Agenda

Review Quiz #6 Preview Quiz #7

Area under curve- world population example

Review Optimizations

Anti-derivatives

Midterm Review

Math = Language

Calc = study of change

Rates = Ave (2pts) = slope

Instantaneous (1pt)

Limits $\rightarrow$ two $\rightarrow$ one.

Definition, Quick + Dirty

Applications.

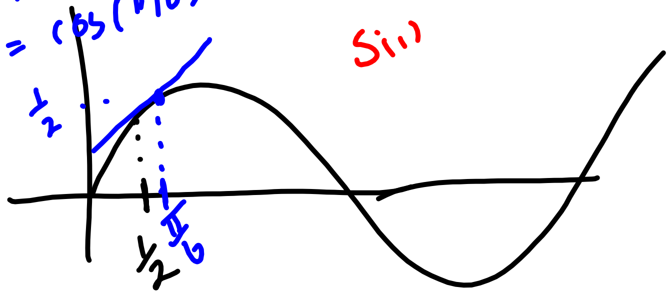
Review Quiz #8 Preview Quiz #9

Round your final answer to four decimal places.

Use linear approximation to estimate the quantity $\sin\left(\frac{1}{2}\right)$.

Pt $(\pi/2, 1/2)$

slope $= \cos(\pi/6) = \sqrt{3}/2$



$\sin\left(\frac{1}{2}\right) =$ ______

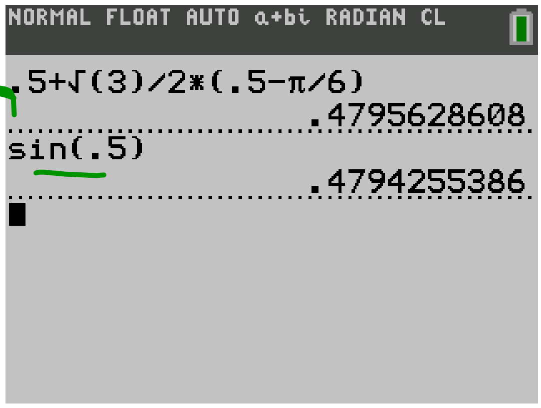
$\frac{\pi}{6} = \frac{3.14}{6} \approx \frac{1}{2}$

Sin	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1

$y - y_1 = m(x - x_1)$

$y - 1/2 = \frac{\sqrt{3}}{2}(x - \pi/2)$

$y(\frac{1}{2}) = \frac{1}{2} + \frac{\sqrt{3}}{2}(\frac{1}{2} + \frac{\pi}{2})$



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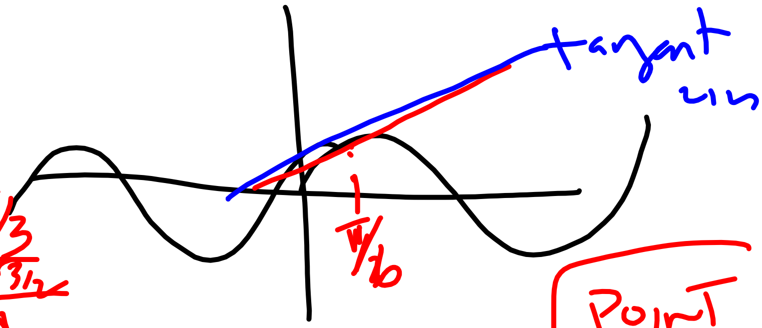
Use linear approximation to estimate the quantity $\sin\left(\frac{1}{2}\right)$.

$\sin\left(\frac{1}{2}\right) =$ ☐



	$0°$	$\pi/6$	$\pi/4$	$\pi/3$
sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
		$\sqrt{3}/2$		

$y - 1/2 = \sqrt{3}/2\,(x - \pi/6)$

$y = \sin x$

$\sin(\pi/6) = 1/2$

$y' = \cos x$

$\cos(\pi/6) = \sqrt{3}/2$

Point $(\pi/6, 1/2)$

slope $\sqrt{3}/2$

$\boxed{y = \sqrt{3}/2\,(x - \pi/6) + 1/2}$

$y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}\left(\frac{1}{2} - \frac{\pi}{6}\right) + 1/2$

$\approx \sin(1/2) = .4796\ldots$

$\sin(1/2) = .4794\ldots$

Round your final answers to four decimal places if necessary.

Suppose that the average yearly cost per item for producing x items of a business product is

$\bar{C}(x) = 11 + \frac{91}{x}$. $\Rightarrow$ $\bar{C}(x(t)) = 11 + 91(x(t))^{-1}$

$\bar{C}' = -91(x(t))^{-2} \cdot x'(t)$

$\bar{C}'(2) = -91(x(2))^{-2} x'(2)$

$-91(9.2)^{-2} \; .5$

The three most recent yearly production figures are given in the table.

Year	0	1	2
Prod. (x)	8.2	8.7	9.2

Ave Rate. $\frac{9.2-8.7}{2-1} = .5$

Estimate the value of $x'(2)$ and the current (year 2) rate of change of the average cost.

$x'(2) =$ .5 ; The rate of change of the average cost is ___ per year.

Round your final answers to four decimal places if necessary.

Suppose that the average yearly cost per item for producing x items of a business product is

$$\overline{C}(x) = 11 + \frac{91}{x}.$$

The three most recent yearly production figures are given in the table.

Year	0	1	2
Prod. (x)	8.2	8.7	9.2

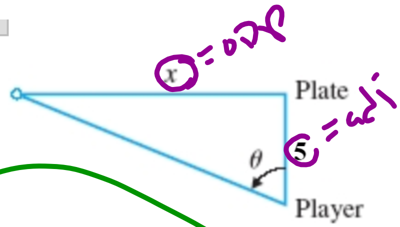
Ave Rate. $\frac{.5}{1} = .5$

Estimate the value of $x'(2)$ and the current (year 2) rate of change of the average cost.

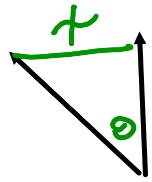
$x'(2) =$ .5 ; The rate of change of the average cost is -1.075 per year.

$$C' = -\frac{91}{x^2}(x') = \frac{-91(.5)}{(9.2)^2} = -1.075$$

A baseball player stands 5 meters from home plate and watches a pitch fly by. In the diagram, x is the distance from the ball to home plate and θ is the angle indicating the direction of the player's gaze. Find the rate θ' at which his eyes must move to watch a fastball with $x'(t) = -45$ m/s as it crosses home plate at $x = 0$.



$\theta' =$ -9 rad/s.



$$\frac{d}{dt} \tan\theta = \frac{x}{5} = \frac{o}{a}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{\frac{dx}{dt}}{5}$$

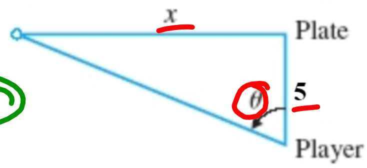
$\theta = 0$

$\cos\theta = 1$

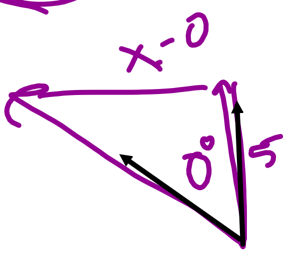
$\sec\theta = \frac{1}{1} = 1$

$$\frac{d\theta}{dt} = \frac{-45}{5} = -9$$

A baseball player stands 5 meters from home plate and watches a pitch fly by. In the diagram, x is the distance from the ball to home plate and θ is the angle indicating the direction of the player's gaze. Find the rate θ' at which his eyes must move to watch a fastball with $x'(t) = -45$ m/s as it crosses home plate at $x = 0$.



$\theta' =$ -9 rad/s.



$\sec 0° = \frac{1}{\cos 0°} = 1$

$\frac{d}{dt} \tan\theta = \frac{x}{5} = \frac{opp}{adj}$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{5} \frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{1}{5}(-45)$

The frequency at which a guitar string vibrates (which determines the pitch of the note we hear) is related to the tension T to which the string is tightened, the density ρ of the string, and the effective length L of the string by the equation $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$. By running a finger along a string, a guitarist can change the distance between the bridge and their finger. Suppose that $L = 1/2$ m and $\sqrt{\frac{T}{\rho}} = 70$ m/s so that the units of f are Hertz (cycles per second).

constant

If the guitarist's hand slides so that $L'(t) = -1$, find $f'(t)$. At this rate, how long will it take to raise the pitch one octave (that is, double f)?

$f'(t) =$

$$f = .5L^{-1} \cdot (70)$$

$$f' = -.5L^{-2} \cdot 70 \cdot L'$$

$$= -.5\left(\tfrac{1}{2}\right)^{-2} 70 \cdot (-1)$$

The frequency at which a guitar string vibrates (which determines the pitch of the note we hear) is related to the tension T to which the string is tightened, the density ρ of the string, and the effective length L of the string by the equation $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$. By running a finger along a string, a guitarist can change the distance between the bridge and their finger. Suppose that $L = 1/2$ m and $\sqrt{\frac{T}{\rho}} = 70$ m/s so that the units of f are Hertz (cycles per second).

If the guitarist's hand slides so that $L'(t) = -1$, find $f'(t)$. At this rate, how long will it take to raise the pitch one octave (that is, double f)?

$f'(t) =$ ______

$= 70$

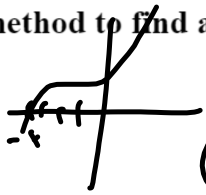
$$\boxed{f = \frac{1}{2L(t)}\sqrt{\frac{T}{\rho}}}$$

$$f(t) = \frac{35}{L(t)} = 35L^{-1}$$

$$\frac{df}{dt} = -35L^{-2}\cdot\frac{dL}{dt}$$

$$= (-35)\left(\tfrac{1}{2}\right)^{-2}(-1) = 35\cdot 4 = 140$$

Use Newton's method to find an approximate root (accurate to six decimal places).



$$x^3 + 4x^2 - 3x + 2 = 0 \quad \rightarrow \quad 3x^2 + 8x - 3$$

$x_0 = 1$

$1 \rightarrow x$

$x - y_1 / \text{nderiv}(y_1, x, x) \rightarrow x$

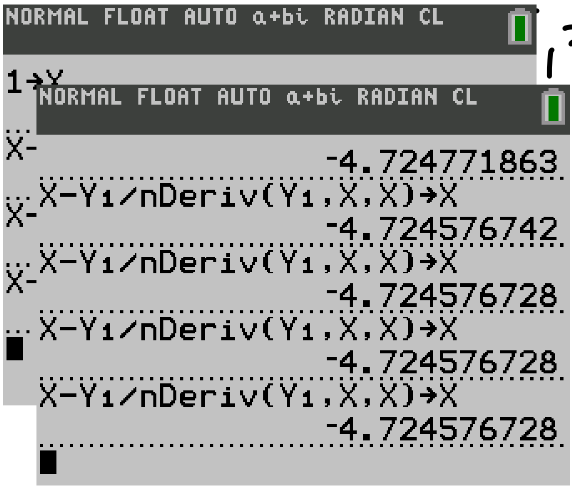
$x =$ $\boxed{-4.72477}$...

$x_1 = 1 - (1+4-3+2) / (3+8-3) =$

$= 1 - \frac{4}{8} = \frac{1}{2}$

$x_2 = \frac{1}{2} - \frac{(1/2)^3 + 4(\frac{1}{2})^2 - 3(\frac{1}{2}) + 2}{3(1/2)^2 + 8(\frac{1}{2}) - 3}$

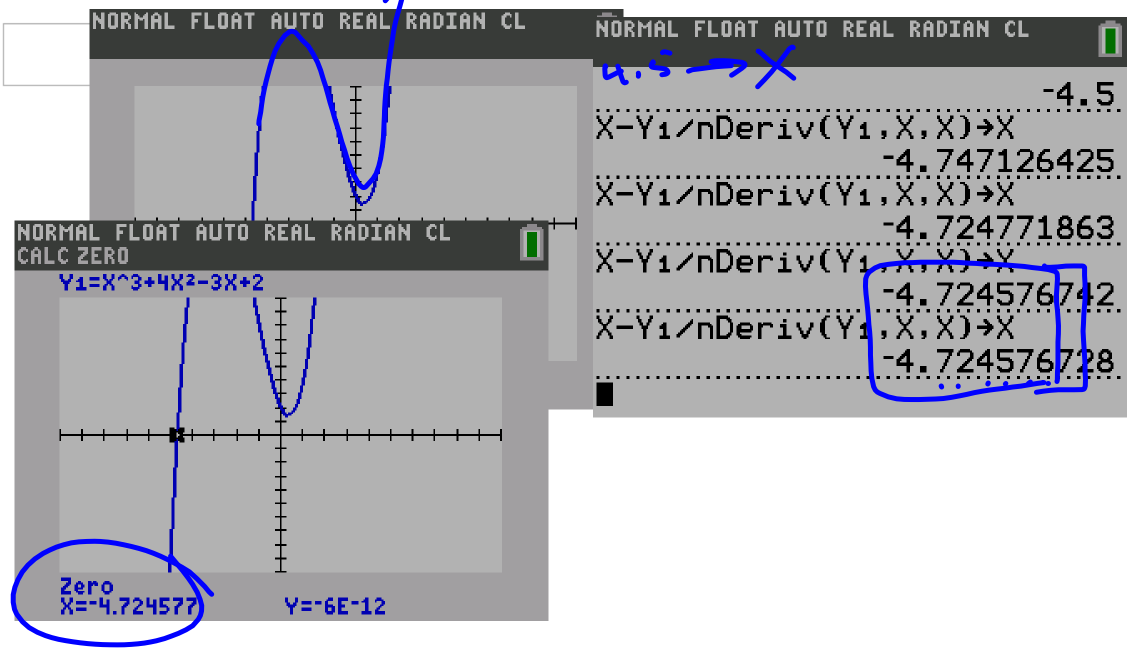
$\approx .6$?



Use Newton's method to find an approximate root (accurate to six decimal places).

$$f(x) = x^3 + 4x^2 - 3x + 2 = 0$$

$x =$



Give your final answers as reduced improper fractions.

Use Newton's method with the given x_0 to compute x_1 and x_2 by hand.

$x^3 + 2x^2 - 7 = 0, x_0 = 1$

$x_1 =$ ☐ and $x_2 =$ ☐

Give your final answers as reduced improper fractions.

$Y_1 = X^\wedge 3 + 2X - 7$

Use Newton's method with the given x_0 to compute x_1 and x_2 by hand.

$y' = 3x^2 + 4x$

$x^3 + 2x^2 - 7 = 0, x_0 = 1$

$1 \to X$

$x_1 =$ $11/7$ and $x_2 =$ $7373/4697$

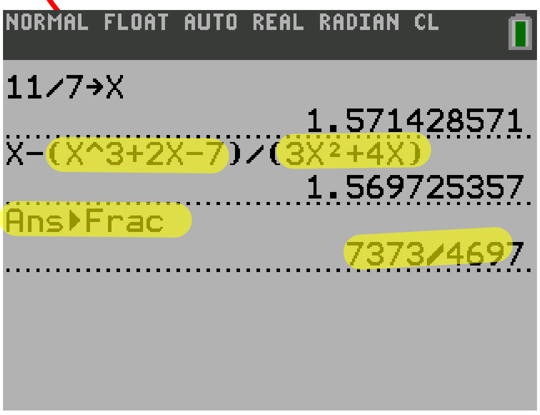
$X - Y_1 / \text{nderiv}(Y_1, X, X) \to X$

<enter>

$$1 - \frac{1+2-7}{3+4} = \frac{11}{7}$$

$$\frac{11}{7} - \frac{(11/7)^3 + 2(11/7)^2 - 7}{3(11/7)^2 + 4(11/7)}$$

<enter>



Determine the intervals where the graph of the given function is concave up and concave down.

$f(x) = 17x^{4/3} + 13x^{1/3}$

$f' = 17 \cdot \frac{4}{3} x^{1/3} + \frac{13}{3} x^{-2/3}$

$f'' = \frac{68}{9} x^{-2/3} - \frac{26}{9} x^{-5/3} = 0$

Concave up: $x >$ 26/68 and $x <$ 0, concave down: 0 $< x <$ 26/68

$x^{-5/3}\left(\frac{68}{9}x - \frac{26}{9}\right)$

$= 0 \qquad = 0$

$x = \frac{26}{68}$

$-5/3 \quad -2/3 \quad 0$

$(x^{-3} - 3x^{2})$

$x^{-3}(1 - 3x)$

Determine the intervals where the graph of the given function is concave up and concave down.

$f(x) = 17x^{4/3} + 13x^{1/3}$

-2 2 3

Concave up: $x >$ 26/68 and $x <$ 0, concave down: 0 $< x <$ 26/68

$f'(x) = 68/3\, x^{1/3} + 13/3\, x^{-2/3}$

$f''(x) = 68/9\, x^{-2/3} + -26/9\, x^{-5/3}$

$\cancel{x^{-5/3}}\ 9\ (68x - 26) = 0$

$x = 0$

$x = 26/68$

- 0 − 26/68 +

Determine the intervals where the graph of the given function is concave up and concave down.

$f(x) = 14x^{4/3} + 8x^{1/3}$

Concave up: $x >$ ☐ and $x <$ ☐, concave down: ☐ $< x <$ ☐

Determine the intervals where the graph of the given function is concave up and concave down.

$f(x) = 14x^{4/3} + 8x^{1/3}$

Concave up: $x >$ [$2/7$] and $x <$ [0], concave down: [0] $< x <$ [$2/7$]

$$f' = \frac{56}{3}x^{1/3} + \frac{8}{3}x^{-2/3}$$

$$f'' = \frac{56}{9}x^{-2/3} - \frac{16}{9}x^{-5/3} = 0$$

$$\frac{4}{9}x^{-5/3}\left(14x - 4\right) = 0$$

0

$\frac{4}{14} = \frac{2}{7}$

f'': $+$ $-$ $+$

0 $2/7$

Determine where the graph of $f(x) = 2x^3 + 21x^2 - 28x - 124$ is concave down.

y''

$f(x)$ is concave down where x $< -\frac{42}{12}$

$f'(x) = 6x^2 + 42x - 28$

$f'' = 12x + 42 < 0$

$-42 \qquad -42$

$12x < -42$

$x < -\frac{42}{12}$

Determine where the graph of $f(x) = 2x^3 + 21x^2 - 28x - 124$ is concave down.

$f(x)$ is concave down where x $< \frac{-21}{6}$

$$f' = 6x^2 + 42x - 28$$

$$f'' = 12x + 42 \leq 0$$

$$\frac{12x}{12} < \frac{-42}{12}$$

Questi

Determine where the graph of $f(x) = 2x^3 + 51x^2 - 45x - 64$ is concave down.

$f(x)$ is concave down where x ☐ .

Questi

Determine where the graph of $f(x) = 2x^3 + 51x^2 - 45x - 64$ is concave down.

$f(x)$ is concave down where x $< \frac{-102}{12} = -\frac{51}{6}$

$y' = 6x^2 + 102x - 45$

$y'' = 12x + 102 < 0$

$\frac{12x}{12} < \frac{-102}{12}$

Find all critical numbers and use the Second Derivative Test to determine all local extrema.

$f(x) = x^4 + 7x^3 - 16$

Critical points: x = 0 and $-\frac{21}{4}$, local minimum: x = $-21/4$

$f'(x) = 0$

$4x^3 + 21x^2 = 0$

$x^2(4x + 21) = 0$

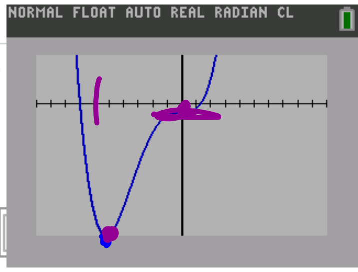
$x = 0 \qquad x = -\frac{21}{4}$

$f''(x) = 12x^2 + 42x$

$f''(0) = 0$?

$f''\left(-\frac{21}{4}\right) = 12\left(-\frac{21}{4}\right)^2 + 42\left(-\frac{21}{4}\right)$

$= +$ MIN

Find all critical numbers and use the Second Derivative Test to 

a.

$f(x) = x^4 + 7x^3 - 16$

Critical points: x = 0 and $-\frac{21}{4}$, local minimum: x =

$y' = 4x^3 + 21x^2 = 0$

$x^2(4x+21) = 0$

$x^2 = 0 \qquad 4x+21 = 0$

$x = 0 \qquad x = -\frac{21}{4}$

$y'' = 12x^2 + 42x$

$y''(0) = 0$

$y''\left(-\frac{21}{4}\right) = 12\left(\frac{-21}{4}\right)^2 + 42\left(\frac{-21}{4}\right)$

$= 110.25$ (+)

up

Find all critical numbers and use the Second Derivative Test to determine all local extrema.

$f(x) = x^4 + 7x^3 - 16$

Critical points: $x = 0$ and $-21/4$, local minimum: $x = -21/4$

$y' = 4x^3 + 21x^2 = x^2(4x+21)$

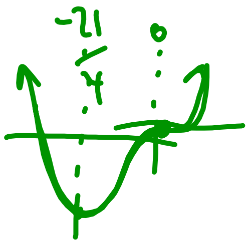
$\Rightarrow 0, -21/4$

$y'' = 12x^2 + 42x$

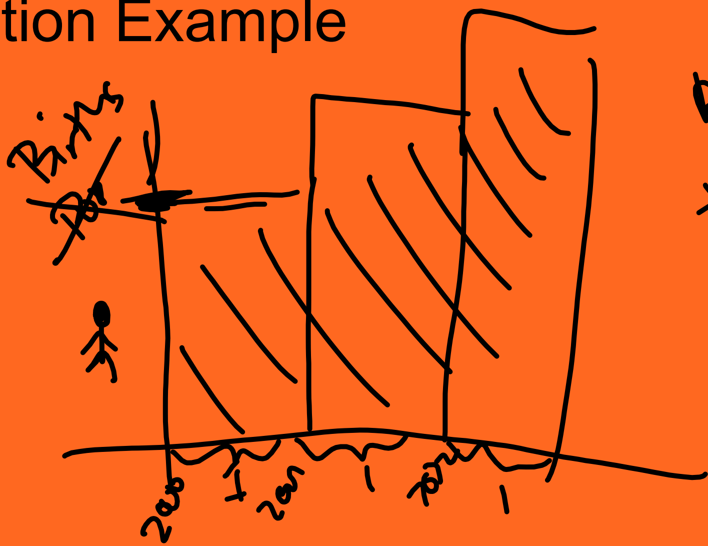
$y''(-21/4) = 12\left(\frac{-21}{4}\right)^2 + 42\left(\frac{-21}{4}\right) = 110.$ $+$

UP

$y''(0) = 0$

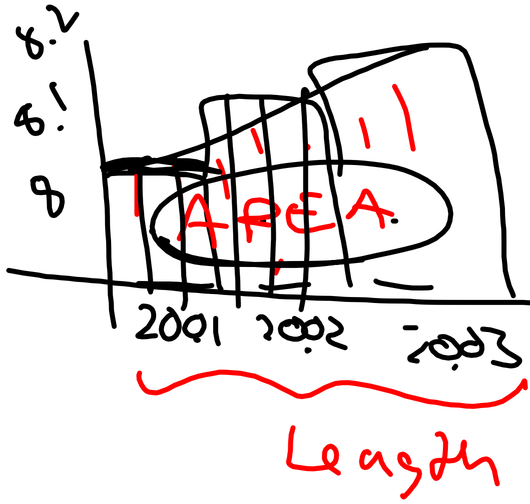


Population Example



Add up to get all population

Population

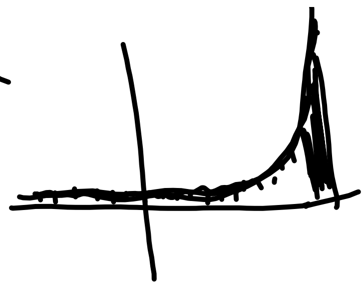


107 billion died

7 billion

3 million maximum

8 billion



8,000,000,000

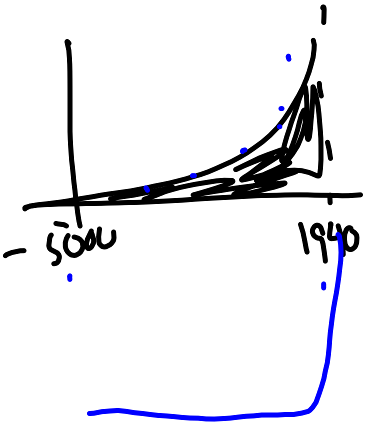
− 3,000,000

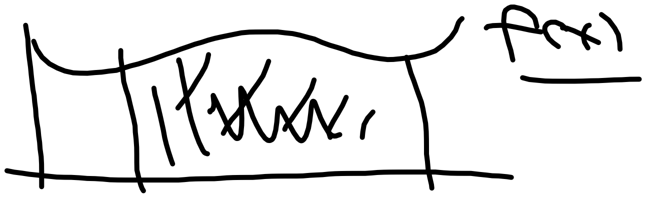
7,994,000,000 ≈ 8 billion

Area Under curve $y = f(x)$

Y_1 = curve.

Adj Window.





Lower Limit = a

b = upper Limit

Calc 7: $\int f(x)\,dx$

Lower: a -5000

upper: b 1940

Srn = Area

world pop

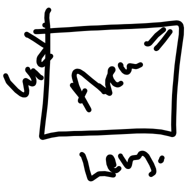
2015: 7.4

2005: 6.5

1995: 5.7

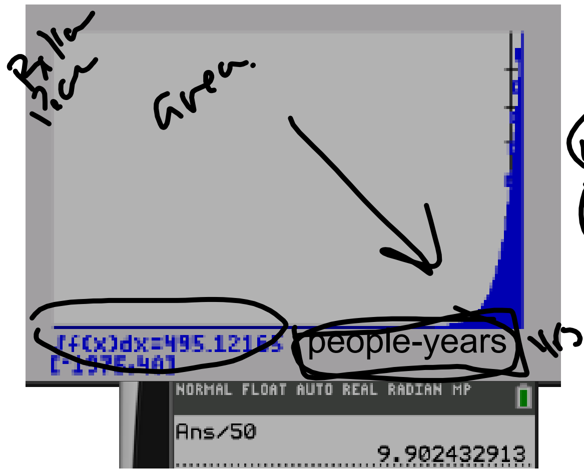
1985: 4.85

1975: 4



area: 386.2billion people-years

divide by life expectancy years (dived by 38... 10 billion people since 1920



$\frac{495}{100} = 5$ bil.

$\frac{495}{20} = 25.$

Area = $\dfrac{\text{People}\cdot\text{yrs}}{\text{yrs}}$

if people lived to be 50 yrs

then the number of people 495/50=9.9 bill

$\frac{500}{20} = 25$

window: xmin: -3000

xmax: 1920

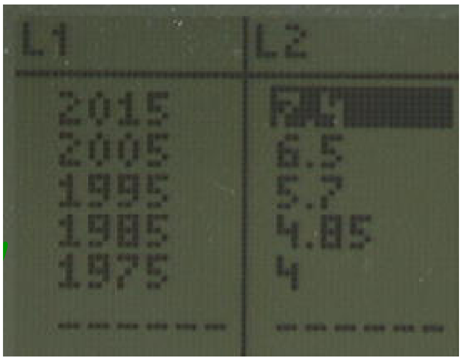
calc 7:

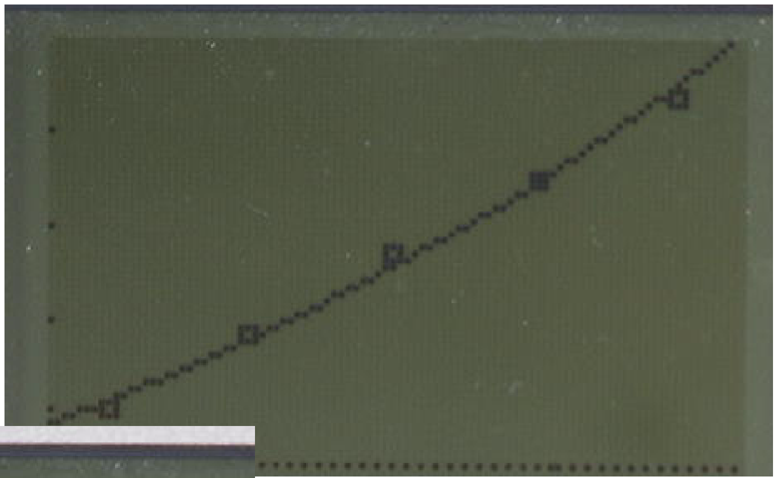
lower: -3000

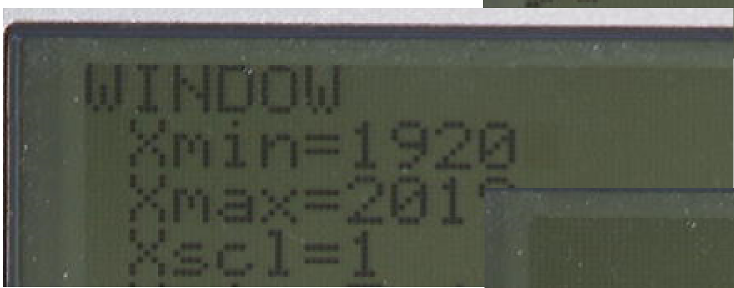
upper: 1920

116 billion people years

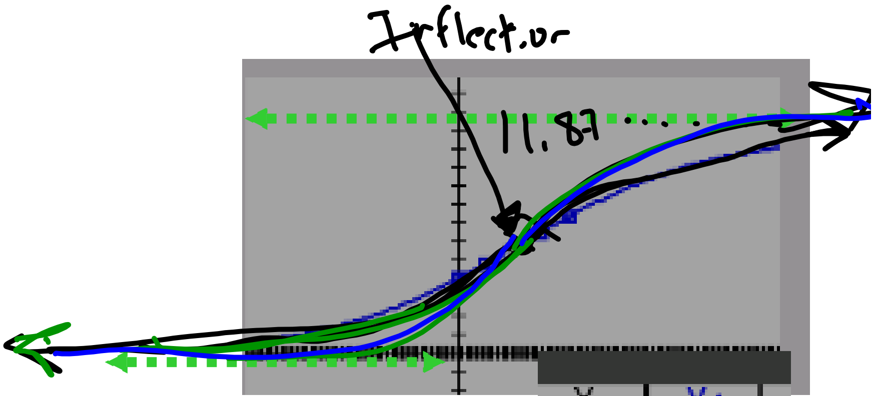
divided by 45...gets 2.5 billion


L1
L2
2015
2005
6.5
1995
5.7
1985
4.85
1975
4




WINDOW
Xmin=1920
Xscl=1


∫f(x)dx=378.63595



Logistic
y=c/(1+ae^(-bx))
a=1.954452013
b=.0291871114
c=11.87175138

$$y = \frac{c}{1 + \frac{a}{e^{bx}}}$$

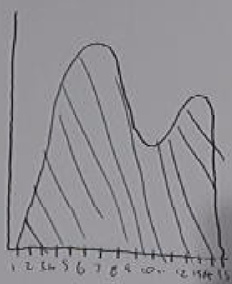
X	Y_1
17	5.421
25	6.1126
2017	11.872
42	7.5441
53	8.3834
63	9.057
143	11.525

x-axis (independant variable): years

Team Leader: Truth

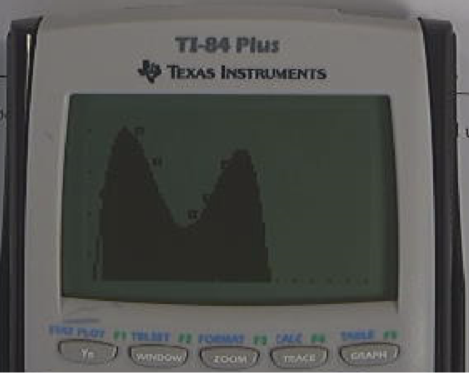
y-axis (dependant variable): revenue of billions of dollars

Conclusion in words:



Between the years 2005 and 2015, the radio industry has ~~[illegible]~~ had 142.56025 billion dollars ~~[illegible]~~ invested in the radio industry.

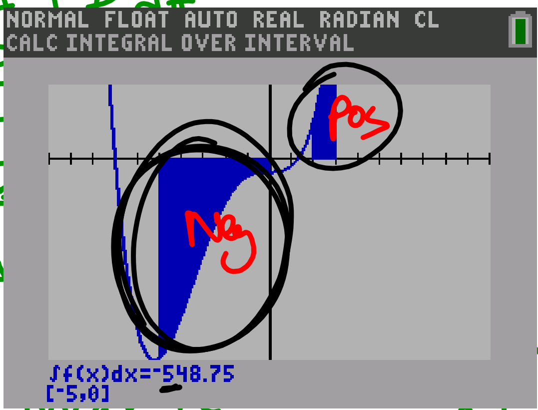
19. Was the area under the b



using calculator?

Calc 7: lower 5 Upper:

Start



Start

$Y_1 =$

hr $= 101.85$ mg

4 mg

$\frac{4074.4}{4}$ hr

0 4

4hr

$\frac{1463.25}{22}$ .90 hr

3 25

22hr

$= 66.51$

DATE: Mar/28/16

Writer: Laura Riley

Team Leader: Alan

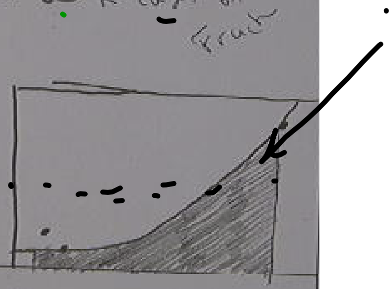
x-axis (independant variable): cases of fracking

y-axis (dependant variable): number of earthquakes

Conclusion in words:

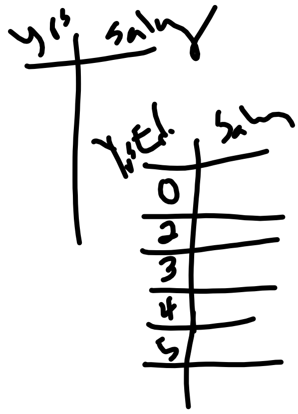
43280

According to the ExpReg there are 43280 thousand fracking-earthquakes between 20 and 80 k cases of frack



9. Was the area under the best regression and between the first and last values found using calculator?

Project Topics

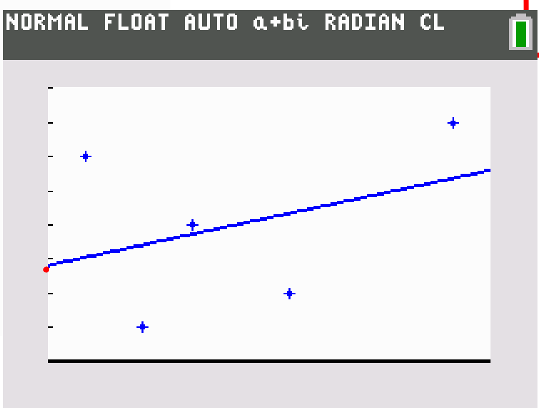


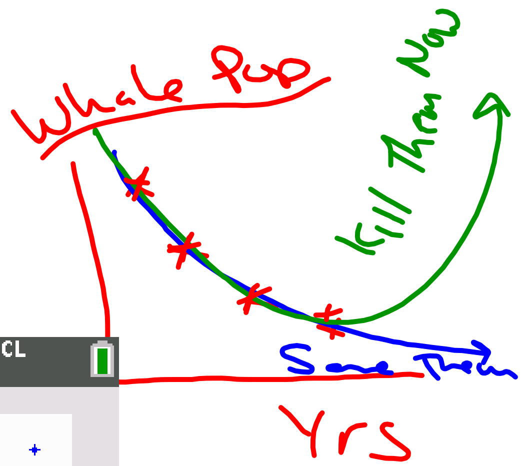
√To get r^2

- Catalog (2nd 0)
- Diagnostic On
- Enter (says DONE)
- redo regressions

NORMAL FLOAT AUTO a+bi RADIAN CL

LinReg
y=ax+b
a=.0102564103
b=2.820512821
r²=.1262327416
r=.3552924733

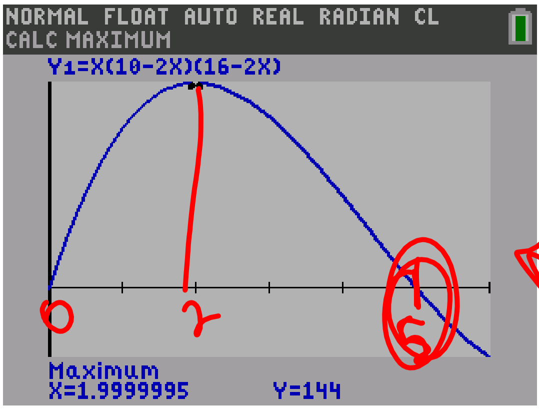


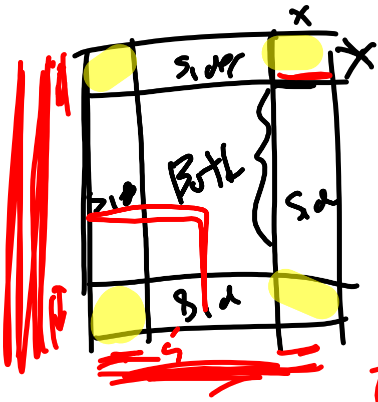


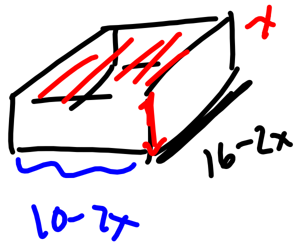
Optimization

A sheet of paper 10''-by−16'' is made into an open box (i.e. there's no top), by cutting $x-$ in. squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.

x = 2 is the max.







Volume = LWH

$= x(10-2x)(16-2x)$

A sheet of paper **10''–by–16''** is made into an open box (i.e. there's no top), by cutting **x– in. squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.**

x = [ ] is the max.

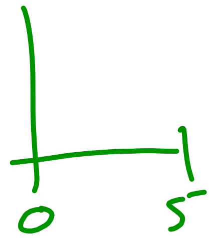
$V(H,L,W)$ $\quad H \cdot L \cdot W$

$v(x) = V = x(10-2x)(16-2x)$

$V = x(160 - 52x + 4x^2)$

$V = 160x - 52x^2 + 4x^3$

$V' = 160 - 104x + 12x^2 = 0$



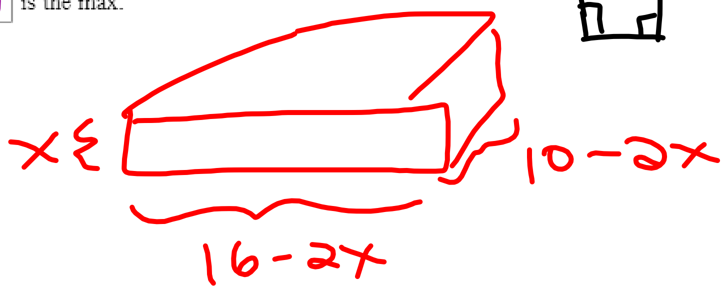
Optimizations

A sheet of paper 10"–by–16" is made into an open box (i.e. there's no top), by cutting x– in. squares out of each corner and folding up the sides. Find the value of x that maximizes the volume of the box.

x = 2 is the max.

$V = lwh$

$= (10-2x) \cdot (16-2x) \cdot x$



zeros: 0, 5, 8

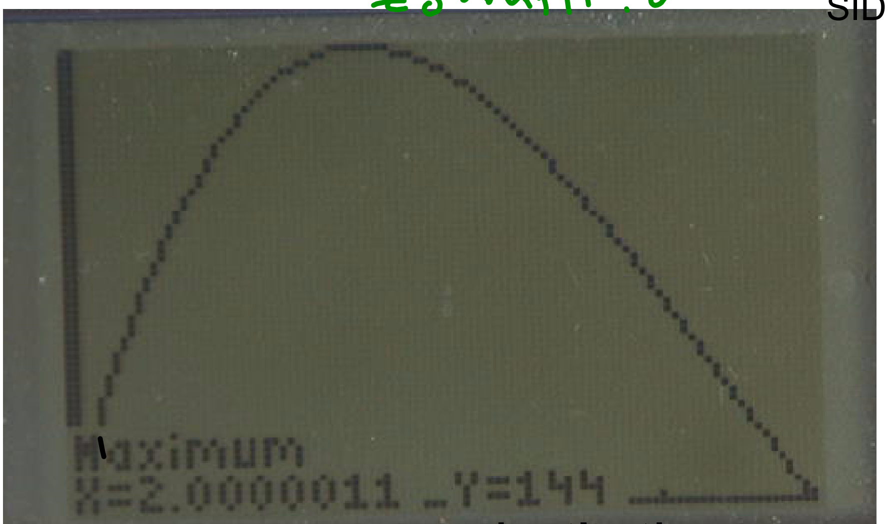
$V(L,W,H) = L \; W \; H$

$V(x) = (10-2x)(16-2x)x$

Window
Xmin: 0
Xmax: 5

HALF THE SMALLER SIDE

Zoomfit: 0



Y= . . . .

calc 4: maximum

LEft:0

Right:5

Guess: 5

where 2in what's the max 144 in^2

when 2in

what x 2in

$y = x(16-2x)(16-2x)$

$x(160 - 32x - 26x + 4x^2)$

by hand

$y = 4x^3 - 52x^2 + 160x$

$y' = 12x^2 - 104x + 160$

$3x^2 - 26x + 40 = 0$

$$\frac{26 \pm \sqrt{26^2 - 4(3)(40)}}{6}$$

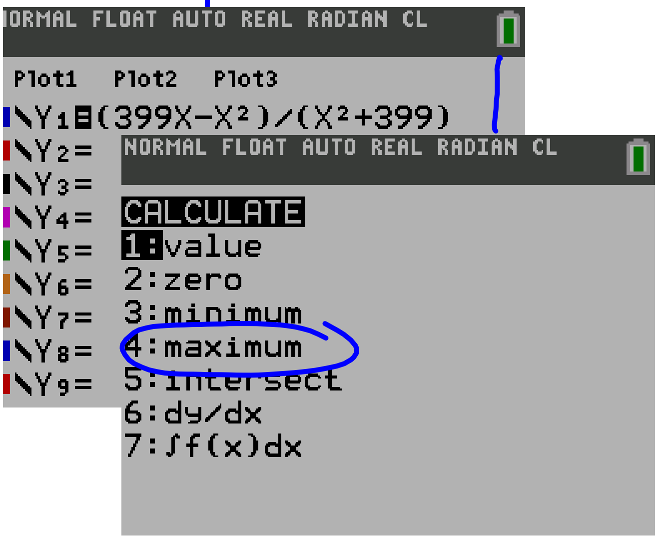
120 <

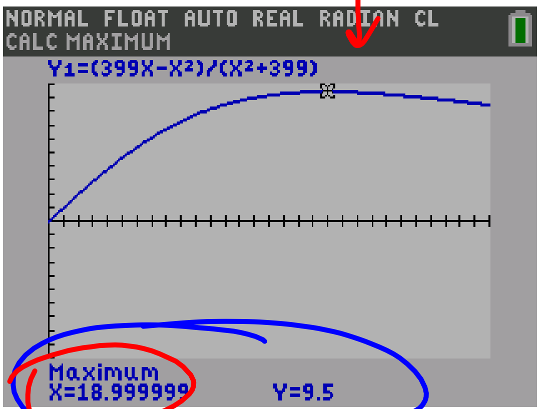
$\overline{26}$

A company's revenue for selling x (thousand) items is given by $R(x) = \dfrac{399x - x^2}{x^2 + 399}$.

Find the value of x that maximizes the revenue and find the maximum revenue.

x = 19 , maximum revenue is \$ 9.5





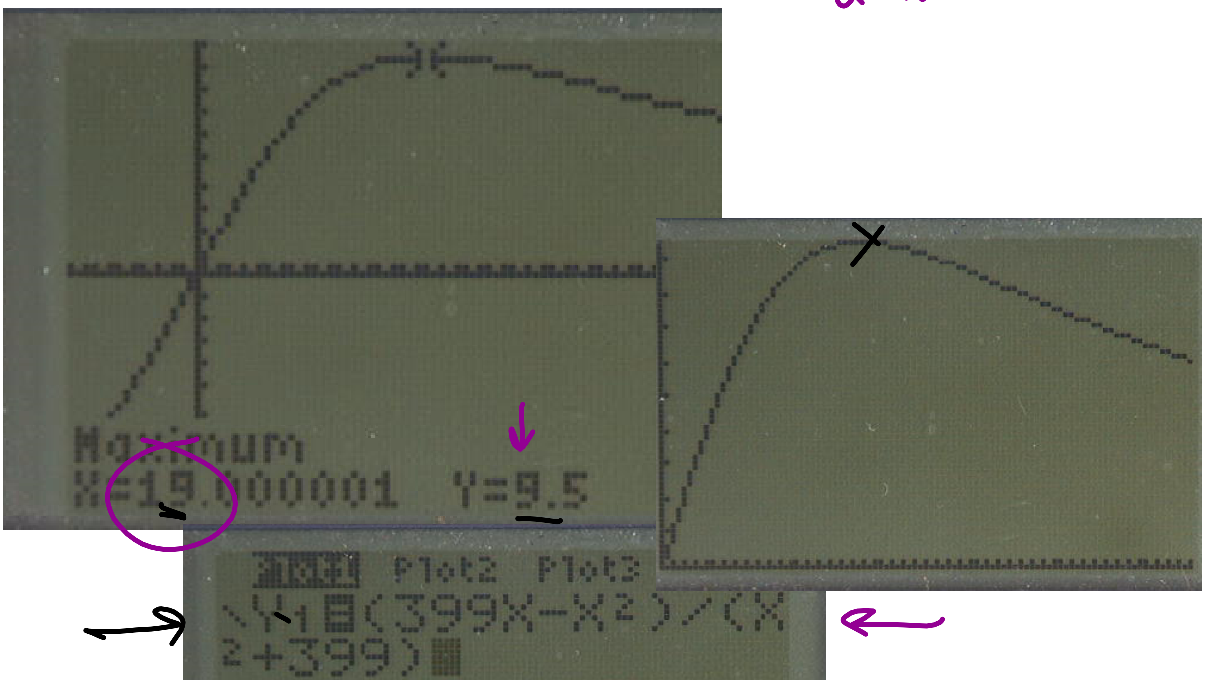
A company's revenue for selling x (thousand) items is given by $R(x) = \dfrac{399x - x^2}{x^2 + 399}$ $\neq 0$

Find the value of x that maximizes the revenue and find the maximum revenue.

x = 19 , maximum revenue is $ 9.5

$$R' = (x^2+399)(399-2x) - (399x - x^2)(2x) = 0$$

Quadratic Formula



L'Hôpital Rule

$$y = \frac{0}{0},\ \frac{\infty}{\infty},\ 0\cdot\infty$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0} \Rightarrow \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Lecture Anti-derivative

$$\frac{d}{dx}\left(x^3 + c^{\text{onstant}}\right) = 3\underline{x^2}$$

$$\int \underline{3x^2}\,dx = \underbrace{x^3 + \text{Constant}}$$

anti

Indefinite Integral

$$\int 2x\,dx = x^2 + \text{constant}$$

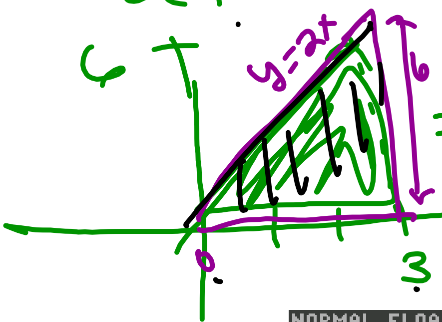
anti-derivative of "2x"

Definite Integrals

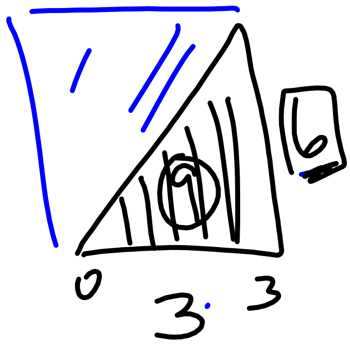
$$\int_0^3 2x\,dx = 9$$

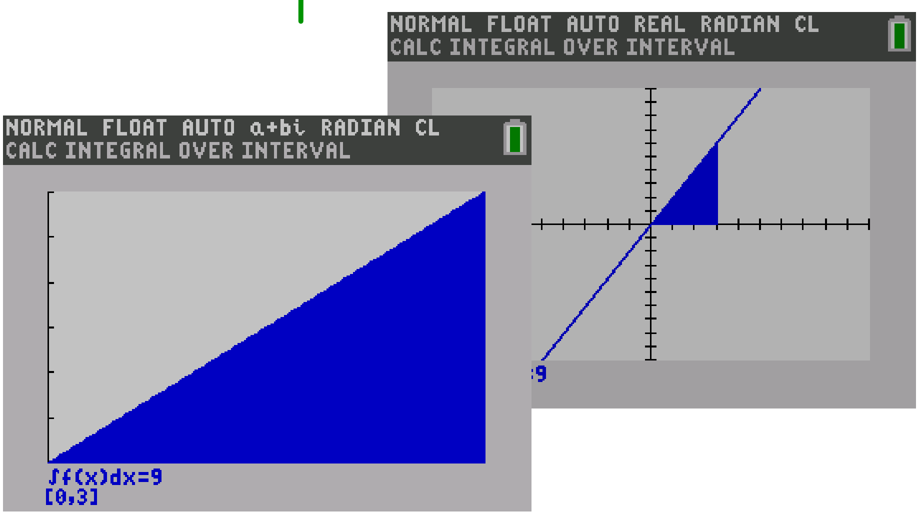
→ area under 2x between 0 & 3 is 9

Area under $y = 2x$ between 0 & 3



Area = 9





$y_1 = 2x$

Rules

Power

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

Ex $\int x^6 dx = \frac{x^7}{7} + c$

Ex $\int x^{-3/4} dx = \frac{x^{1/4}}{1/4} + c = 4x^{1/4} + c$

Rules

sum/differ

$$\int f(x) + g(x) \, dx$$
$$\int f(x)\,dx + \int g(x)\,dx$$

Ex $\int x^3 + x^2 \, dx = \int x^3 dx + \int x^2 dx$

$$= \frac{x^4}{4} + \frac{x^3}{3} + c$$

Rule

m.lt. by constant

$$\int A f(x) \, dx = A \int f(x) \, dx$$

Take constant out.

Ex $\int 3x^5 dx = 3 \int x^5 dx$

$$= \frac{3x^6}{6} + c$$

$$= \frac{x^6}{2} + c$$

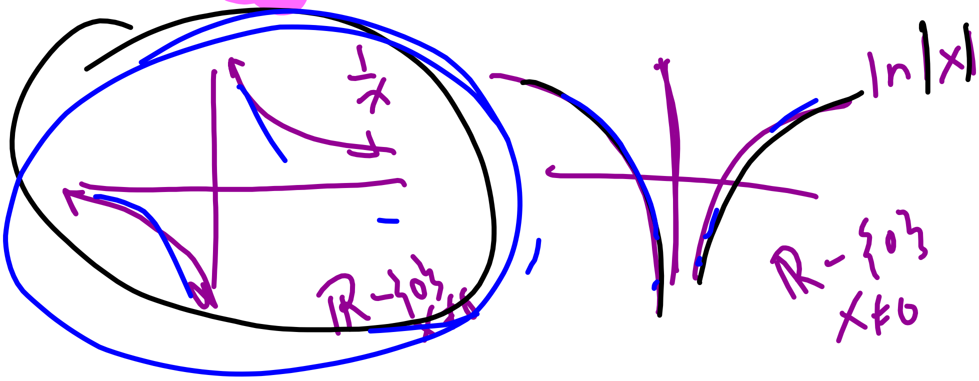
$\frac{x^6}{2}$

Trans:

$$\int e^x\,dx = e^x + c$$

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

$$\int \cos x\,dx = \sin x + c$$

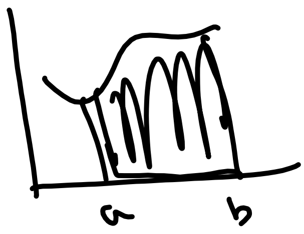


Project

Area under curve/Ave Value

Present: Max/Mins/Inflection Pts

Prepare: Find area under curve



area=