<u>Agenda</u>

Review and Examples of L'Hopitals Rule

Preview Analysis of Functions

Project: Finding Critical Points

Review

Find sin(9/4) using Local Linear Approx.

$0, \pi/6, \pi/4, \pi/3, \pi/2 \approx 3/2$

$\frac{9}{4} \approx \frac{3\pi}{4}$

Ex $\sin\left(\frac{9}{4}\right)$ $\quad \frac{3\pi}{4} \approx \frac{9}{4}$

$x = a = \frac{3\pi}{4}$ $\quad y = \sin x$

$y = \sin\left(\frac{3\pi}{4}\right)$ $\quad y' = \cos x$

$= \frac{\sqrt{2}}{2}$ $\quad m = y'\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\left(\frac{3\pi}{4}, \frac{\sqrt{2}}{2}\right)$ $\quad m = -\frac{\sqrt{2}}{2}$

$y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right)$

$y\left(\frac{9}{4}\right) = \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(\frac{9}{4} - \frac{3\pi}{4}\right)\right) = .7821$

$y = x^3 - 4x^2 - 7 = 0 \rightarrow y' = 3x^2 - 8x$

Guess $x = 1$

$$X_1 = 1 - \frac{1-4-7}{3(1)^2 - 8(1)} = -1$$

$1 - \frac{-10}{-5}$

$1 - 2 = -1$

$$X_2 = -1 - \frac{-1-4-7}{3+8} =$$

$-1 + \frac{+12}{11} = \frac{1}{11}$

Your response:

The frequency at which a guitar string vibrates (which determines the pitch of the note we hear) is related to the tension T to which the string is tightened, the density ρ of the string, and the effective length L of the string by the equation $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$.

By running a finger along a string, a guitarist can change the distance between the bridge and their finger. Suppose that $L = 1/2$ ft and $\sqrt{\frac{T}{\rho}} = 300$ ft/s so that the units of f are Hertz (cycles per second).

If the guitarist's hand slides so that $L'(t) = -4$, find $f'(t)$.

At this rate, how long will it take to raise the pitch one octave (that is, double f)?

$f'(t) =$ 2400

$$f'(t) = \frac{d}{dt} f(t) = \frac{d}{dt}\left(\frac{1}{2L(t)}\sqrt{\frac{T}{\rho}}\right)$$

$$= \frac{1}{2}\sqrt{\frac{T}{\rho}} \cdot \frac{d}{dt} L(t)^{-1} = \frac{1}{2}\sqrt{\frac{T}{\rho}} \cdot (-1) L^{-2} \cdot L'$$

$$= \frac{1}{2}(300)(-1)(1/2)^{-2} \cdot (-4) = 2400$$

$$=$$

$$f = \frac{1}{2} L^{-1} \cdot \sqrt{\frac{T}{\rho}}$$

$$f' = \frac{1}{2}(-1)(L)^{-2}\left(\frac{dL}{dt}\right) \cdot \sqrt{\frac{T}{\rho}}$$

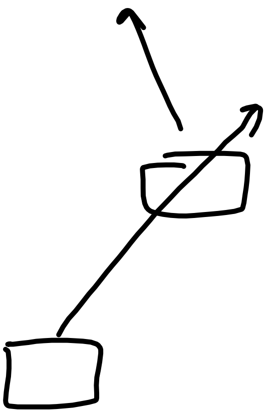
room

…cy at which a guitar string vibrates (which determines the pitch of the note we …ted to the tension T to which the string is tightened, the density ρ of the string, and the effective length L of the string by the equation $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$. By running a finger along a string, a guitarist can change the distance between the bridge and their finger.

Suppose that $L = 1/2$ m and $\sqrt{\frac{T}{\rho}} = 100$ m/s so that the units of f are Hertz (cycles per second).

If the guitarist's hand slides so that $L'(t) = -1$, find $f'(t)$. At this rate, how long will it take to raise the pitch one octave (that is, double f)?

$f'(t) =$ 200



Talking:

$$f = \frac{1}{2} L^{-1} \sqrt{\frac{T}{\rho}}$$

$$f' = -\frac{1}{2} L^{-2} L' \sqrt{T/\rho}$$

$$f' = -\frac{1}{2}(1/2)^{-2} L' \sqrt{T/\rho}$$

$$f' = -\frac{1}{2} \cdot 4 \cdot (-1) \cdot 100$$

$$= -\frac{1}{2} \cdot 4 \cdot (-1)\, 100$$

$$200$$

$$y = x^{-1}$$

$$y' = -1x^{-1-1} = -x^{-2}$$

$$\frac{dy}{dx} = -x^{-2}$$

$$\frac{dy}{dt} = -x^{-2} \cdot \frac{dx}{dt}$$ (chain rule)

Review L'Hopitals Rule

Give your final answer as a reduced fraction.

Evaluate $\lim_{x \to 4} \frac{7\sqrt{x} - 14}{x - 4}$.

$= \frac{7\sqrt{4} - 14}{4 - 4} \begin{matrix} = 0 \\ = 0 \end{matrix}$ Use L'Hopital Rule

$\lim_{x \to 4} \frac{7 \cdot \frac{1}{2} \cdot x^{-1/2}}{1} = 3.5 \cdot \frac{1}{\sqrt{4}_{=2}} = \frac{7}{4}$

$\frac{d}{dx}\sqrt{x} = \frac{d}{dx} x^{1/2} = \frac{1}{2}x^{-1/2}$

Give your final answer as a reduced fraction.

Evaluate $\lim_{x \to 4} \frac{7\sqrt{x} - 14}{x - 4}$. $= \frac{0}{0}$

Use LHR

$\lim_{x \to 4} x^{-1/2} = 4^{-1/2}$

$= 1/2$

$\lim_{x \to 4} \frac{7\left(\frac{1}{2}\right)x^{-1/2}}{1} = 7 \cdot \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{7}{4}$

$\downarrow$

$\lim_{x \to 4} \frac{7}{2\sqrt{x}}$

OR

$$\frac{7(\sqrt{x}-2)}{x-4} \cdot \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)}$$

$$\frac{7\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x}+2}$$

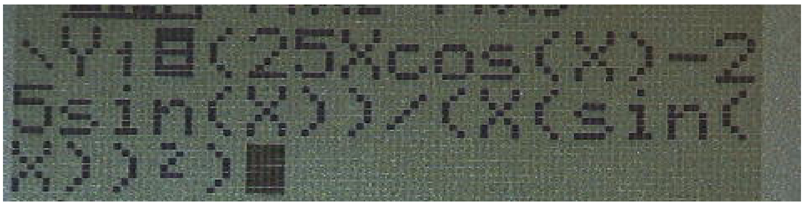
$$\lim_{x \to 4} \frac{7}{\sqrt{x}+2} = \frac{7}{4}$$

Find the limit $\lim_{x \to 0} \frac{25x \cdot \cos x - 25\sin x}{x . \sin^2 x}$. $= \frac{0 \cdot 1 - 0}{0 \cdot 0} = \frac{0}{0}$

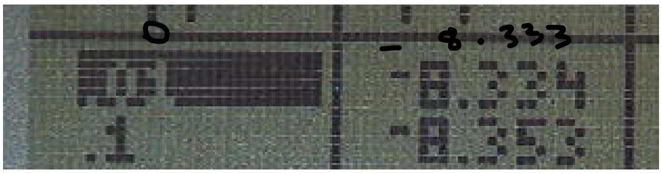
Use L'Hôpital

$$\lim_{x \to 0} \frac{25x \cdot (-\sin x) + 25 \cdot \cos x - 25 \cos x}{x \cdot 2\sin x \cos x - \sin^2 x} = \frac{0}{0}$$

DR



0 – 8.333



$-8\frac{1}{3}$

Find the limit $\lim_{x \to 0} \frac{\underbrace{25x \cos x} - \underbrace{25\sin x}}{x \sin^2 x}$. $= \frac{0}{0}$ USE LHR

$(\sin x)^2$

$$\frac{\underbrace{-25x\sin x + \cancel{25\cos x}} \underbrace{-\cancel{25\cos x}}}{\sin^2 x + x(2)(\sin x)\cos x}$$

$$\lim_{x \to 0} \frac{-25x}{\sin x + 2\underbrace{x\cos x}} = \frac{0}{0} \text{ USE LHR}$$

$$\lim_{x \to 0} \frac{-25}{\cos x + -2\underbrace{x\sin x + 2\cos x}} = \frac{-25}{1+2}$$

$$= -\frac{25}{3} = -8\tfrac{1}{3}$$

Evaluate $\lim_{x \to \infty} \left(\sqrt{25x^2 + 6} - 5x \right) \cdot \dfrac{\sqrt{25x^2+6} + 5x}{\sqrt{25x^2+6} + 5x}$

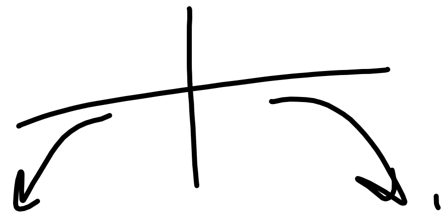
$$\lim_{x\to\infty} \frac{\cancel{25x^2}+6-\cancel{25x^2}}{\sqrt{25x^2+6}+5x} = \lim_{x\to\infty} \frac{6}{\infty+\infty} = \frac{6}{\infty}$$

$$= 0$$

$$\lim_{x \to \infty} \boxed{x^3 - 1x^4} = -\infty$$

① Quartic P4

Lead : -1



Evaluate $\lim\limits_{x \to \infty} \left(\sqrt{25x^2 + 6} - 5x \right)$.

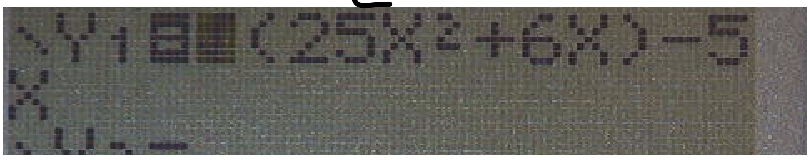
$\infty - \infty$

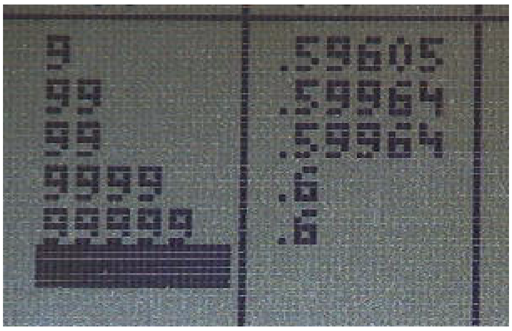
$$\lim_{x \to \infty} \frac{\sqrt{25x^2+6} - 5x}{1} \cdot \frac{\sqrt{25x^2+6} + 5x}{\sqrt{25x^2+6} + 5x}$$

$$\lim_{x \to \infty} \frac{\cancel{25x^2}+6 - \cancel{25x^2}}{\sqrt{25x^2+6} + 5x}$$

$$\lim_{x \to \infty} \frac{6}{\sqrt{\infty} + 5\infty} = \frac{6}{\infty} = 0$$

50	.012
999	6E-4
999999	0





$$\lim_{x\to\infty} \frac{6x}{\sqrt{25x^2+6x}+5x}$$

like

like "0"

$$\Rightarrow \frac{6x}{\sqrt{25x^2}+5x}$$

$$\Rightarrow \frac{6x}{10x}$$

$$\rightarrow .6$$

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$P = Q e^{RT}$$

$$\lim_{N \to \infty} \boxed{Q \left(1 + \frac{R}{N}\right)^{NT}}$$

$$\lim_{x\to\infty}\left(1+\frac{R}{x}\right)^{x} = 1^{\infty}$$

$$e^{\ln\left(\lim_{x\to\infty}\left(1+\frac{R}{x}\right)^{x}\right)} = e^{\lim_{x\to\infty}\ln\left(1+\frac{R}{x}\right)^{x}}$$

$$= e^{\lim_{x\to\infty} x \ln\left(1+\frac{R}{x}\right)} \quad \infty \cdot 0$$

$$= e^{\lim_{x\to\infty}\frac{\ln\left(1+\frac{R}{x}\right)}{x^{-1}}} \quad \frac{0}{0} \text{ use L'H}$$

$$= e^{\lim_{x\to\infty}\frac{\frac{1}{(1+R/x)}\cdot R \cdot \cancel{\frac{d}{dx}\left(\frac{1}{x}\right)}}{\cancel{\frac{d}{dx}\left(\frac{1}{x}\right)}}}$$

$$= e^{\lim_{x\to\infty} R\left(\frac{1}{1+R/\infty}\right)} = R$$

$$= e^{R}$$

$$\boxed{P = O\left(1+\frac{R}{N}\right)^{NT}}$$

$$P = O e^{RT}$$

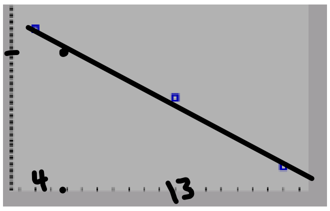
$$\lim_{N\to\infty}\left(1+\frac{R}{N}\right)^{N} = e^{R}$$ Comp continuously $N = \infty$

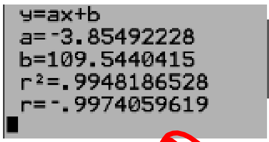
$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$,

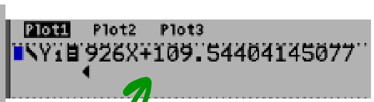
1^{∞}, $\infty - \infty$. $0, 1, \infty$

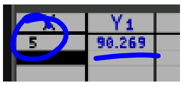
$e^{\ln 1^{\infty}}$ = $\infty - \infty$. $\frac{\infty + \infty}{\infty + \infty}$

Table.









Stat Edit

L1	L2
4	93
13	62
20	31

The price of an item affects consumer demand for that item. Suppose that based on market research, a company estimates that $f(x)$ thousand small flashlights can be sold at the price of $\$x$, as given in the table.

As

L1	x	4	13	20
L2	$f(x)$	93	62	31

Estimate the number of flashlights that can be sold at \$5.

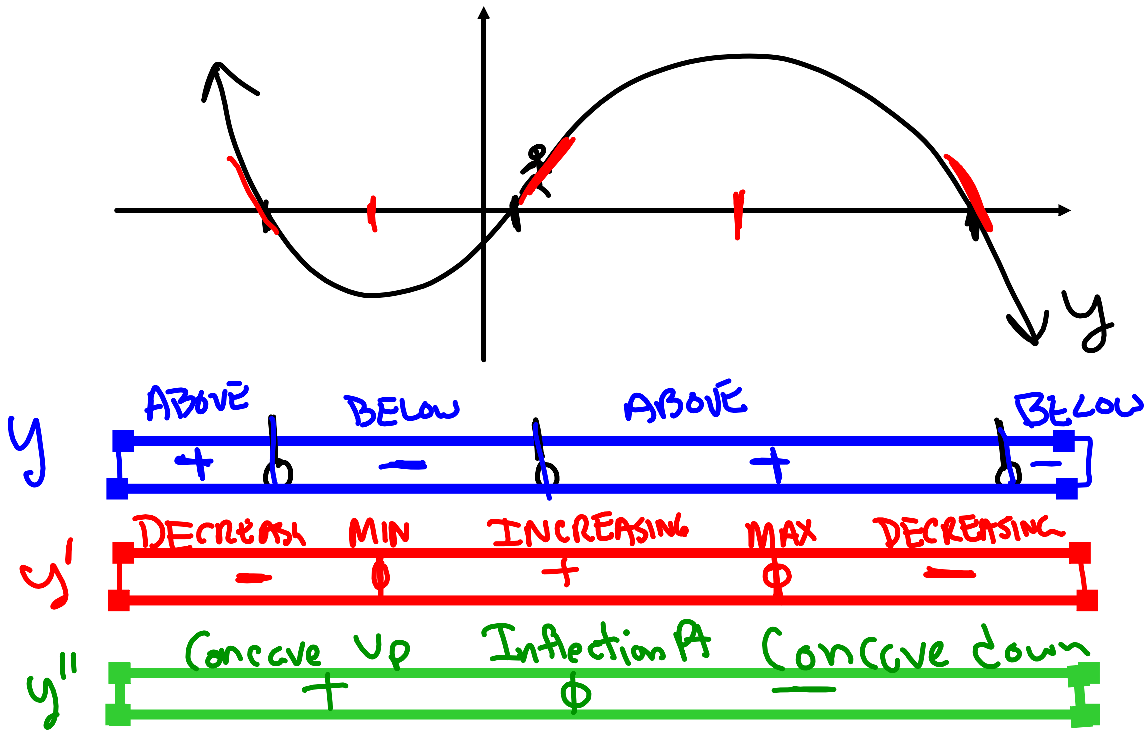
Round your final answer to the nearest integer.

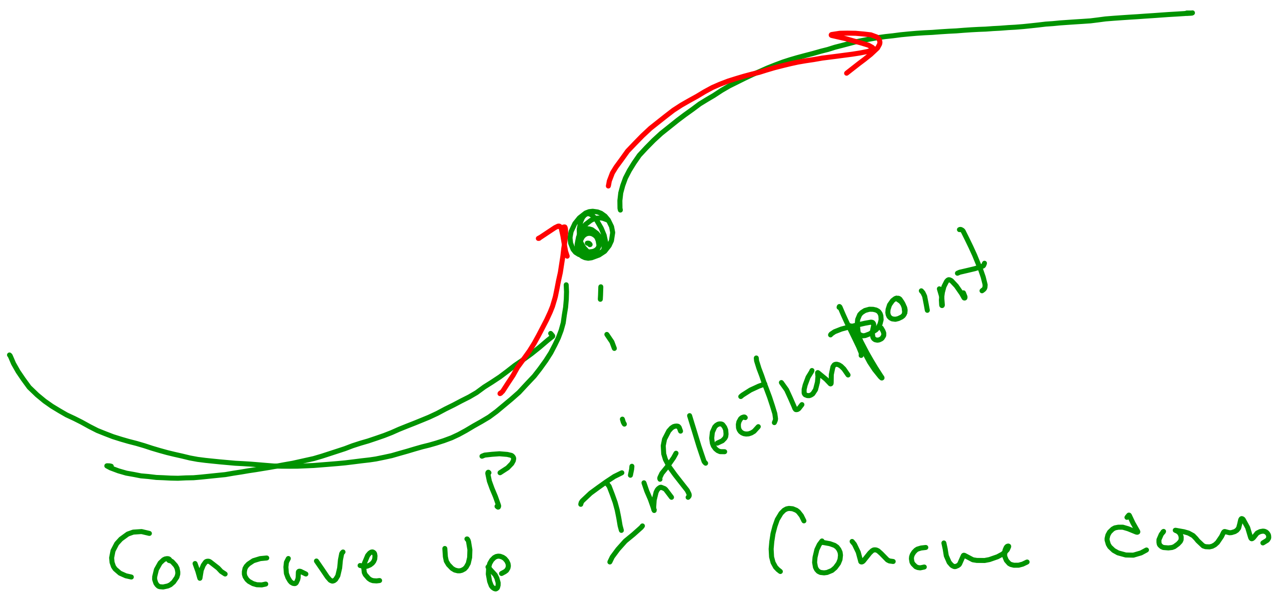
We would expect to sell approximately 90.26 thousand flashlights at a price of \$5.

Stat Calc: 4 LinReg

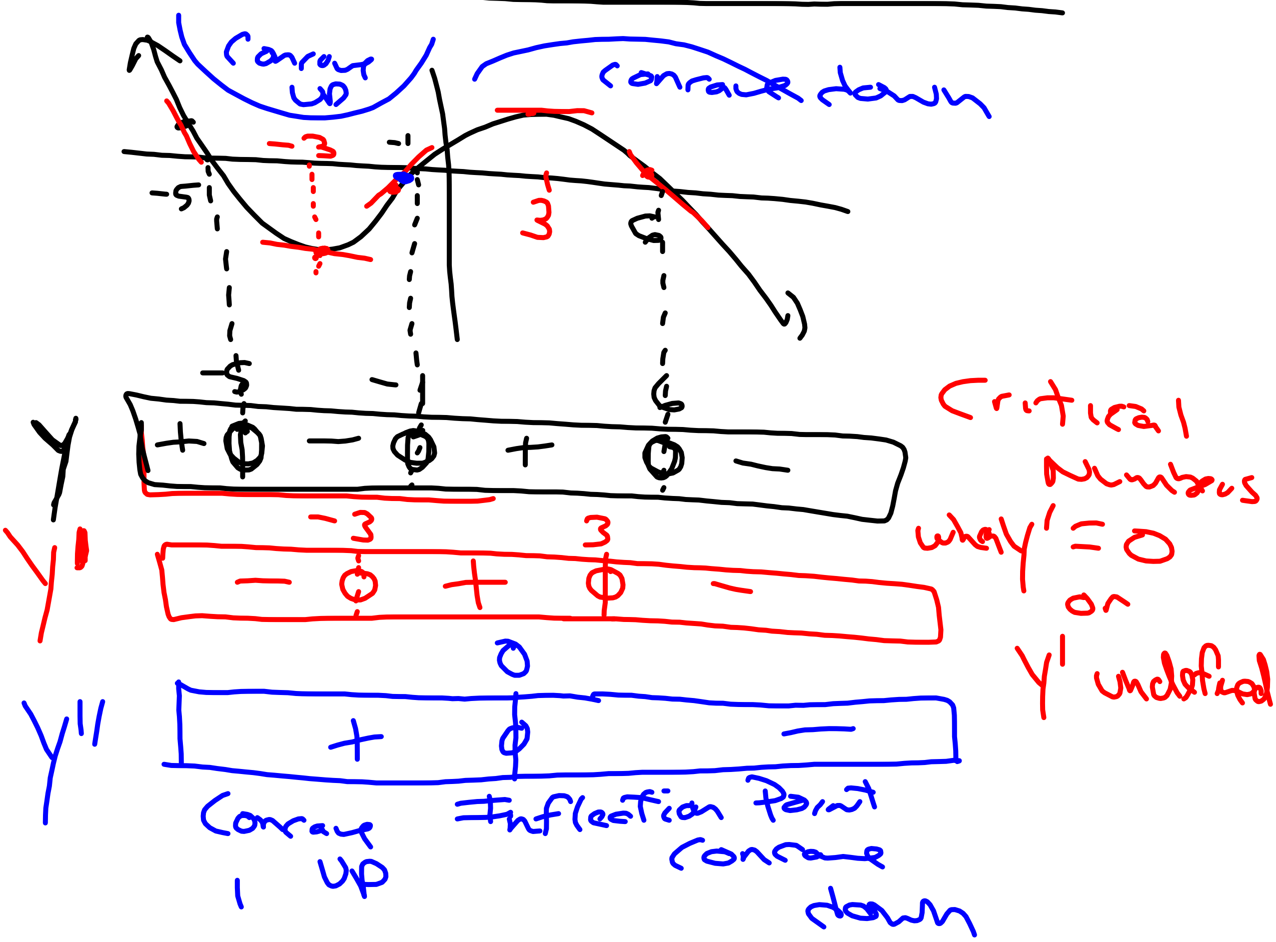
Y= Y1= VARS 5: >> 1:

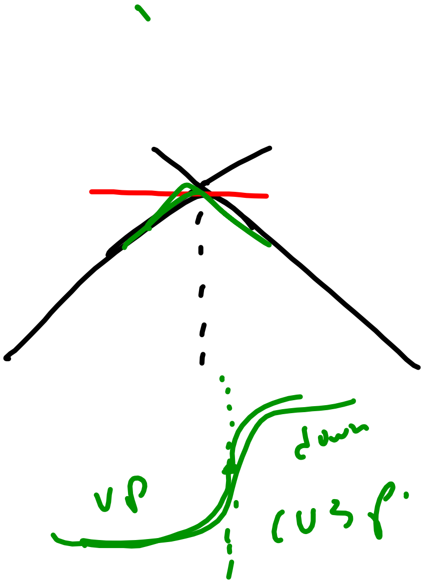
Analysis of Curves

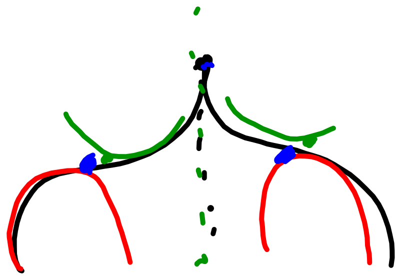



P
Inflection point
Concave up
Concave down

Analysis of curves

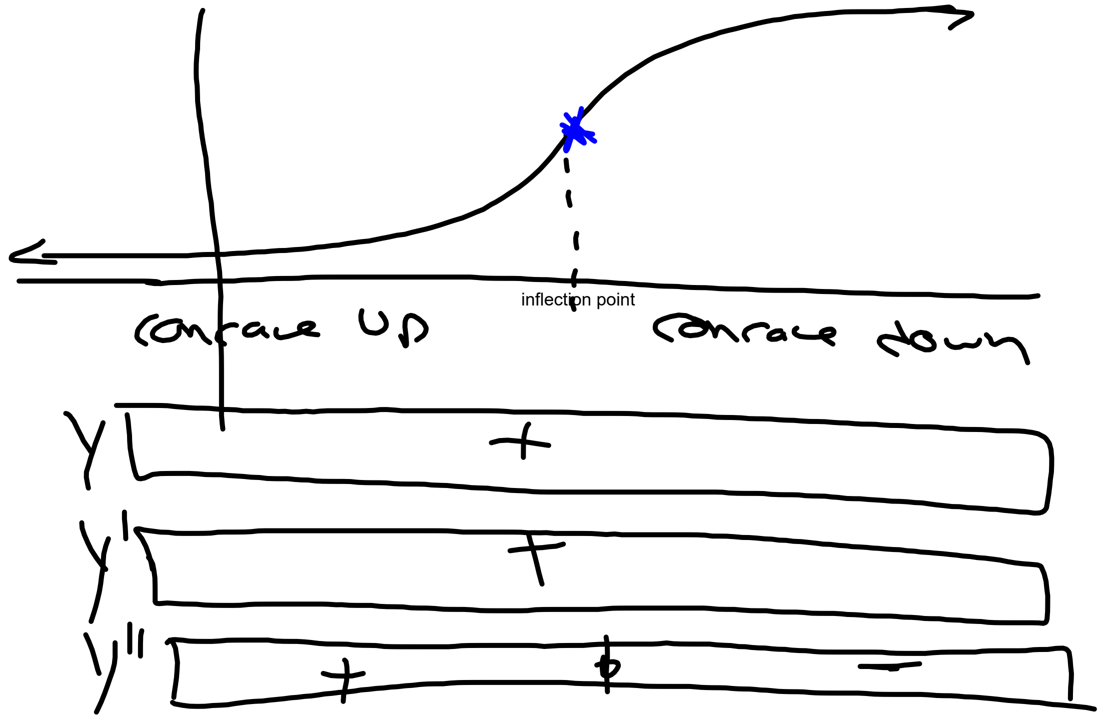


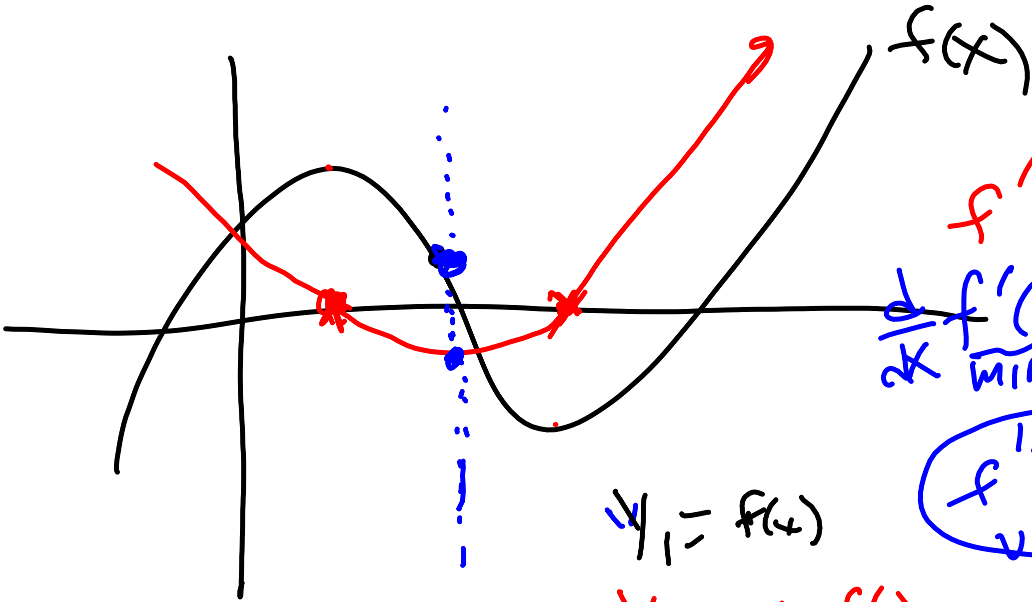




$$y = -|x|$$

$$y' = \cancel{1 \text{ or } -1} \quad \text{UNDEFINED !?}$$


inflection point
concave up
concave down
y
+
y'
+
y''
+
0
-



$f'(x) = 0$

$\frac{d}{dx} f'(x) = 0$

MIN ?

$f''(x) = 0$

$Y_1 = f(x)$

$Y_2 = \text{nDer } f(x)$

Calc max/min → Inflect.

$\rightarrow Y_1 =$

$\rightarrow Y_2 =$

$\rightarrow Y_3 =$ 2nd deriv Y_1
or
1st deriv Y_2

X	Y_1	Y_2	Y_3
7	3.7	.825	-.0917
8	4.5	.79167	.00833
9	5.3	.80833	.00833
10	6.1	.775	-.0917

$5 \rightarrow X$

$X - Y_2/\text{nDeriv}(Y_2, X, X) \rightarrow X$

(enter)

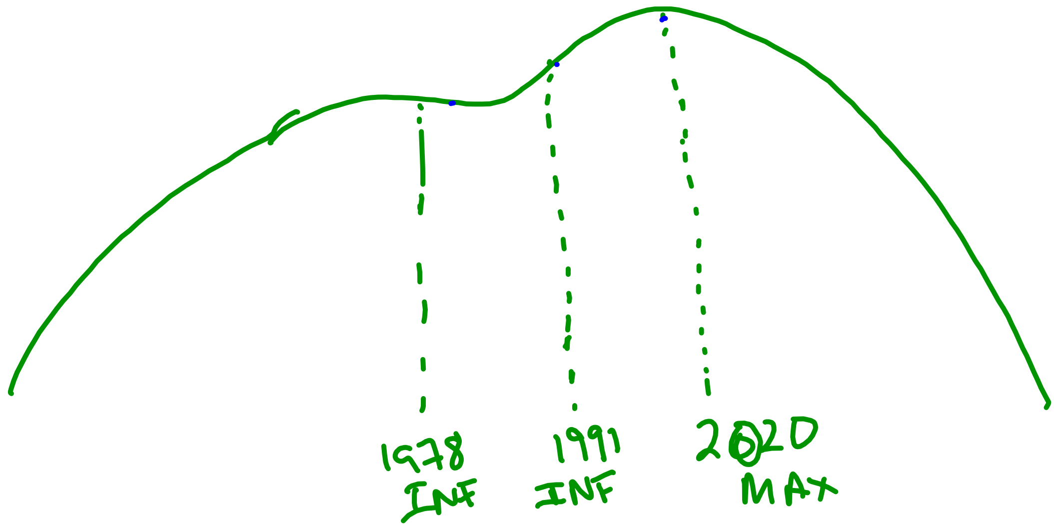
zero for Y' critical #.

$5 \rightarrow X$

$X - Y_3/\text{nDeriv}(Y_3, X, X) \rightarrow X$

zero for Y'' Inflection pt

According to the quartic regresion....



X	Y1	Y2	Y3
20	11.641	.10104	-.0122
40	11.627	-.0832	-.0064
60	9	-.1643	-.0019
80	5.5771	-.1669	.00142
120	1.1463	-.0351	.00435
150	2	.08682	.00332
180	5.6299	.13615	-5E-4
200	8.0043	.0877	-.0046
240	5.033	-.3152	-.0164
260	-5.049	-.7189	-.0242

max

Inf

min

Inf

max

down

decel. to accelerat

Inf

up

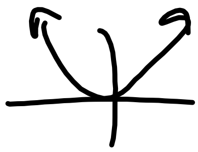
Inf point

Inflc

≈170 accelerating to Decelerating

down

$y = x^4$



$y' = 4x^3 = 0$ $= 0$

$y'' = 12x^2 = 0$

+	0	+

write

$y''(170) = 0$ 170

$y''(160) = -$ $y''(180) = +$

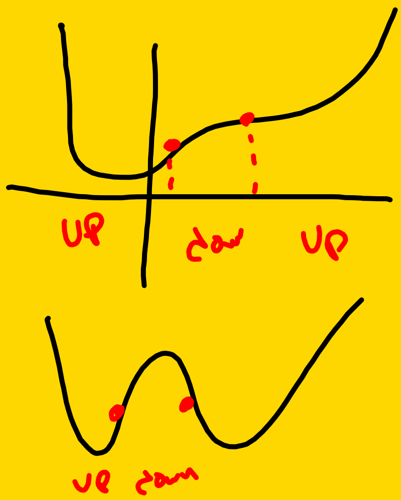
decel. accel

speaker According to Quartic Regression.

we expect at a price of $1.70 the

sale change from deleraty to accelerating

Project



Find critical numbers....Max and Mins

According to the quartic regression, the max/and minimums occur at __ __ and __

The inflection points occur at __ and __

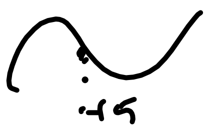
$Ax^4 + Bx^3 + Cx^2 + Dx + E$

$Y_1 =$ Quartic Reg

$Y_2 = 4*Ax\wedge 3 + 3*Bx^2 + 2Cx^1$

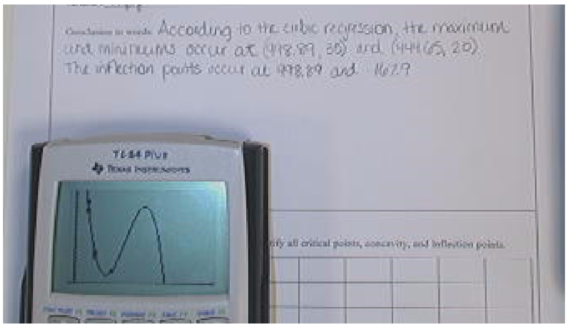
$+ D$

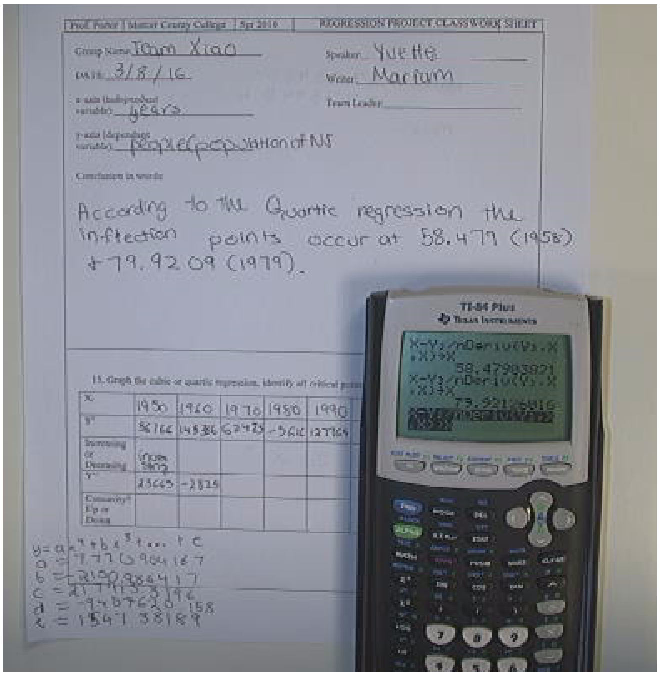
$Y_3 =$ (second deriv)

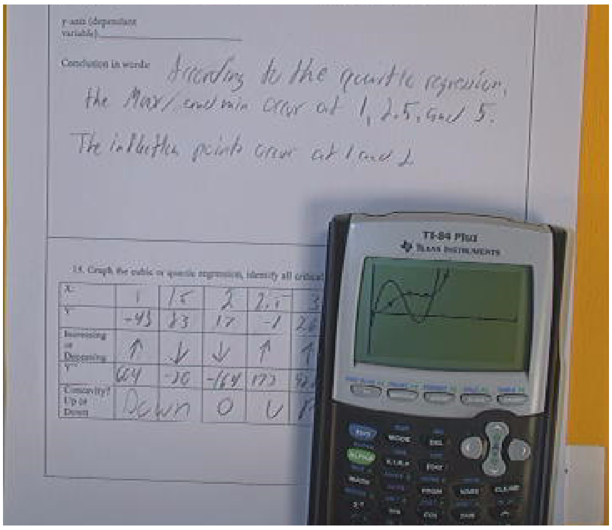


45 → X

X − Y_3 / nDeriv(Y_3,X,X) → X


According to the cubic regression, the maximum
and minimums occur at
The inflection points occur at
TI-84 Plus
Texas Instruments
all critical points, concavity, and inflection points.


REGRESSION PROJECT CLASSWORK SHEET
Group Name Team Xiao
Speaker Yvette
Writer Marium
Team Leader
Conclusion in words
According to the Quartic regression the inflection points occur at 58.479 (1958) + 79.9209 (1979).
TI-84 Plus
Texas Instruments
58.47983821


Conclusion in words:
TI-84 Plus
Texas Instruments

Conclusion in words: According to the inflection points of the quartic regressions, years ~~[illegible]~~ 2010.43 and ~~[illegible]~~ 2020.54 show a shift

15. Graph the cubic or quartic regression, identify all critical points, concavity, and inflection points.

X	5	8	9	10	11	15	25	26	30
Y	49.3	39.8	43.6	46.3	46.2	8.3	464.8	674.7	2,357.8
Increasing or Decreasing	−	+	+	−	−	+	+	+	+
Y''	-15.64	3.6	3.59	1.53	-1.87	-15.06	200.53	260.96	607.64
Concavity? Up or Down	up	up	up	up	down	down	up	up	up

Prof. Porter	Mercer County College	Spr 2016	REGRESSION PROJECT CLASSWORK SHEET

Group Name: Blitz

Speaker: Nick

DATE: 3/8/16

Writer: RJ

x-axis (independent variable): year

Team Leader: Jenys

Steve

y-axis (dependent variable): kg weight

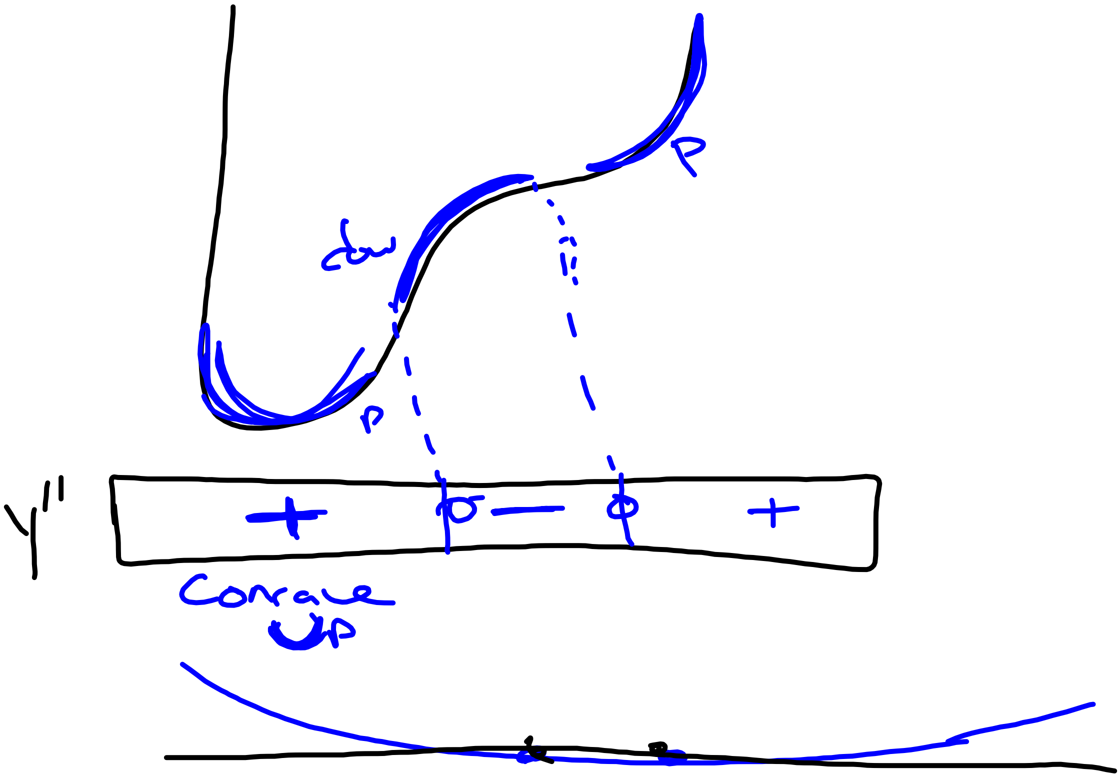
Conclusion in words:

According to the quartic regression
max/mins occur at $x = 121 = 1779$
&
inflection point occurs at: (-46.6377, 0) ; (72.810716, 0)
1846 1998

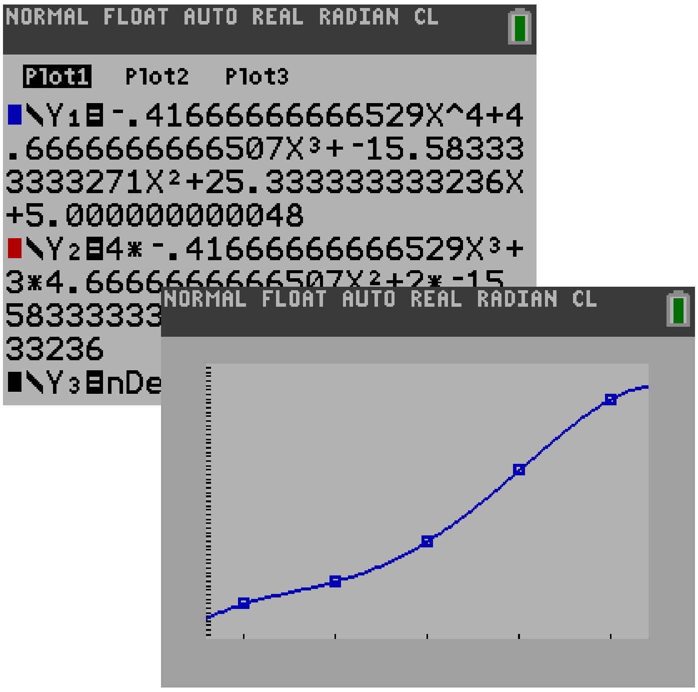
11. Graph the cubic or quartic regression, identify all critical points, concavity, and inflection points.

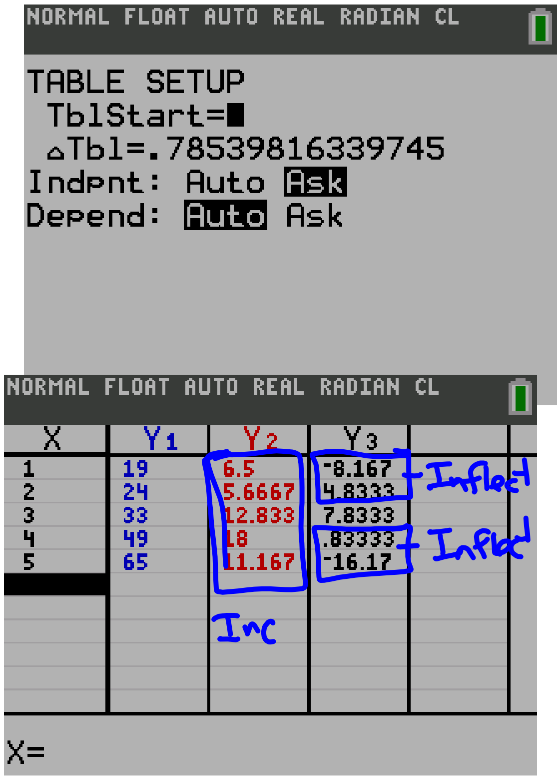
X	90	100	110	116	117	120	122	125	130
Y	7829	66269	81623	11351	12982	14684	1686	19202	24937
Increasing or Decreasing	Incr	Incr	Incr	Incr	Incr	Incr	Incr	Incr	Incr
Y'	.2174	3.9	34.5	63.5	67.1	79.6	88.1	101.31	144.2
Concavity? Up or Down	down	up	up	up	up	up	up	up	up

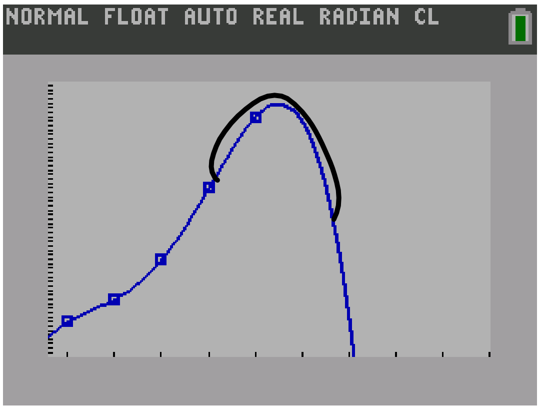
(-46.6377, 0)
(72.810716, 0)



down
p
P
y''
+
0
−
0
+
Concave up


NORMAL FLOAT AUTO REAL RADIAN CL
Plot1 Plot2 Plot3
Y1=-.41666666666529X^4+4.666666666507X³+-15.583333333271X²+25.333333333236X+5.000000000048
Y2=4*-.41666666666529X³+3*4.666666666507X²+2*-15.58333333
33236
Y3=nDe
NORMAL FLOAT AUTO REAL RADIAN CL


NORMAL FLOAT AUTO REAL RADIAN CL
TABLE SETUP
TblStart=
ΔTbl=.78539816339745
Indpnt: Auto Ask
Depend: Auto Ask
NORMAL FLOAT AUTO REAL RADIAN CL
X Y1 Y2 Y3
1 19 6.5 -8.167
2 24 5.6667 4.8333
3 33 12.833 7.8333
4 49 18 .83333
5 65 11.167 -16.17
Inflect
Inflect
Inc
X=



NORMAL FLOAT AUTO REAL RADIAN CL

X	Y_1	Y_2	Y_3
1	19	6.5	⁻8.167
2	24	5.6667	4.8333
3	33	12.833	7.8333
4	49	18	.83333
5	65	11.167	⁻16.17
6	64	⁻17.67	⁻43.17

X=7

max/min

+ /\ − 1st der. max

− 2nd der.

concave down