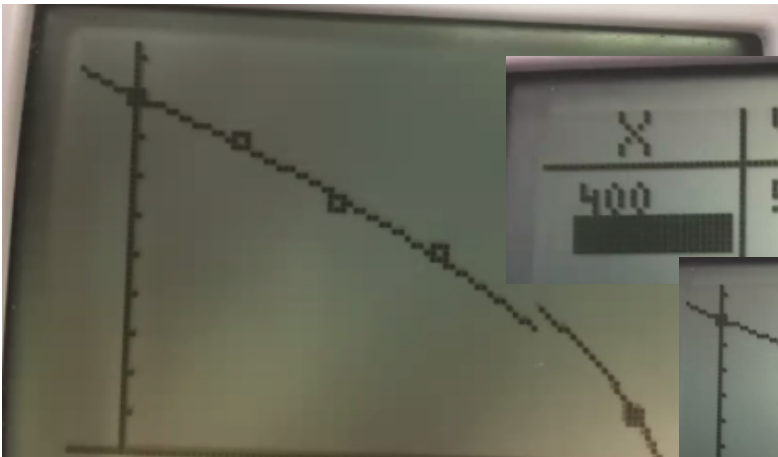
$$Y^2 = -0.01 \dots + 14.5$$

Dwad

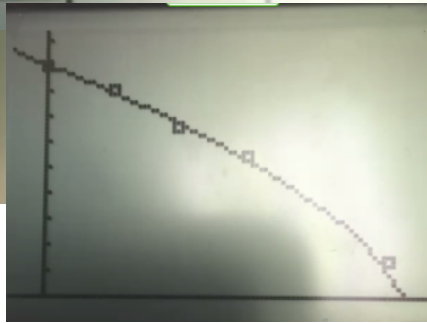
)  
)

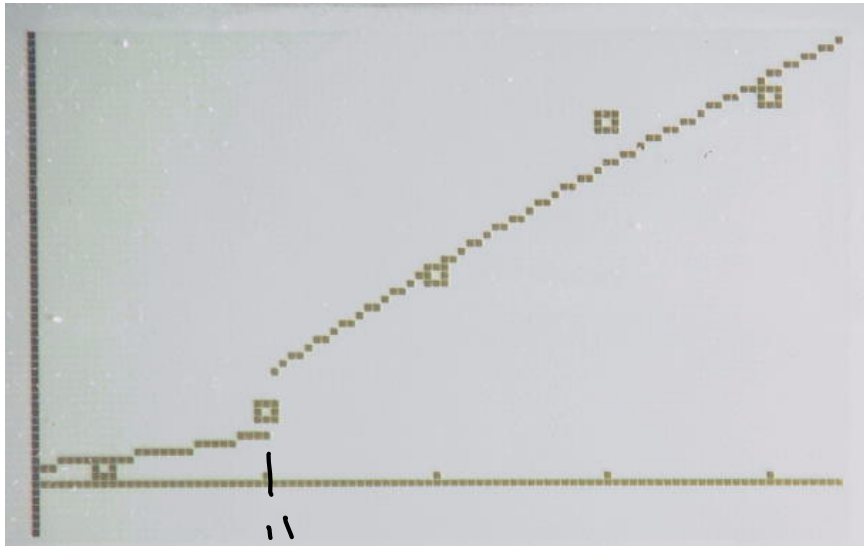


According to the split regressions,  
as we approach 250,000 in  
property costs, the number of  
available homes is 5.3402



X	Y <sub>1</sub>	Y <sub>2</sub>
400	99.43	99.43



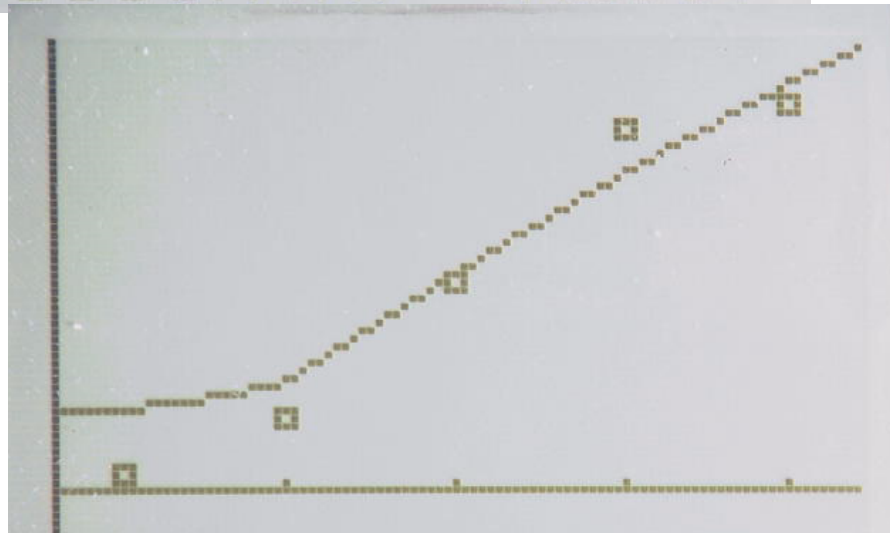


$$Y_2(11) - Y_1(11) = 14574.46\dots$$

```

Plot2 Plot3
\Y1# .96129924517
059*2.3676061244
206^X/(X<=11)+145
74.46729
\Y2# -717189.1888
8321+310422.4497
75831n(X)/(X>=11)

```



What is  $f(11) =$ 

10	19895	ERROR
11	22121	22121
12	ERROR	54102



Team Mathletes

Ami Sanders

MAT 151

10/26/14

$Y_2(5) - Y_1(5)$

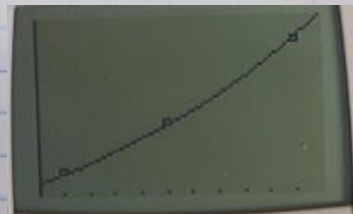
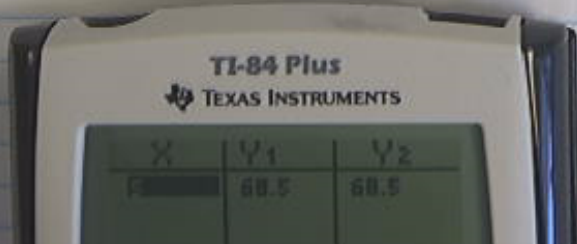
M-VARS  $\rightarrow$  Y-VARS  $\rightarrow$  1:Function  $\rightarrow$   $Y_2$   $Y_2(5) - Y_1(5) = -1.499797018$

-1.499797018  $\rightarrow$  Subtract from  $Y_1$  to make function continuous.

AB25

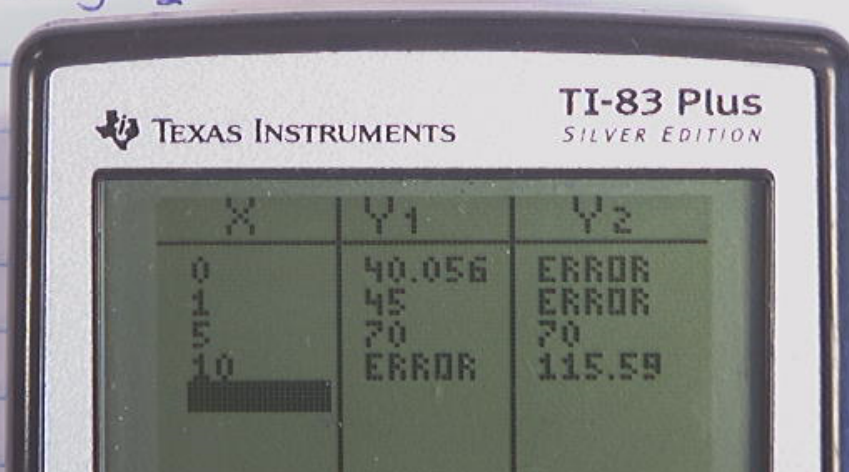
At exactly 5 years of employment as an FBI special agent,  
an FBI agent would make \$68,500.00.

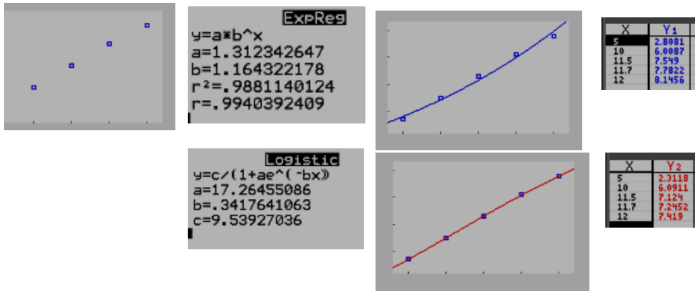
\* Subtract from  $Y_1$  or adding  $Y_2$



5 years of employment as an FBI special  
t would make \$68,500.00.

$Y_1$  or adding  $Y_2$





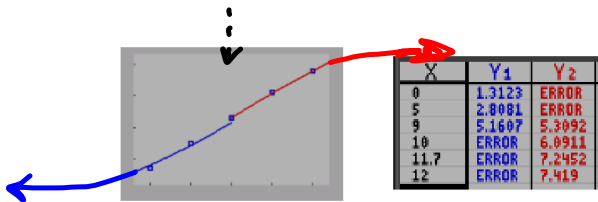
Inner

$$\lim_{x \rightarrow 9^-} y_1 = 5.16$$

$$\lim_{x \rightarrow 9^+} y_2 = 5.30 \dots$$

decade v world population

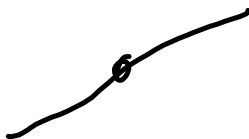
### Split Regressions



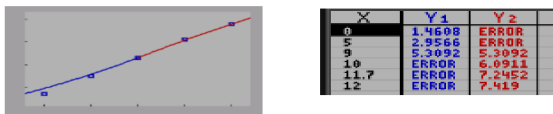
Outer

$$\lim_{x \rightarrow \infty} y_2 = 9.53 \dots$$

$$\lim_{x \rightarrow -\infty} y_1 = 0$$



### Limits of Continuous Functions



Continuous

$$\lim_{x \rightarrow 9} S(x) = 5.30$$

According to the continuous regression above, as we approach 1990, we expect world population to be 5.3 billion