Agenda

Review of Midterm

Project Discussion: topics

and area under curve calculated by summation

Preview Summation Notation

Midterm Review

Evaluate $\lim\limits_{x\to\infty}\left(\sqrt{25x^2+5}-5x\right)$.

$$\lim_{x\to\infty}\left(\sqrt{25x^2+5}-5x\right)\frac{\left(\sqrt{25x^2+5}+5x\right)}{\left(\sqrt{25x^2+5}+5x\right)}$$

$$\lim_{x\to\infty}\frac{\cancel{25x^2}+5-\cancel{25x^2}}{\underbrace{\sqrt{25x^2+5}}_{\infty}\underbrace{+5x}_{+\infty}}=\frac{5}{\infty}=0$$

NORMAL FLOAT AUTO REAL DEGREE CL

X	Y_1			
1	.47723			
100	.005			
1000	5E-4			
1E7	0			

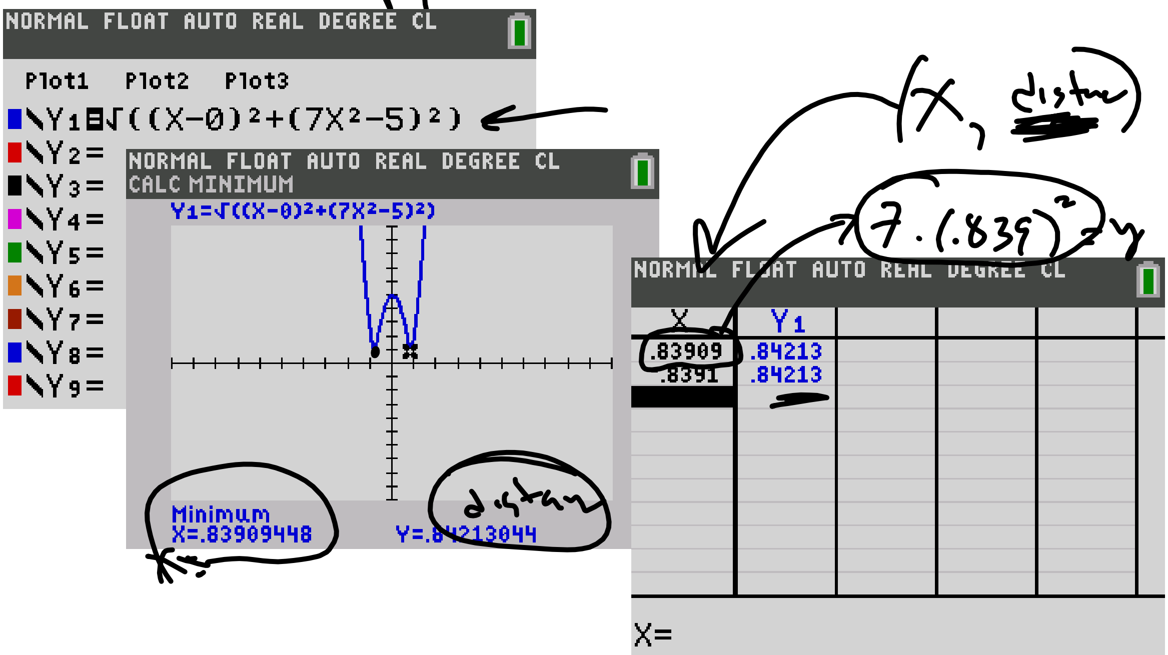
X=

Find the points on the curve $y = 7x^2$ closest to the point (0, 5).

$(x, 7x^2)$

($-.83909$, 4.92855) and ($.83909$, 4.92855)

$$d = \sqrt{(x-0)^2 + (7x^2-5)^2}$$



A company's revenue for selling x (thousand) items is given by $R(x) = \dfrac{360x - x^2}{x^2 + 360}$.

Find the value of x that maximizes the revenue and find the maximum revenue.

$x =$ ☐ , maximum revenue is \$ ☐

$$y' = \frac{(x^2+360)(360-2x) - (360x - x^2)(2x)}{\cancel{(x^2+360)^2}} = 0$$

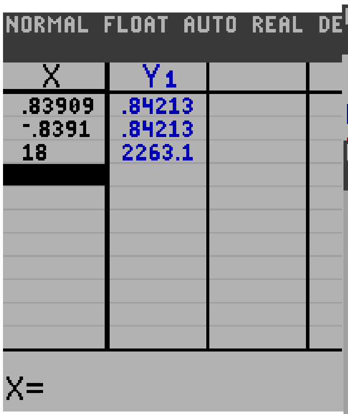
$$360x^2 + 360^2 - \cancel{2x^3} - 720x - 720x^2 + \cancel{2x^3}$$

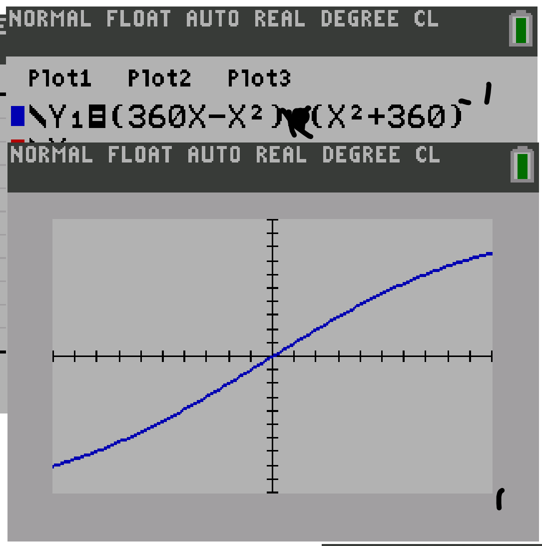
$$-360x^2 - 720x + 360^2 = 0$$

$$x^2 + 2x - 360 = 0$$

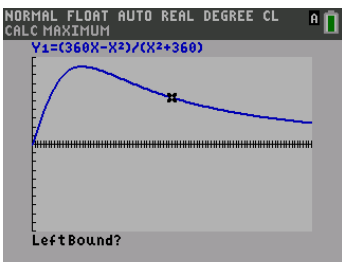
$$x = \frac{-2 \pm \sqrt{4 + 4(360)}}{2} = 18$$

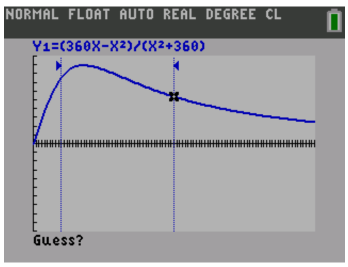
$$y = \frac{360x - x^2}{x^2+360} = 2263$$

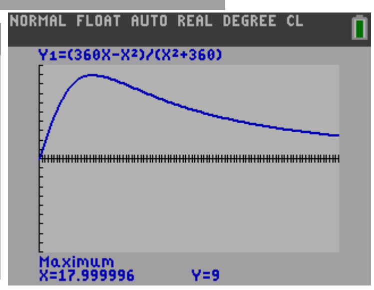




calc : 4 Max

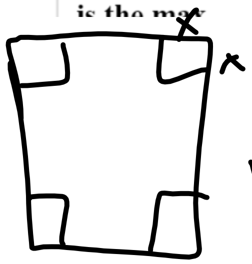






A sheet of paper 72 cm-by-78 cm is made into an open box (i.e. there's no top), by cutting x-cm squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box. Give your answer in the simplified radical form.

$x =$ ☐ is the max



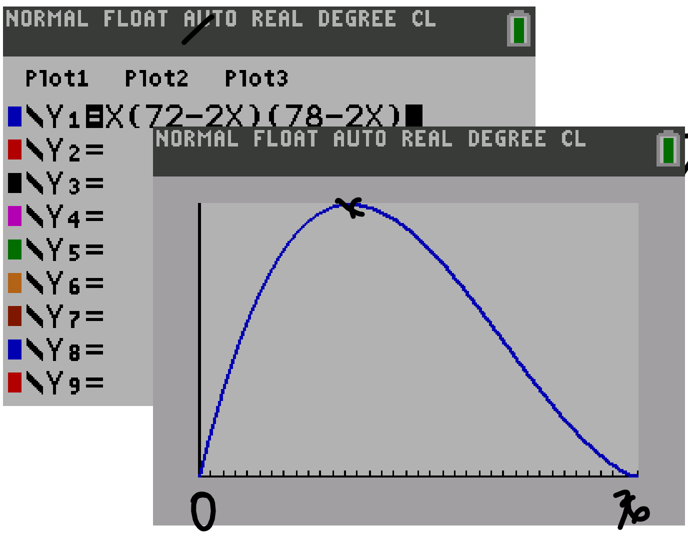
$W = 72 - 2x$

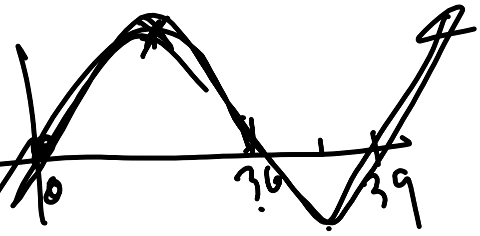
$L = 78 - 2x$

$H = x$

$y = x(72 - 2x)(78 - 2x)$

zeros: $\underline{0}$ $\underline{36}$ $\underline{39}$





Xmin: 0

Xmax: 36

Zoom 0: fit

$25 - \sqrt{157}$

A sheet of paper ~~72~~ 64 cm-by-~~78~~ 66 cm is made into an open box (i.e. there's no top), by cutting *x*-cm squar out of each corner and folding up the sides. Find the value of *x* that maximizes the volume of the box. Give your answer in the simplified radical form.

$$y = x(72-2x)(78-2x)$$

$$y = 5616x - 150x^2 + x^3$$

$$y' = 3x^2 - 300x + 5616 = 0$$

$$\frac{65}{3} =$$

$$3\overline{)65} = 21\tfrac{2}{3}$$

t $x = \frac{65}{3} - \frac{\sqrt{1057}}{3}$

$$21\tfrac{2}{3} - \frac{\sqrt{1057}}{3}$$

Find the derivative of $f(x) = 4 \ln x^{10}$. $= 40 \ln x$

$4 \cdot \frac{1}{x^{10}} \cdot \frac{d}{dx}(x^{10})$

$\frac{4 \cdot 1 \cdot 10x^9}{x^{10}} = \frac{40}{x}$

Determine values of a and b that make the given function continuous.

$$f(x) = \begin{cases} \dfrac{15\sin x}{x} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ b\cos x & \text{if } x > 0 \end{cases}$$

$$\lim_{x \to 0^-} \frac{15\sin x}{x} = \frac{0}{0}$$

L'HR

$$\lim_{x \to 0^-} \frac{15\cos x}{1} = 15$$

$$\lim_{x \to 0^-} \frac{15\sin x}{x} = 15 \rightarrow a = 15$$

$$\lim_{x \to 0^+} b\cos x = b \cdot 1 = 15 \qquad b = 15$$

Find the derivative of the function.

$$f(x) = \frac{x^2}{7} + \frac{2}{\cancel{x^2}} \quad x^{-2} = \boxed{\frac{1}{7}x^2 + 2x^{-2}}$$

$$\frac{2}{7}x + (-4)x^{-3}$$

Compute the derivative of $f(x) = \dfrac{10}{\sqrt{6x^3 + 1}}$.

$$\left(\frac{10}{(6x^3+1)^{1/2}}\right)' = \frac{(6x^3+1)^{1/2}(0) - (10)\left(\frac{1}{2}(6x^3+1)^{-1/2}\right)(18x^2)}{6x^3+1}$$

$$10(6x^3+1)^{-1/2}$$

$$-5(6x^3+1)^{-3/2}(18x^2)$$

Find the derivative of the function $f(x) = 7e^{4x+3}$.

$7e^{4x+3}(4)$

$28e^{4x+3}$

Compute the derivative of $f(x) = 5\cos x^3$.

$= -5\sin(x^3)(3x^2)$

$5\cos^3 x$

$15\cos^2 x(-\sin x)$

Find the slope of the tangent line at the point $(2, 3)$ for the ellipse $7x^2 + 4y^2 = 64$.

$$14x + 8y\,dy/dx = 0$$

$$14(2) + 8(3)\,dy/dx = 0$$

$$\cancel{28} + \frac{24\,dy/dx}{24} = \frac{0 - 28}{24}$$

Find the derivative $y\,'(x)$ implicitly for the equation

$$\frac{5x+4}{y} = 2x + y^2. \Rightarrow \frac{d}{dx}\,5x+4 = \frac{d}{dx}\left(2xy + y^3\right)$$

$$5 = 2xy' + 2y + 3y^2y'$$

$$5 - 2y = 2xy' + 3y^2y'$$

$$5 - 2y = (2x + 3y^2)\,y'$$

$$\frac{5-2y}{2x+3y^2} = y'$$

Find a value of c satisfying the conclusion of the Mean Value Theorem. Write your answer in radical form.

$f(x) = x^3 + 3x^2$, [0, 4]

$f(4) = 4^3 + 3(4)^2 = 112$

$f(0) = 0$

$\frac{f(4) - f(0)}{4 - 0} = \frac{112}{4} = 28$

$f' = 3x^2 + 6x = 28$

$\frac{-6 \pm \sqrt{36 - 4(3)(-28)}}{6}$

2.21

$\frac{-6 - \sqrt{\ldots}}{6}$ = NEG

$\frac{-6 + \sqrt{\ldots}}{6}$ = POS?

[0, 4]

Find a value of c satisfying the conclusion of the Mean Value Theorem. Write your answer in radical form.

$f(x) = x^3 + 3x^2$, [0, 4]

$f'(x) = 3x^2 + 6x$

$\frac{f(4) - f(0)}{4 - 0} = \frac{64 + 3(16)}{4 - 0} = \frac{112}{4} = 28$

$3x^2 + 6x = 28$

on [0, 4]

$3x^2 + 6x - 28 = 0$

$\frac{-6 \pm \sqrt{36 - 4(3)(-28)}}{6}$

$y_1 = 3x^2 + 6x$

$y_2 = 28$

Calc 5: Intersect

Math 0: solver

$0 = 3x^2 + 6x - 28$

$x = 2$ guess

$x = 2.4 \ldots$

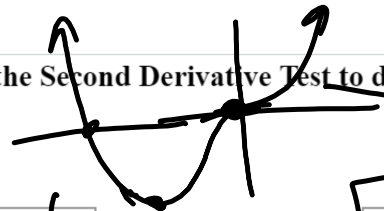
Find the limit $\lim_{x \to \infty} \frac{15x^3}{e^x}$. $= \frac{\infty}{\infty}$

Use LHR

$\rightarrow \frac{45x^2}{e^x} \rightarrow \frac{90x}{e^x} \rightarrow \lim_{x\to\infty} \frac{90}{e^x}$

0

Find all critical numbers and use the Second Derivative Test to determine all local extrema.



$f(x) = x^4 + 8x^3 - 58$

Critical points: x = 0 and -6, local minimum: x = $\boxed{-6}$

A

$y' = 4x^3 + 24x^2$

$y' = 0 = 4x^2(x+6)$

$4x^2 = 0 \qquad x + 6 = 0$

$0, -6$

$y'' = 12x^2 + 48x$

$y''(0) = 0$ Not min or max

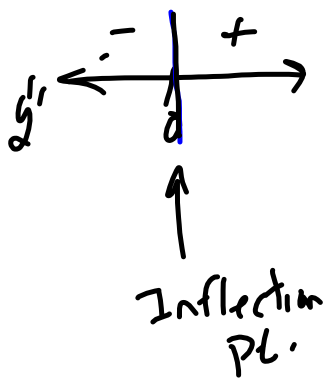
$y''(-6) = +$

$12 \cdot 36 - 48 \cdot 6 = +$

Determine the intervals where the graph of $f(x) = 3x + 4/x$ is concave up and concave down.

$4x^{-1}$

$f(x)$ is concave up on $x>0$ and concave down on $x<0$.



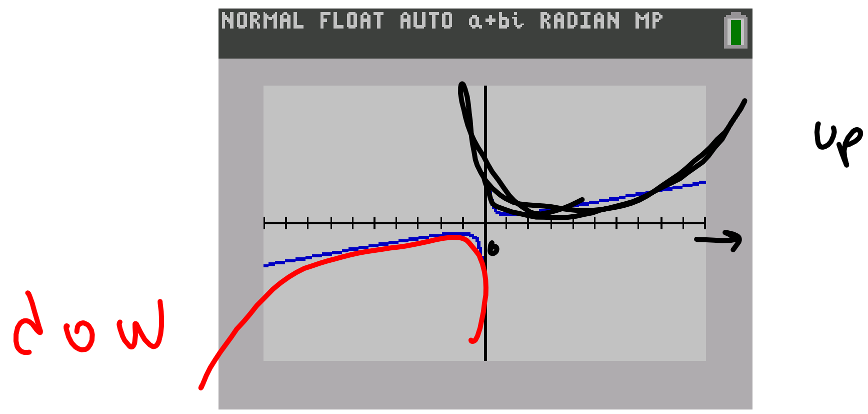
✓ $y = 3x + \frac{4}{x} = 3x + 4x^{-1}$

✓ $y' = 3 - 4x^{-2}$

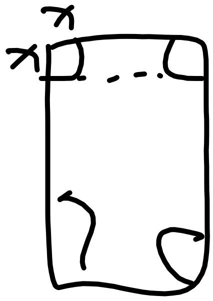
✓ $y'' = 8x^{-3} \neq 0$ Undefined $x = 0$.

$8x^{-3} > 0$ $\frac{8}{x^3} > 0$ $x^3 > 0$

$x > 0$

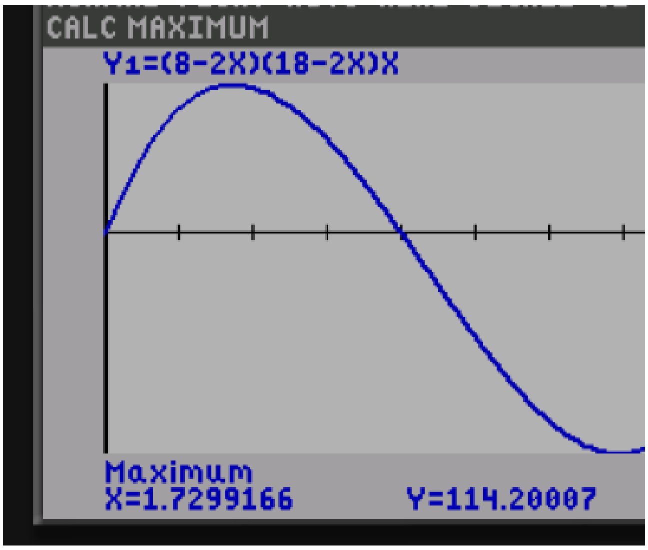


A sheet of paper $8''$–by–$18''$ is made into an open box (i.e. there's no top), by cutting $x-$ in. squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.



$L = 8-2x \quad W = 18-2x \quad H = x$

$V = LWH = (8-2x)(18-2x)(x)$



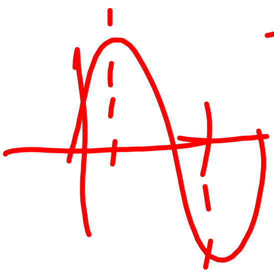
$$(8-2x)(18-2x)x$$

$$(144-36x-16x+4x^2)x$$

$$y = 144x - 52x^2 + 4x^3$$

$$y' = 144 - 104x + 12x^2 = 0$$

$$\frac{104 \pm \sqrt{104^2 - 4(12)(144)}}{2(12)}$$



Find the general antiderivative. Use c as the constant of integration.

$$\int x^{1/4}(x^{5/4} - 112)\,dx$$

$$\int x^{6/4} - 112x^{1/4}\,dx$$

$$\frac{x^{6/4+1}}{6/4+1} - \frac{112\,x^{1/4+1}}{1/4+1} + C$$

$$\frac{4}{10}\cdot x^{10/4} - 112\left(\frac{4}{5}\right)x^{5/4} + C$$

Evaluate $\int (9\cos x + 10x^{10})\,dx$. Use c as the constant.

$$\int 9\cos x\,dx + \int 10x^{10}\,dx$$

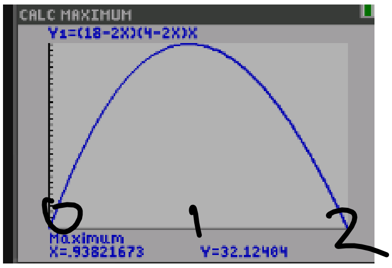
$$9\int \cos x\,dx + 10\int x^{10}\,dx$$

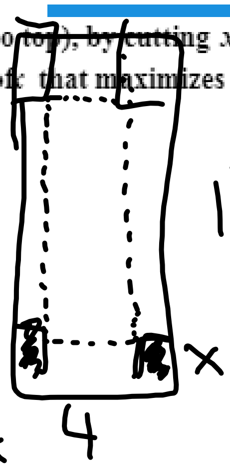
$$\longrightarrow 9\sin x + \frac{10x^{11}}{11} + C$$

check $\rightarrow$ $9\cos x + 10x^{10}$

A sheet of paper 4"–by–18" is made into an open box (i.e. there's no top), by cutting x– in. squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.

$x = \dfrac{11-\sqrt{67}}{3}$ is the max.





V=LwH

V=(18-2x)(4-2x)x

v=72x-44x^2+4x^3

Length 18-2x

Width 4-2x

Height = x

V'=72-88x+12x^2=0

$x=(88+\sqrt{7744-4(12)(72)})/2(12)$

$$\frac{88 \pm \sqrt{2^6}\cdot\sqrt{67}}{24}$$

$$\frac{88 \pm 8\sqrt{67}}{24} = \frac{11-\sqrt{67}}{3} =$$

4288

2 2144

2 1072

2 536

2 268

2 134

2 67

Determine where the graph of $f(x) = 2x^3 + 15x^2 + 39x - 67$ is concave down.

$f(x)$ is concave down where x $< -\frac{30}{12}$

y'=6x^2+30x+39

y''=12x+30 <0

x<-30/12=-2.5

Compute the derivative of $f(x) = \sinh^2(9x)$.

$f'(x) =$ ☐

$$y = (\sinh(9x))^2$$

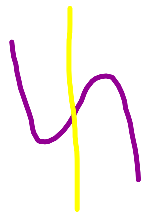
$$y' = 2(\sinh(9x))^1 \cdot \frac{d}{dx}(\sinh(9x))$$

$$= 2\sinh(9x) \cdot \cosh(9x) \cdot 9$$

16

Determine where the graph of $f(x) = 2x^3 + 39x^2 - 18x - 31$ is concave down.

$y' = 6x^2 + 78x - 18$



$y'' = 12x + 78$

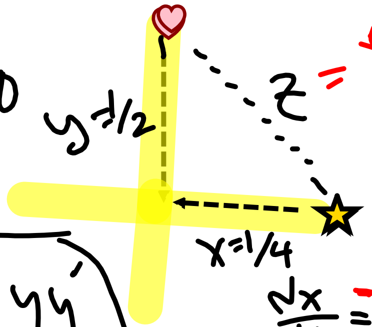
$12x + 78 < 0$

$\frac{12x}{12} < \frac{-78}{12}$ $\quad x < -\frac{78}{12}$

19

A car is traveling at 50 mph due south at a point 1/2 mile north of an intersection. A police car is traveling at 40 mph due west at a point 1/4 mile due east of the same intersection. At that instant, the radar in the police car measures the rate at which the distance between the two cars is changing. What does the radar gun register? Round your answer to three decimal places.

$\frac{dy}{dt} = -50$



$z = \frac{\sqrt{5}}{4}$

$x = 1/4$

$y = 1/2$

$z^2 = (1/4)^2 + (1/2)^2$

$z^2 = 1/16 + 1/4 = 5/16$

$z = \sqrt{5}/4$

$\boxed{z^2 = x^2 + y^2}$

$\frac{dx}{dt} = -40$

want $\frac{dz}{dt}$

$\frac{x x' + y y'}{\sqrt{x^2 + y^2}}$

$\frac{d}{dt} z^2 = \frac{d}{dt}(x^2 + y^2)$

$2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$

$\frac{\sqrt{5}}{4} z' = \frac{1}{4}(-40) + \frac{1}{2}(-50)$

$\frac{dz}{dt} = \frac{-10 + -25}{\frac{\sqrt{5}}{4}}$

$= -62.6$

20

Find the general antiderivative. Use c as the constant of integration.

$\int (6\cos x - 41)\, dx$

$$\int 6\cos x \, dx - \int 41 \, dx$$

$$6\int \cos x \, dx - 41x + C$$

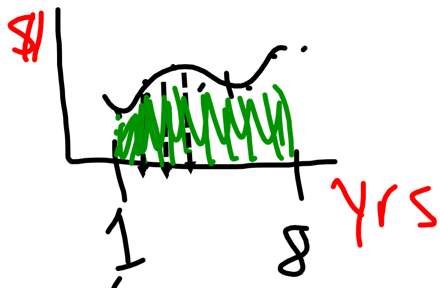
$$\boxed{6\sin x - 41x + C}$$

Project Discussion

1. STAT EDIT
STAT CALC
Regression

2. Y_1 = VARS 5 > > 1

3. Zoom 9:



4. Calc : 7

$\int f(x)dx$

Lower : 1
Upper : 8

An: 167 $·yrs.

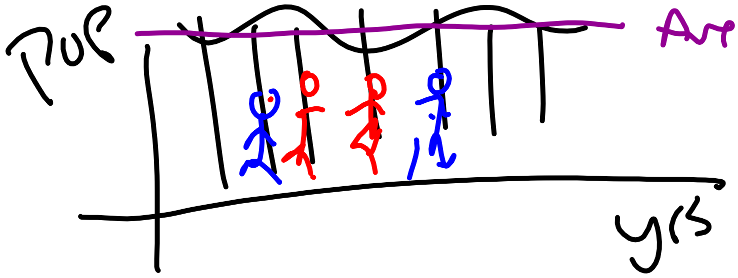
between 2001 and 2008
Lady G. Made $167 mil ! ~~$.40~~

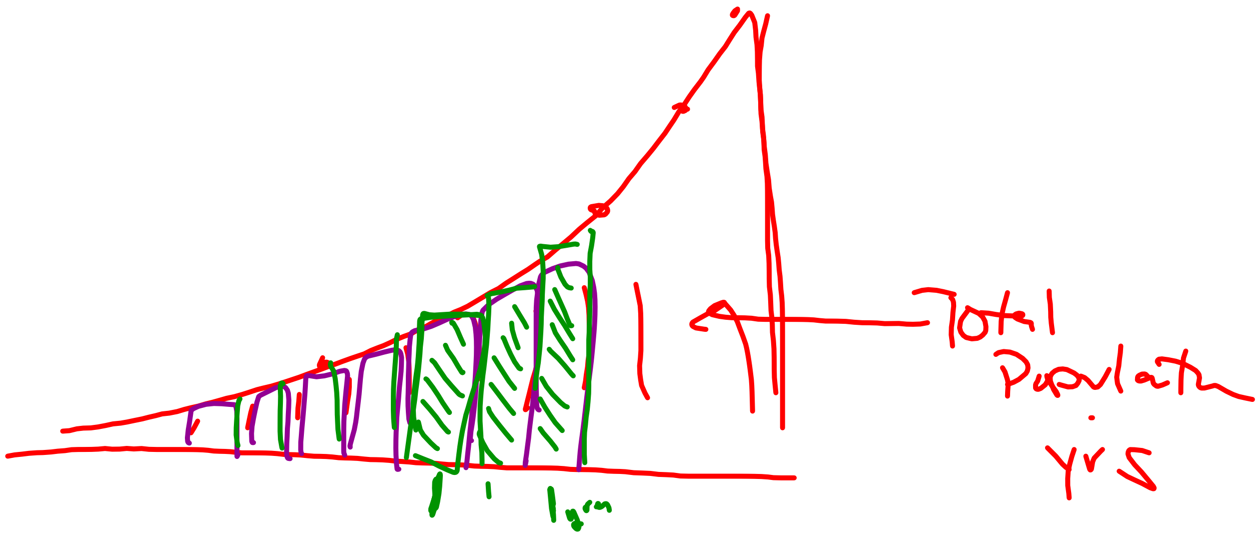
$\frac{167}{2008-2001}$ = AVE VALUE

x40

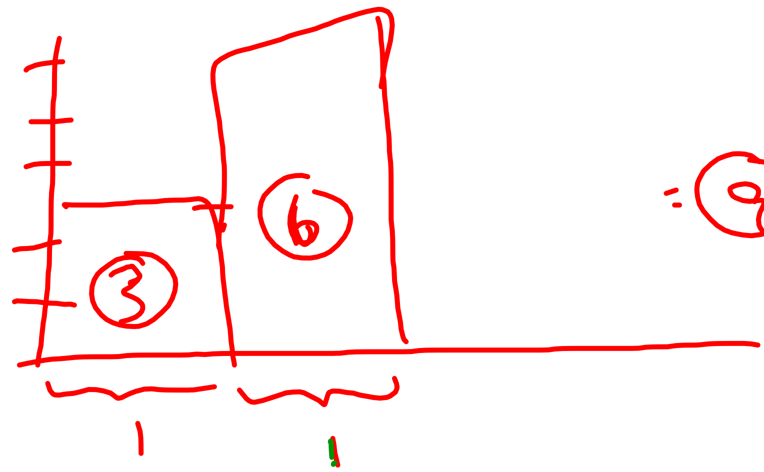
pop · yrs

years

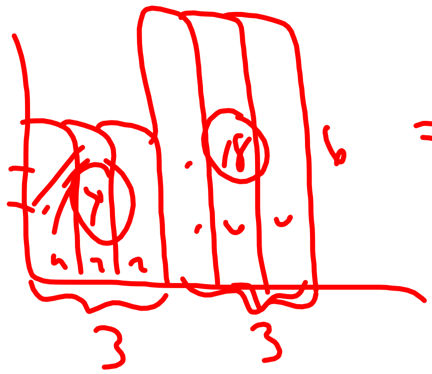




$$\text{Pop yrs} \quad \frac{1}{\text{yrs}} = \text{PoD}$$



= 9



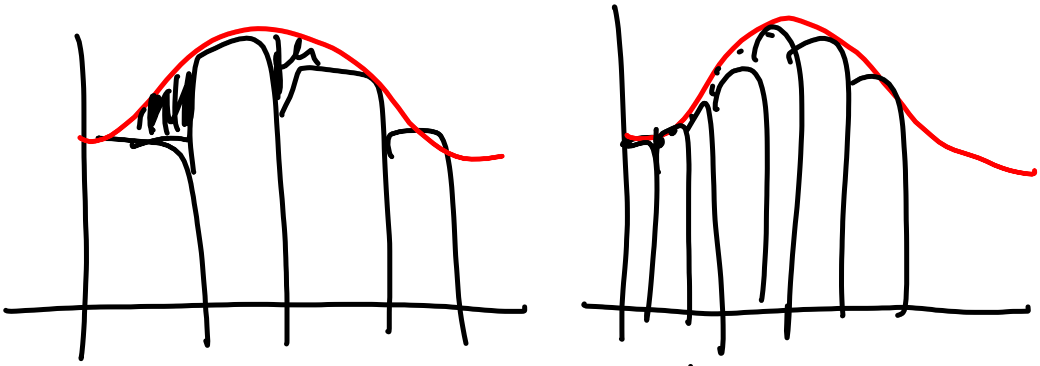
= 9+18 = 27

Lecture Summation Notation

$$\sum = f(n)$$

$$\lim_{n \to \infty} f(n) = A$$

$$\int f(x)$$



Summation Notation

$\sum_{i}^{n}$ Adds up rectangles $= f(n)$

$\lim_{n \to \infty} f(n) = \text{Area}$

Summation Notation

$$\sum_{i=a}^{b} \boxed{f(i)} = f(a) + f(a+1) + f(a+2) + f(a+3) + f(a+4) \cdots + f(b-1) + f(b)$$

End

sigma

index

start

First 10 even numbers,

$2+4+6+8+10+12+14+16+18+20$

$$\sum_{i=1}^{10} 2i = 2(1) + 2(2) + 2(3) \cdots + \quad + 2(10)$$

$1 + 2 + 3 + 4 + 5 + 6$

$$\sum_{i=1}^{6} i = 21$$

Summation!

$$\sum_{i=1}^{n} i = (n+1)\left(\frac{n}{2}\right)$$

value

# of pairs

$f(n)$

rule

$$\sum_{i=1}^{n} i = (1+n)\left(\frac{n}{2}\right)$$

$1+2+3+\dots n$

First 4

1 2 3 4

$$\sum_{i=1}^{4} i = (1+4)\left(\frac{4}{2}\right) = 10$$

~~First 500~~ numbers

$$\sum_{i=1}^{500} i = (501)\left(\frac{500}{2}\right) = 125{,}250$$

$$\frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + x_3 + x_4}{n}$$

$$\sum_{i=2}^{4} i^2 = 2^2 + 3^2 + 4^2$$

$$4 + 9 + 16 = 29$$

Properties

$$\sum_{i=1}^{n} c f(i) = c \sum_{i=1}^{n} f(i)$$

Prop $$\sum_{i=1}^{n} A+B = \sum_{i=1}^{n} A + \sum_{i=1}^{n} B$$

$2 + 4 + 6 + 8 + 10$

$$\sum_{i=1}^{5} 2i = 2\sum_{i=1}^{5} i = \cancel{2}(1+5)\left(\frac{5}{\cancel{2}}\right)$$

$$= 30$$

Write in summation notation: the sum of the first 360 odd positive integ

A. $\sum_{i=1}^{360}(2i-1) = 1+3+5+\dots+719$

B. $\sum_{i=1}^{360}(2i+1) = 3+$

C. $\sum_{i=0}^{359}(2i-1) = -1+$

D. $\sum_{i=1}^{360}(i-1) = 0+$

$$\sum_{i=1}^{360} 2i - \boxed{\sum_{i=1}^{360} 1}$$

$1+1+1+1+1+1+1+1\dots$

$$2\boxed{\sum_{i=1}^{360} i} - (1)(360)$$

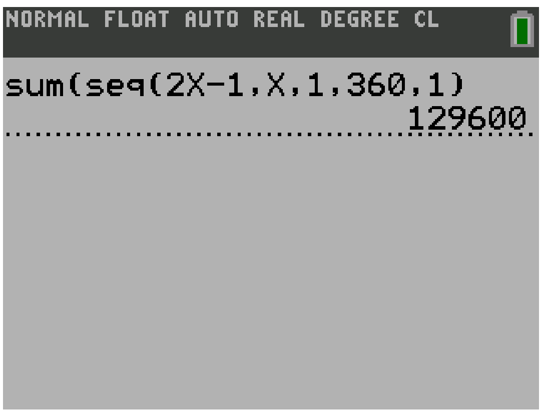
$$2\,\frac{(1+360)(360)}{2} - 360$$

$$(361)(360) = 129{,}960$$

$$\frac{-360}{129{,}600}$$

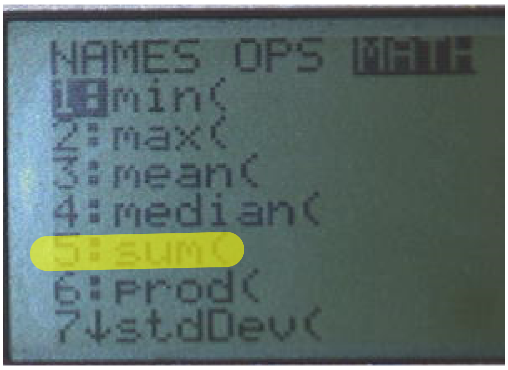
sum(seq(2X−1, X, 1, 360, 1))

f(x), x, a, b, Δx = step



2nd stat > > 5 Sum

sum(seq(



2nd stat > 5 seq

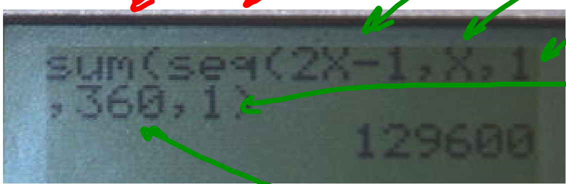
2nd stat

2nd stat

2x-1

variable X

start



changes by

end

Compute.

$$\sum_{i=1}^{17} (8i + 1) = \boxed{1241}$$

$$8\sum_{i=1}^{17} i + \sum_{i=1}^{17} 1$$

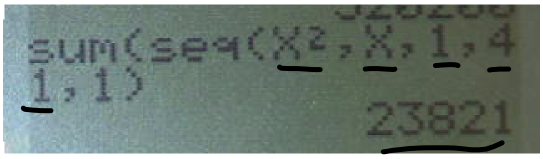
$$8(17+1)\left(\frac{17}{2}\right) + 17 = 1241$$

sum(seq(8x+1, X, 1, 17, 1))

```
sum(seq(8X+1,X,1,17,1)
                     1241
```

Compute $\sum_{i=1}^{41} i^2$. $= 23821$

Calculator



We have

$$\sum_{i=1}^{41 \leftarrow n} i^2 = \frac{41(42)(83)}{6} = 23821.$$

$\sum i^2$

$$\frac{(n)(n+1)(2n+1)}{6}$$

Ex

$1^2 + 2^2 + 3^2 = 14$

$\frac{3 \cdot 4 \cdot 7}{6} = 14$

Sum the values of $f(x) = x^2 + 2$ evaluated at $x = 0.125$, $x = 0.25$, $\cdots$, $x = 0.75$.

Your Answer:

$$\begin{array}{r} .25 \\ -\ .125 \\ \hline .125 \end{array}$$

$\frac{.75}{.125} = 6$

sum(seq(X^2 +2,X,.125,.75,.125)=13.42...

$$= \sum_{i=1}^{6} [(0.125\,i)^2 + 2]$$

.015625

$$= 0.015625 \sum_{i=1}^{6} i^2 + \sum_{i=1}^{6} 2$$

$.125^2$

$$= (0.015625)\left[\frac{6(7)(13)}{6}\right] + (2)(6)$$

$$= 1.42188 + 12 = 13.421875.$$

x^3-2x+8

Compute the sum $\sum_{i=1}^{10}(i^3-2i+8)$. $\doteq 2995$

```
sum(seq(X³-2X+8,X,1,10,1))
                      2995
```

$$\sum_{i=1}^{10}(i^3-2i+8)=\sum_{i=1}^{10} i^3-2\sum_{i=1}^{10} i+\sum_{i=1}^{10} 8$$

$$=\frac{100(11)^2}{4}-2\frac{10(11)}{2}+10\bullet 8$$

$$=2995$$

$$\sum_{i=1}^{n} i^3=\frac{n^2\cdot(n+1)^2}{4}$$

N=3 1+8+27=36

3*3*4*4/4=36

Compute the sum $\sum_{i=1}^{100} (i^3 - 2i + 8)$.

sum(seq(X³−2X+8,X,1,100,1))
2995

$$\sum_{i=1}^{100} (i^3 - 2i + 8) = \sum_{i=1}^{100} i^3 - 2\sum_{i=1}^{100} i + \sum_{i=1}^{100} 8$$

$$= \frac{(100)^2(101)^2}{4} - 2\frac{100(101)}{2} + 100 \cdot 8 \quad 800$$

$$= 2995$$

N=3 1+8+27=36

3*3*4*4/4=36

width $\quad f(x) \leftarrow$ height

Compute the sum and the limit of the sum as $n \to \infty$ of $\sum_{i=1}^{n} \frac{3}{n}\left[\left(\frac{i}{n}\right)^2 + 2\left(\frac{i}{n}\right)\right]$. $= A(n)$

$$\frac{3}{n}\frac{i^2}{n^2} + \frac{6}{n}\cdot\frac{i}{n}$$

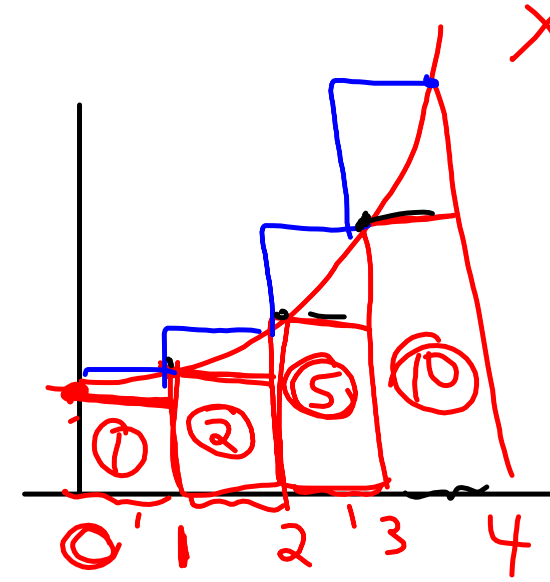
$$\frac{3}{n^3}\sum_{i=1}^{n} i^2 + \frac{6}{n^2}\sum_{i=1}^{n} i$$

$$\frac{3}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{6}{n^2}(n+1)\frac{n}{2}$$

$$\frac{6n^3 + \cdot n^2 \ldots}{6n^3} + \frac{3n^2+3n}{n^2}$$

$$\lim_{n\to\infty} \quad 1 + 3 = 4$$

x^2+1



Using left side rectangles of equal width

18

$1 = 0^2+1$ $\quad 2^2+1$

$1+1=2$

$5:1$

Right

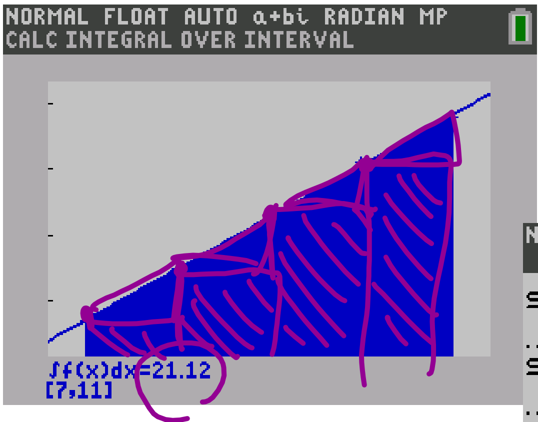
$2+5+10+17$

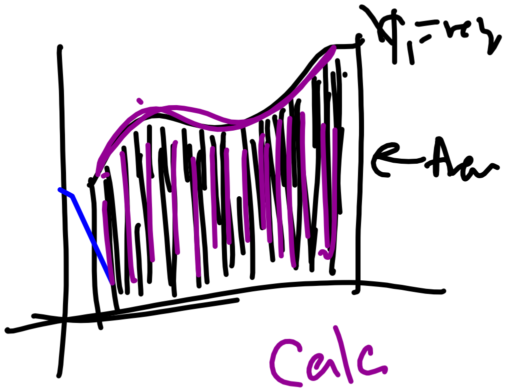
$x^2 + \cancel{x}$

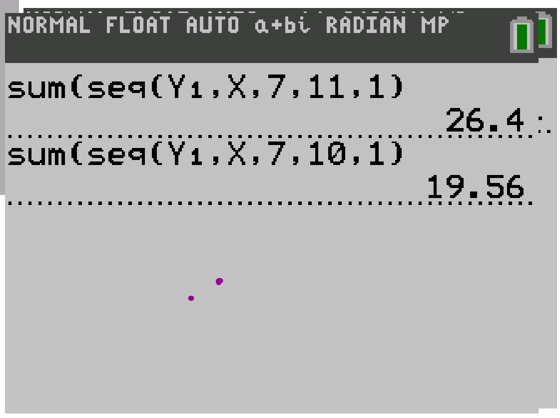
$$\sum f\left(\frac{x}{4}\right)^2 + \left(\frac{x}{4}\right)$$

34

Group work







NORMAL FLOAT AUTO a+bi RADIAN MP

sum(seq(Y1,X,7,11,1)
26.4

sum(seq(Y1,X,7,10,1)
19.56