

## Agenda

Homework Review

Lecture

Groupwork

## Question # 3 Q#1 ALKS

### Union and intersection of intervals

The sets  $D$  and  $E$  are defined as follows.

$$D = \{z \mid z \geq 4\}$$

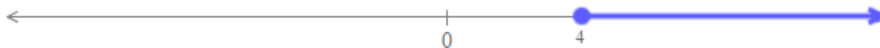
$$E = \{z \mid z < 8\}$$

Write  $D \cap E$  and  $D \cup E$  using interval notation.

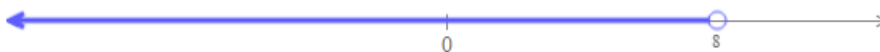
If the set is empty, write  $\emptyset$ .

" " intersection  $[4, 8)$   
 " " Union  $(-\infty, \infty)$

The set  $D$  is the interval  $[4, \infty)$ . It is graphed below.



The set  $E$  is the interval  $(-\infty, 8)$ . It is graphed below.



Q 6.

### Determining a parameter to make a function continuous

Find a value of  $k$  such that the following function is continuous at all real numbers.

$$h(x) = \begin{cases} kx^2 - 3 & \text{if } x < -1 \\ 2x + 3 & \text{if } x \geq -1 \end{cases}$$

If there is no such value  $k$ , click on "None."

$$\begin{aligned} \lim_{x \rightarrow -1^-} kx^2 - 3 &= k - 3 \\ \lim_{x \rightarrow -1^+} 2x + 3 &= 1 \\ k - 3 &= 1 \end{aligned}$$

For  $x < -1$ ,  $h$  is continuous because it acts as the quadratic function  $kx^2 - 3$ .

For  $x \geq -1$ ,  $h$  is continuous because it acts as the linear function  $2x + 3$ .

[More](#)

$$k = 4$$

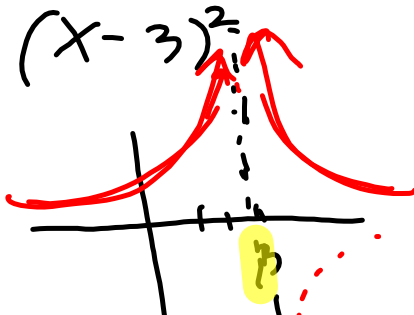
So, the only possible point of discontinuity for  $h$  is  $x = -1$ .


For  $h$  to be continuous at  $-1$ , we must have the following.

(1)  $h$  is defined at  $-1$

(2)  $\lim_{x \rightarrow -1} h(x)$  exists

(3)  $\lim_{x \rightarrow -1} h(x) = h(-1)$

$$\lim_{x \rightarrow 3} \frac{1}{(x-3)^2} = \infty$$


$$\lim_{x \rightarrow 3} \frac{1}{x-3} = \text{DNE}$$


Finding a limit by using the limit laws: Problem type 3

Find the following limit.

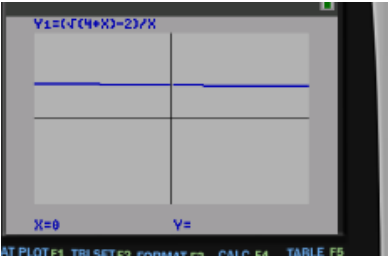
$$\lim_{h \rightarrow 0} \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} = \lim_{h \rightarrow 0} \frac{4+h-4}{h(\sqrt{4+h}+2)}$$

If the limit does not exist, click on "Does Not Exist."

$$\lim_{h \rightarrow 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{4}$$

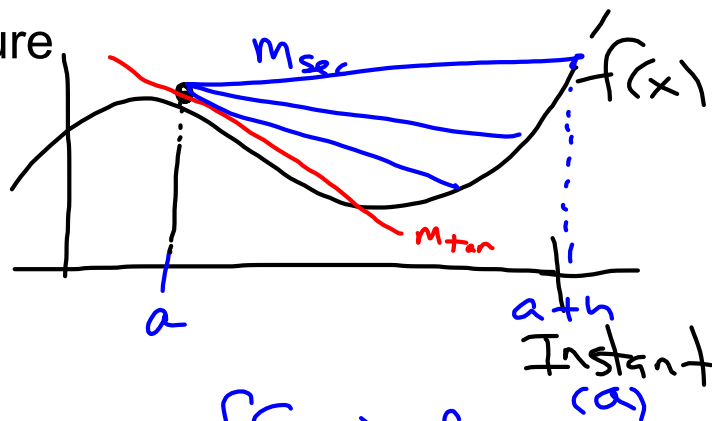
Plot1 Plot2 Plot3  
Y1=( $\sqrt{4+X}-2$ )/X

NORMAL FLOAT AUTO RE	
X	Y1
0	ERROR
.1	.24846
1E-4	.25
-1E-4	.25



## Lecture

Lecture



$$m_{sec} = \frac{f(a+h) - f(a)}{h}$$

$$m_{tan} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Ex  $f(x) = 2x^2 - 5x + 7$

x	Instant $\frac{f(x)-5}{x-5}$
0	-5 down
5/4	0 MIN
2	3 up

$m_{tan}$  at "a"  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$



$$f(a+h) = 2(a+h)^2 - 5(a+h) + 7$$

$$f(a+h) = 2a^2 + 4ah + 2h^2 - 5a - 5h + 7$$

$$f(a) = 2a^2 - 5a + 7$$

dy:  $\frac{f(a+h) - f(a)}{h} = \frac{4ah + 2h^2 - 5h}{h}$

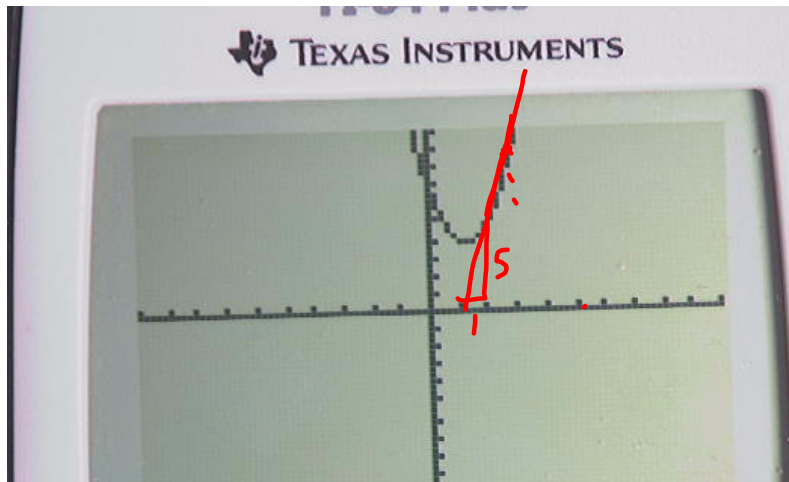
$$\frac{f(a+h) - f(a)}{h} = 4a + 2h - 5$$

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} 4a + 2h - 5 = 4a - 5$$



Plot2 Plot3  
Y1 = 2X<sup>2</sup> - 5X + 7

$$m_{\text{tangent}} = 4a - 5$$



at  $a = 3$   
7

at  $a = 1$   
 $4(1) - 5 = -1$

at  $a = 5/4$   
 $4(5/4) - 5 = 0$

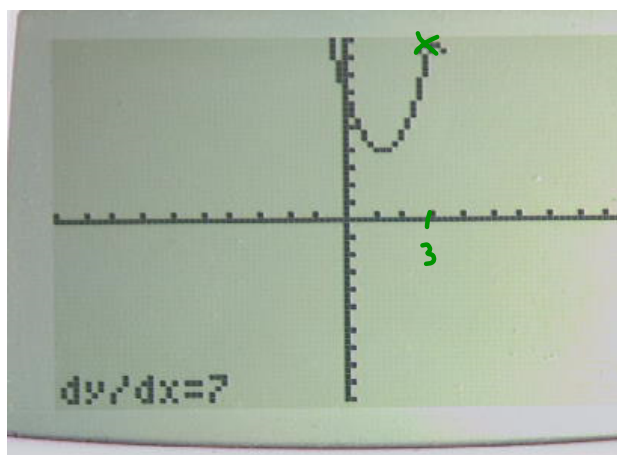
at  $a = 0$   
-5

slope of tangent line at  $x=3$  is about 5

1:value  
2:zero  
3:minimum  
4:maximum  
5:intersect  
6:dy/dx  
7:∫f(x)dx

← 2<sup>nd</sup> Trace

$x=3$



function  
derivative  
 $f'(x) = 4x - 5$

$f(x) = 2x^2 - 5x + 7$   
 $f'(a) = 4a - 5$

$f'(\underline{3}) = 4(3) - 5 = 7$

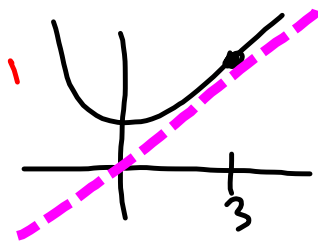
FUNCTIONS

Evaluated Function

**VARs** →

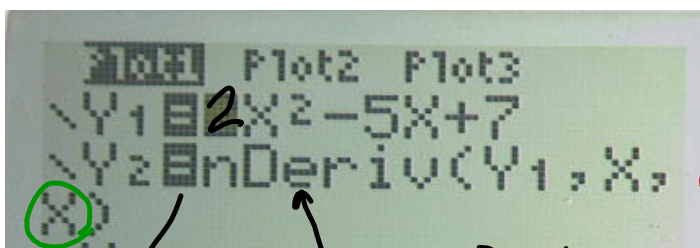
$\frac{d}{dx} \left( \boxed{Y_1} \right)$

$x = \boxed{3}$



Equation of  
Tangent  
Line

4x-5  
4(3)-5 = 7

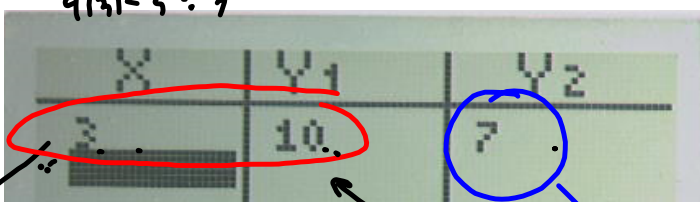


math 8:ndwn

POINT  
**(3,10)**

Slope  
**7**

$a=3$



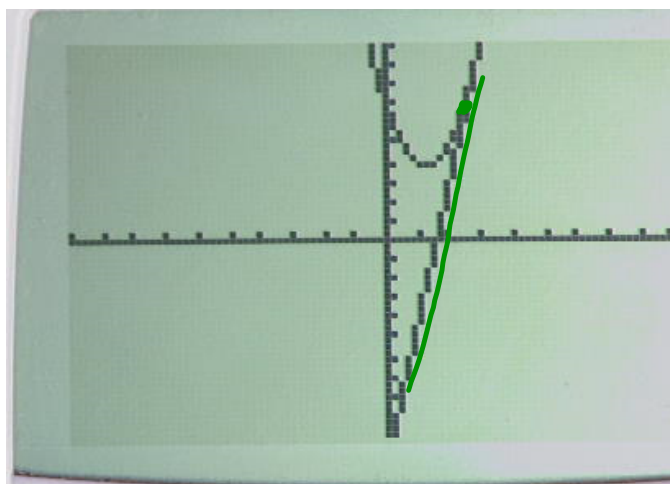
$Y(3)$

Slope of  
Tan. line

Equation of Tangent Line  $y - y_1 = m(x - x_1)$

$$y - 10 = 7(x - 3)$$

$$y = 7(x - 3) + 10$$



$$y_3 = 7(x - 3) + 10$$

Find  $f'(x)$ . Use the definition

$$f(x) = \frac{1}{x-3}$$

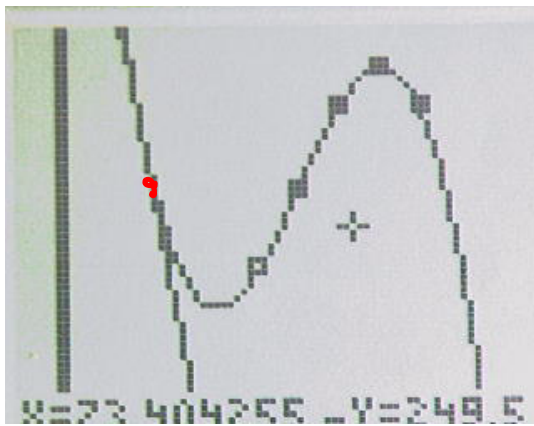
$$f(a+h) = \frac{1}{a+h-3}$$

$$f(a) = \frac{1}{a-3}$$

$$f(a+h) - f(a) = \frac{1}{a+h-3} - \frac{1}{a-3}$$

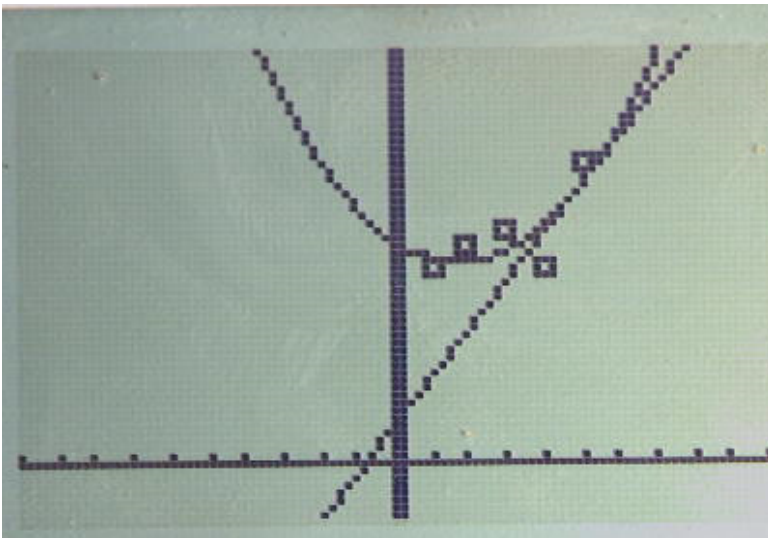
$$\frac{a-3 - (a+h-3)}{(a+h-3)(a-3)} = \frac{-h}{(a+h-3)(a-3)}$$

$$\lim_{h \rightarrow 0} \left[ \frac{-h}{(a+h-3)(a-3)} \right] = \lim_{h \rightarrow 0} \frac{-1}{(a+h-3)(a-3)}$$
$$= \frac{-1}{(a-3)^2}$$



Tangent  
Line

Equation  
+  
Graph  
←



Get regression  
Stat Calc

VARs 5 >> 1

$Y =$

$Y_1 = \text{Quand reg}$

$Y_2 = \text{ndew } (Y_1, X, X)$

Table

X	$Y_1$	$Y_2$
7	$y_{\text{ave}}$	$m$

$$Y_3 = m(X - 7) + y_{\text{ave}}$$

## Equation of the <sup>or</sup> Tangent Line

$$y = m(x-a) + y_0$$

$y_1 = \text{regression}$

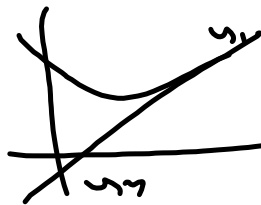
$$y_2 = \text{ndev}(y, X) \quad \text{or}$$

$$\left. \frac{d}{dx} [y] \right|_{x=[x]}$$

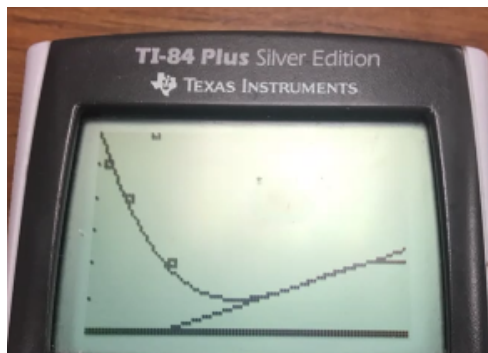
$$y_3 = \underline{m_1} (\underline{x-a}) + \underline{y_0}$$

table

$a$	$y_0$	$m_1$
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Group:



$$\begin{aligned} y_1 &= n \text{ deriv. } (y_1, x, x) \\ y_2 &= \text{Calculate (from table)} \\ &= m_1 (x - a) + y_0 \\ x &= 1000 \\ y_1 &= 1.2857 \\ y_2 &= -0.00236 \text{ (slope)} \\ &= -0.00236(x - 1000) + 1.2857 \end{aligned}$$



# business

Group Name: Rattlesnakes Speaker: Christina  
DATE: \_\_\_\_\_ Writer: Mattia Colombo  
x-axis (independent variable): \_\_\_\_\_ Team Leader: Bill Burrows  
y-axis (dependent variable): \_\_\_\_\_

Conclusion in words:  
In 2007, the amount of people below the poverty line was increasing by 1.5 million. In 2008, after the housing market crash, the rate of the amount of people going into poverty more than doubled to 3.4 million

3. Roughly split the graph into  
Plot data and regressions.

TI-84 Plus  
TEXAS INSTRUMENTS

X	Y1	Y2
7	37.28	1.5
8	38.88	3.4
9	40.48	5.3
10	42.08	7.2
11	43.68	9.1
12	45.28	11.0
13	46.88	12.9
14	48.48	14.8
15	50.08	16.7
16	51.68	18.6
17	53.28	20.5
18	54.88	22.4
19	56.48	24.3
20	58.08	26.2
21	59.68	28.1
22	61.28	30.0
23	62.88	31.9
24	64.48	33.8
25	66.08	35.7
26	67.68	37.6
27	69.28	39.5
28	70.88	41.4
29	72.48	43.3
30	74.08	45.2
31	75.68	47.1
32	77.28	49.0
33	78.88	50.9
34	80.48	52.8
35	82.08	54.7
36	83.68	56.6
37	85.28	58.5
38	86.88	60.4
39	88.48	62.3
40	90.08	64.2
41	91.68	66.1
42	93.28	68.0
43	94.88	69.9
44	96.48	71.8
45	98.08	73.7
46	99.68	75.6
47	101.28	77.5
48	102.88	79.4
49	104.48	81.3
50	106.08	83.2
51	107.68	85.1
52	109.28	87.0
53	110.88	88.9
54	112.48	90.8
55	114.08	92.7
56	115.68	94.6
57	117.28	96.5
58	118.88	98.4
59	120.48	100.3
60	122.08	102.2
61	123.68	104.1
62	125.28	106.0
63	126.88	107.9
64	128.48	109.8
65	130.08	111.7
66	131.68	113.6
67	133.28	115.5
68	134.88	117.4
69	136.48	119.3
70	138.08	121.2
71	139.68	123.1
72	141.28	125.0
73	142.88	126.9
74	144.48	128.8
75	146.08	130.7
76	147.68	132.6
77	149.28	134.5
78	150.88	136.4
79	152.48	138.3
80	154.08	140.2
81	155.68	142.1
82	157.28	144.0
83	158.88	145.9
84	160.48	147.8
85	162.08	149.7
86	163.68	151.6
87	165.28	153.5
88	166.88	155.4
89	168.48	157.3
90	170.08	159.2
91	171.68	161.1
92	173.28	163.0
93	174.88	164.9
94	176.48	166.8
95	178.08	168.7
96	179.68	170.6
97	181.28	172.5
98	182.88	174.4
99	184.48	176.3
100	186.08	178.2

X=16

DATE: 1/26/16

x-axis (independent variable): year

y-axis (dependent variable): New Jersey population

Writer: Mariam Struh

Team Leader: Yvette

Conclusion in words:

X	y
1950	4,835,329
1960	6,065,782
1970	7,171,112
1980	7,363,011
1990	7,730,188

The population of NJ is steadily increasing over the years. Based on the following points we have determined that the population in NJ in 2020 will be  $5.84 \times 10^6$  with a change of  $-1.2 \times 10^5$ . This indicates, according to the quadratic regression, the population will decrease

people  
per year

TI-84 Plus  
TEXAS INSTRUMENTS

X	Y1	Y2
129	5.85E6	-1.2E5
159	8.34E6	-2.4E5
190	-1.2E5	-5.8E5
219	-2E5	-4.8E5
B	-9.8E6	304051

X=170

perform different regressions on each side. Substantially in the years to come

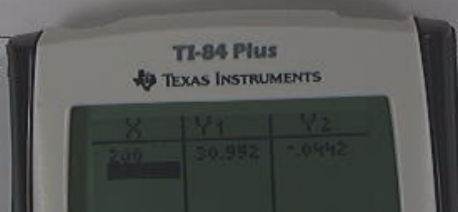


y-axis (dependant variable): \_\_\_\_\_

Conclusion in words: When the horsepower is 200, the gas consumption reduces to  $\approx 31$  mpg. The instantaneous rate of gas consumption reduces to  $-0.044$  mpg per horsepower.

3. Roughly split  
Plot data and

ions on each side.



Group Name: SPP

Speaker: Cierra

DATE: 1/26/16

Writer: Ranika

x-axis (independent variable): Yrs

Team Leader: Shiv

y-axis (dependent variable): \$

Conclusion in words:

In 2017, the rate of change in salary increases to \$146.5 thousand. The instantaneous rate of an increase in salary is \$25.5 thousand/year which

3. Roughly split the  
Plot data and re;

ons on each side.

TI-84 Plus CE

	NORMAL	Float	AUTO	REAL	RADIAN	MP
X	Y1	Y2				
17	146.48	25.519				
18	124.36	20.378				
19	207.54	36.157				

X=17

plot plot f1 tblset f2 format f3 - calc f4 - table f5

## Groupwork

# Limits

$$\lim_{x \rightarrow a} f(x)$$

Try first  $f(a) = ??$

Ex  $\lim_{x \rightarrow 3} 2x^2 - 5x + 7 = 10$

$$f(3) = 10$$

Ex  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} =$

$$f(2) = \frac{0}{0}$$

$$= \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{\cancel{(x-2)}}$$

$$= \lim_{x \rightarrow 2} x + 2 = 4$$

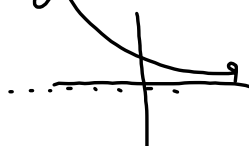
Ex  $\lim_{x \rightarrow \infty} \frac{3x^2 - 8}{5x^2 + 7} = \frac{3}{5}$

$$\frac{DU=2}{DD=2}$$

Right End Behavior.



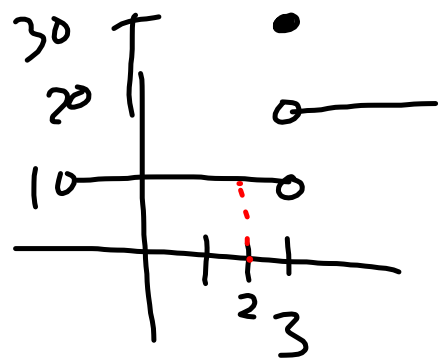
Ex  $\lim_{x \rightarrow \infty} e^{-x} = 0$



Ex

$$\lim_{x \rightarrow 2} f(x) = 10$$

$$\lim_{x \rightarrow 3} f(x) = \begin{matrix} \text{Does} \\ \text{not} \\ \text{exist} \\ \hline \hline \end{matrix}$$



$$f(3) = 30$$

$$f(2.9999) = 10$$

$$f(3.0001) = 20$$

## One Sided Limit.

$$\lim_{x \rightarrow a^+} f(x) = \text{Right Side Limit}$$

$$\lim_{x \rightarrow a^-} f(x) = \text{Left Sided Limit}$$



### Finding limits for a piecewise-defined function

The function  $g$  is defined piecewise as follows.

$$g(x) = \begin{cases} (x-1)^2 - 2 & \text{if } x < 2 \\ \frac{x-5}{x+1} & \text{if } x > 2 \end{cases}$$

Find the following limits.

If a limit does not exist, click on "Does Not Exist."

(a)  $\lim_{x \rightarrow 2^-} g(x)$

(b)  $\lim_{x \rightarrow 2^+} g(x)$

(c)  $\lim_{x \rightarrow 2} g(x)$

### Finding limits for a piecewise-defined function

The function  $g$  is defined piecewise as follows.

$$g(x) = \begin{cases} (x-1)^2 - 2 & \text{if } x < 2 \\ \frac{x-5}{x+1} & \text{if } x > 2 \end{cases}$$

Left  $(2-1)^2 - 2 = 1 - 2 = -1$   
Right  $\frac{2-5}{2+1} = \frac{-3}{3} = -1$

Find the following limits.

If a limit does not exist, click on "Does Not Exist."

(a)  $\lim_{x \rightarrow 2^-} g(x)$

Left end

(b)  $\lim_{x \rightarrow 2^+} g(x)$

Right end

(c)  $\lim_{x \rightarrow 2} g(x)$

$= -1$

---

$$\lim_{x \rightarrow a^+} r_1(x) = \#$$

$$\lim_{x \rightarrow a^-} r_2(x) = \#$$

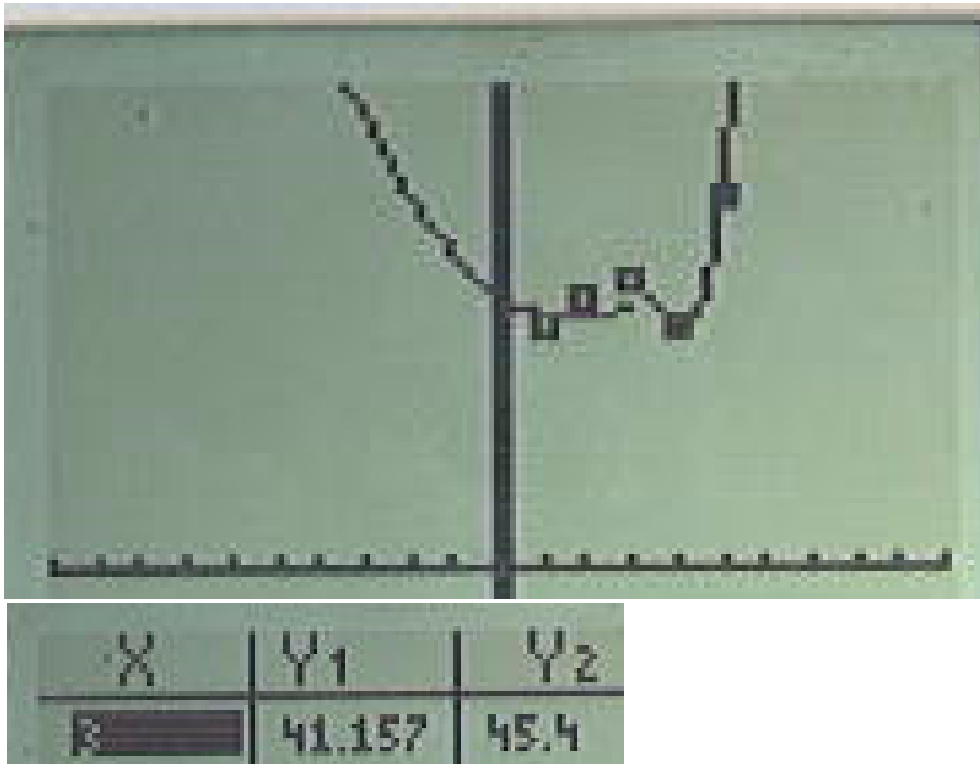
$$\lim_{x \rightarrow \infty} r_1(x) = \infty?$$

$$\lim_{x \rightarrow \infty} r_2(x) = \infty?$$

TEXAS INSTRUMENTS TI-83 Plus

X	Y <sub>1</sub>	Y <sub>2</sub>
25	434.38	368.13





As progress to the third quarter, revue is expected \$41.157 million

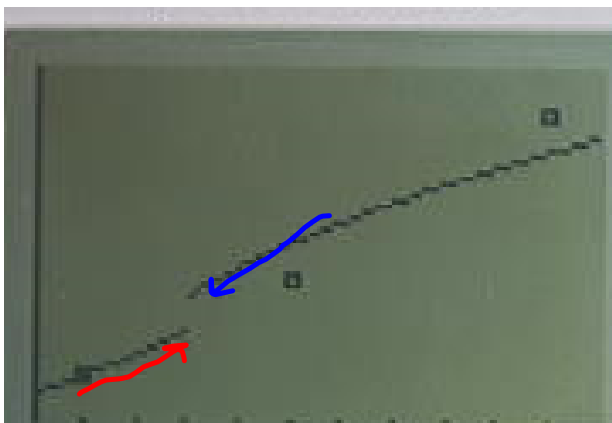
$$\lim_{x \rightarrow 3^-} Y_1(x) = 41.157$$

$$\lim_{x \rightarrow 3^+} Y_2 = 45.4$$

As we look bavk to the third quarter, revenue is expected to be 45.4 mil

As we approach 75 years of age, there will be  $\approx 434.38$  thousand cases of breast cancer.

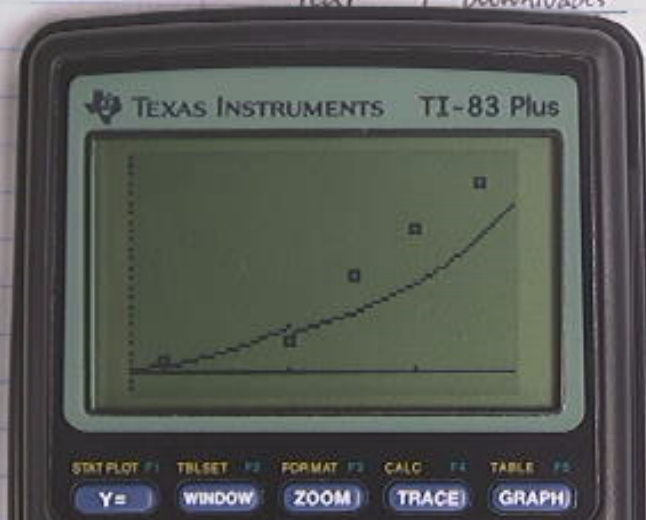
$x$	$y_1$	$y_2$
3	66.456	65.579



$$\lim_{x \rightarrow 3^+} y_2$$

## App Downloads

Year | downloads



$$\lim_{x \rightarrow 11^-} y = 4.8$$

$$\lim_{x \rightarrow 11^+} y = 4.0$$

In the year 2011, the app downloads is expected 4.8 billion.

As we look back to 2011, the app downloads is expected 4.0 billion.

