# Agenda

Review Quiz 3

Lecture Local Linear Approximation

Lecture Newton's Method

Project: Using Newtons Method

Review Quiz 5

**Find the derivative $y\,'(x)$ implicitly.**

$$\sqrt{xy} - 6y^2 = 87$$

Find the derivative $y\,'(x)$ implicitly.

$\sqrt{xy} - 6y^2 = 87$

$$\frac{d}{dx}\left[(xy)^{1/2} - 6y^2 = 87\right]$$

$$\frac{1}{2}(xy)^{-1/2} \cdot \frac{d}{dx}(xy) - 12yy' = 0$$

$$2\sqrt{xy}\left[\frac{1}{2\sqrt{xy}} \cdot (xy' + y) - 12yy' = 0\right]$$

$$xy' + y - 24y\sqrt{xy}\,y' = 0$$

$$(x - 24y\sqrt{xy})\,y' = -y$$

$$y' = \frac{y}{-x + 24y\sqrt{xy}}$$

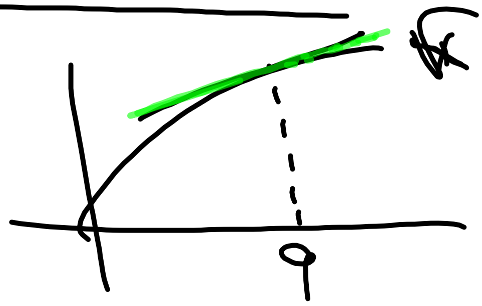
Lecture

Local Linear

# Local Linear Approx

$f(x) = \sqrt{x}$

$f(10) = \sqrt{10}$




Center - can evaluate function easily

" $a = 9$

Find the tangent Line (at 9)

$L(x) =$ Local Linear Approx

Point

$y = \sqrt{x}$

$(9, 3)$

$m$

$y' = \frac{1}{2} x^{-1/2}$ at $x = 9$

$y' = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

$9^{-1/2} = \frac{1}{\sqrt{9}} = \frac{1}{3}$

$y - 3 = \underbrace{\frac{1}{6}}_{m}(x - 9)$

$L(x) = y = 3 + \frac{1}{6}(x - 9)$

$L(10) = 3 + 1/6(10 - 9) = 3\,1/6$

Plot1 Plot2 Plot3
\Y1 = √(X)
\Y2 = 3+(1/6)(X−9)

local $L(x)$

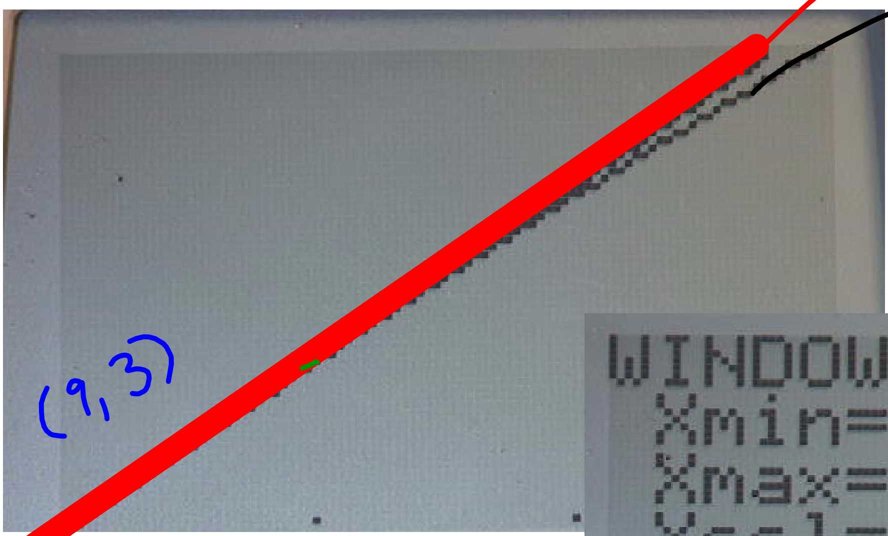
linear $\sqrt{x}$

approximation

$y=\sqrt{x}$ $\quad y'=\frac{1}{2}x^{-1/2}$

$y'(9)=\frac{1}{2}\frac{1}{\sqrt{9}}$

$=1/6$




WINDOW
Xmin=9
Xmax=12
Xscl=1
Ymin=3
Ymax=3.4641016
Yscl=1

| X | Y1 $\sqrt{x}$ | Y2 $=3+\frac{1}{6}(x-9)$ |
|---|---|---|
| 9 | 3 | 3 |
| 10 | 3.1623 | 3.1667 |
| 11 | 3.3166 | 3.3333 |
| 0 | 0 | 1.5 |
| 16 | 4 | |

3±(1/6)(10-9)

3+(1/6)(16-9)=4.16667

$\sqrt{92} = ???$ $a = 100$

$f(x) = \sqrt{x}$ Point $(100, 10)$

$f'(x) = \frac{1}{2}(x)^{-1/2}$ Slope of Tangent Line at 100 $= \frac{1}{20}$

$f'(100) = \frac{1}{2}\frac{1}{\sqrt{100}}$

$= \frac{1}{20}$

$y - y_1 = m(x - x_1)$

$y - 10 = \frac{1}{20}(x - 100)$

$y(x) = 10 + \frac{1}{20}(x - 100)$

$\sqrt{92} = y(92) = 10 + \frac{1}{20}(92 - 100)$

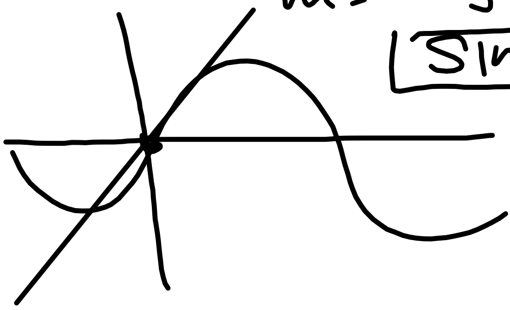
$= 10 - .4$

$= 9.6$

$m=1 \quad y=x$

$\boxed{\sin(x)} \sim \boxed{x}$

as long as X close to 0



$x=0$

Tangent Line to $y=\sin(x)$ at $x=0$

$y' = \cos(x)$

$m = y'(0) = \cos(0) = 1$

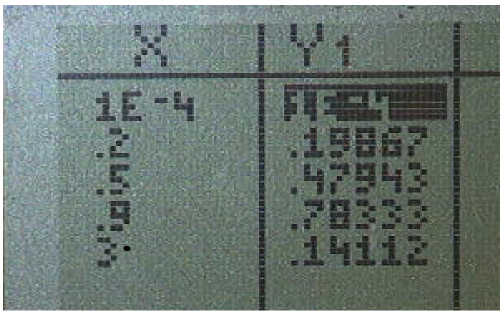
Point $= (0, 0)$

$\sin(0) = 0$

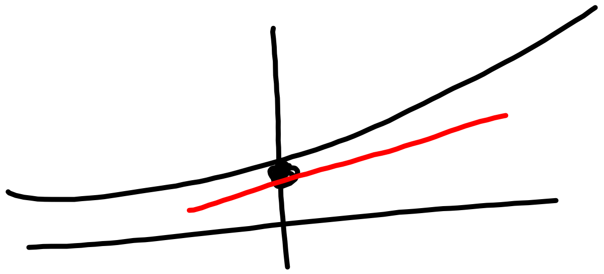
$y - 0 = 1(x - 0)$

$y = x$

sinx



| X | $Y_1$ |
|---|---|
| 1E-4 | [illegible] |
| .2 | .19867 |
| .5 | .47943 |
| .9 | .78333 |
| 3 | .14112 |



$e^x$

$e^{.01} \approx ?$

$f(x) = e^x$ Point $(0,1)$ $1.01$

$f'(x) = e^x$ $m = 1$

$f'(0) = 1$

$$y - 1 = 1(x - 0)$$

$$\boxed{y = x + 1}$$

$$e^{.01} \approx 1.01$$

```
e^(.01)
                 1.010050167
```

$f(x) = \sqrt[3]{x}$ $\qquad \sqrt[3]{16}$

center $\sqrt[3]{8}$ $\cancel{(\text{or } \sqrt[3]{27})}$

$x = 8$

POINT $(8, 2)$ $\qquad$ slope

$y = x^{1/3}$

$y - 2 = \frac{1}{12}(x - 8)$ $\qquad y' = \frac{1}{3}x^{-2/3}$

$= \frac{1}{3}(8)^{-2/3}$

$y = 2 + \frac{1}{12}(x - 8)$

$= \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

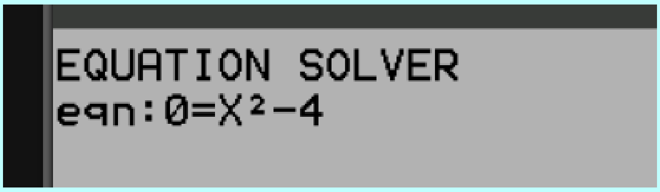
$y(16) = 2 + \frac{1}{12}(16 - 8)$

$= 2 + \frac{3}{4} = 2.75$

$2.52$

NEWTON's METHOD

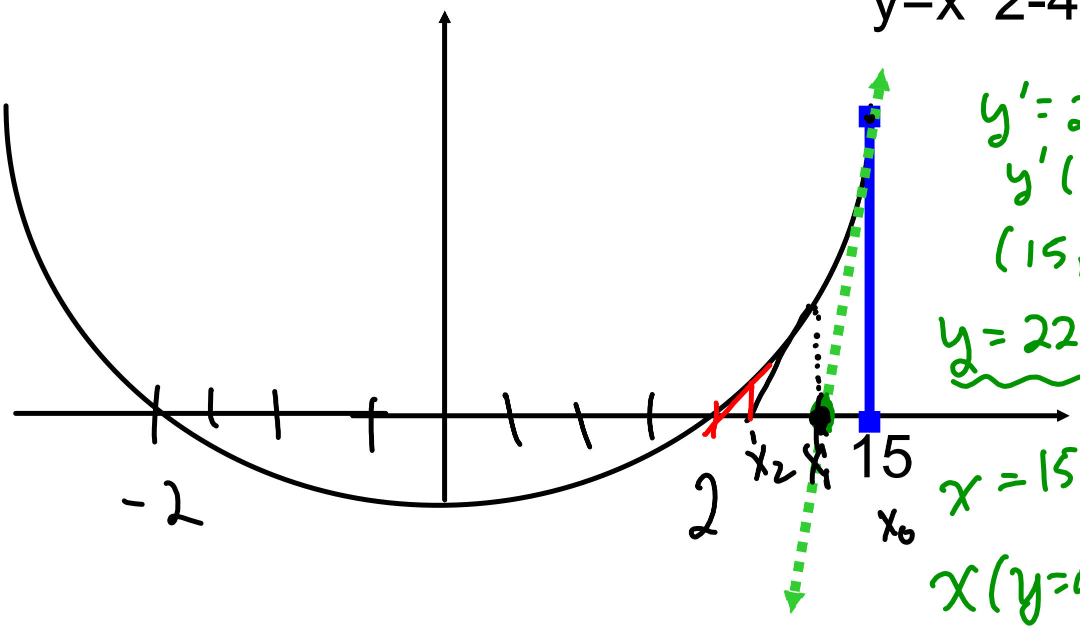
---used to find the zeros of a function




Need: Function (to be zeroed)

and an initial guess

y=x^2-4




$y' = 2x$

$y'(15) = 30 = m$

$(15, 221)$

$y = 221 + 30(x-15)$

zero

$x = 15 + \frac{y-221}{30}$

$x(y=0) = 15 - \frac{221}{30}$

$= 7.636$

$$\boxed{x_1 = x_0 - y_0/y_0'}$$

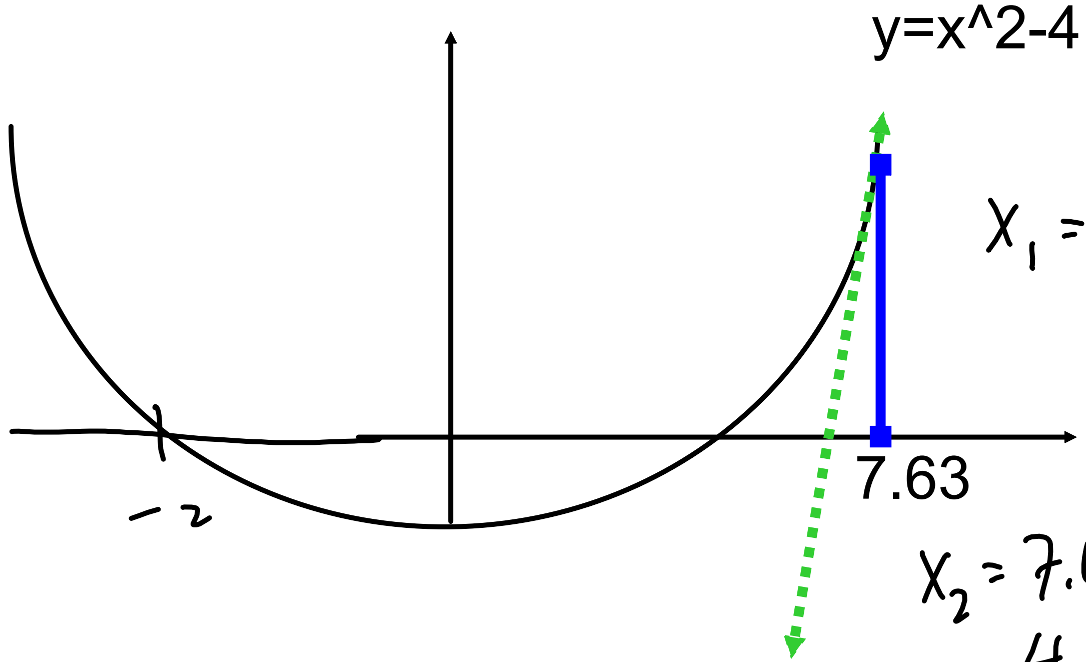
$$\underbrace{X_{n+1}}_{\text{next}} = \underbrace{X_n}_{\text{Early}} - \frac{y(x_n)}{y'(x_n)}$$

$x_2 = x_1 - y_1/y_1'$

$y = x^2 - 4$

```
NORMAL FLOAT AUTO REAL RADIAN CL
15→X
                               15
X-Y1/nDeriv(Y1,X,X)→X
                      7.633333333
X-Y1/nDeriv(Y1,X,X)→X
                5.2   4.0786754
```

```
NORMAL FLOAT AUTO REAL RADIAN CL
                      2.529692976
X-Y1/nDeriv(Y1,X,X)→X
                      2.055456265
X-Y1/nDeriv(Y1,X,X)→X
                      2.000748106
X-Y1/nDeriv(Y1,X,X)→X
                       2.00000014
X-Y1/nDeriv(Y1,X,X)→X
                                2
```

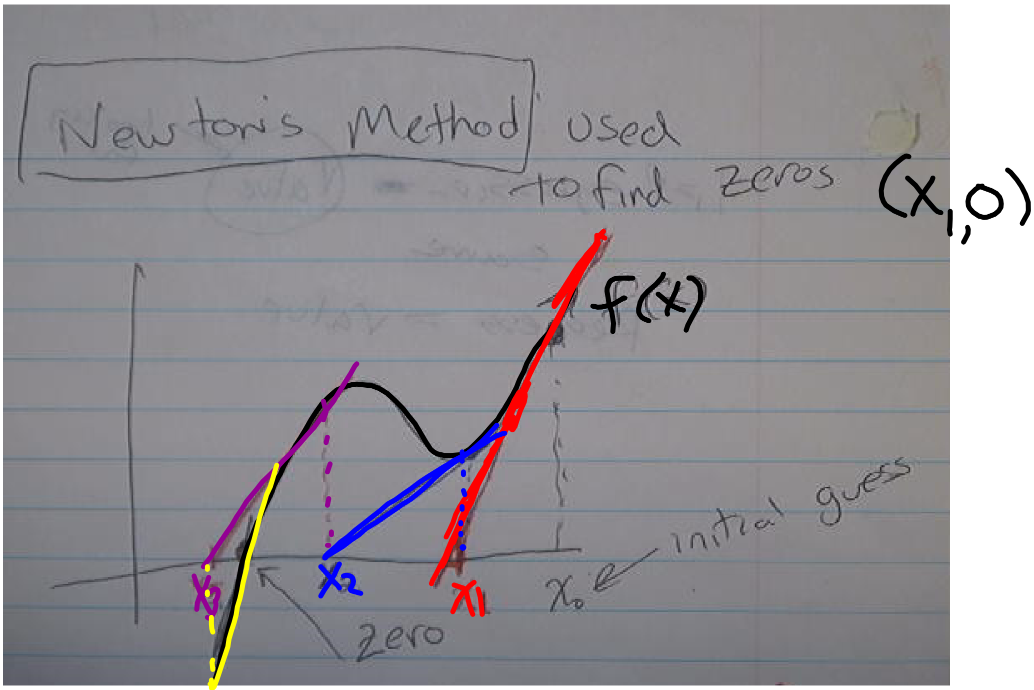


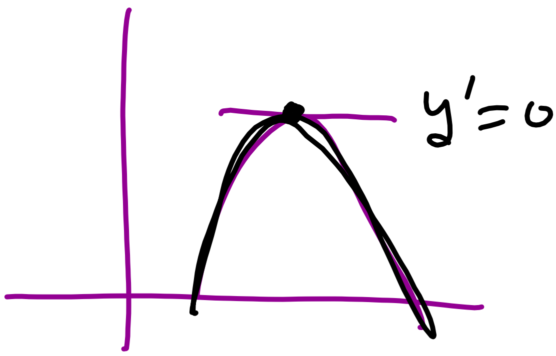

$$X_1 = 7.63$$

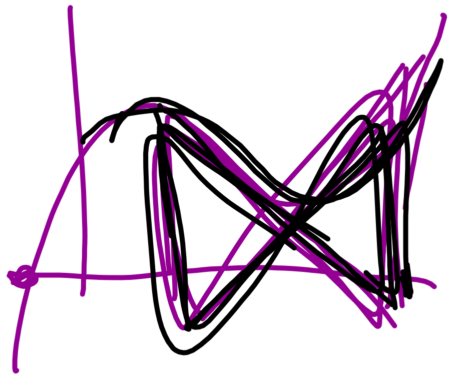
$$X_2 = 7.63 - \frac{y(7.63)}{y'(7.63)}$$

$$= 4.077\ldots$$

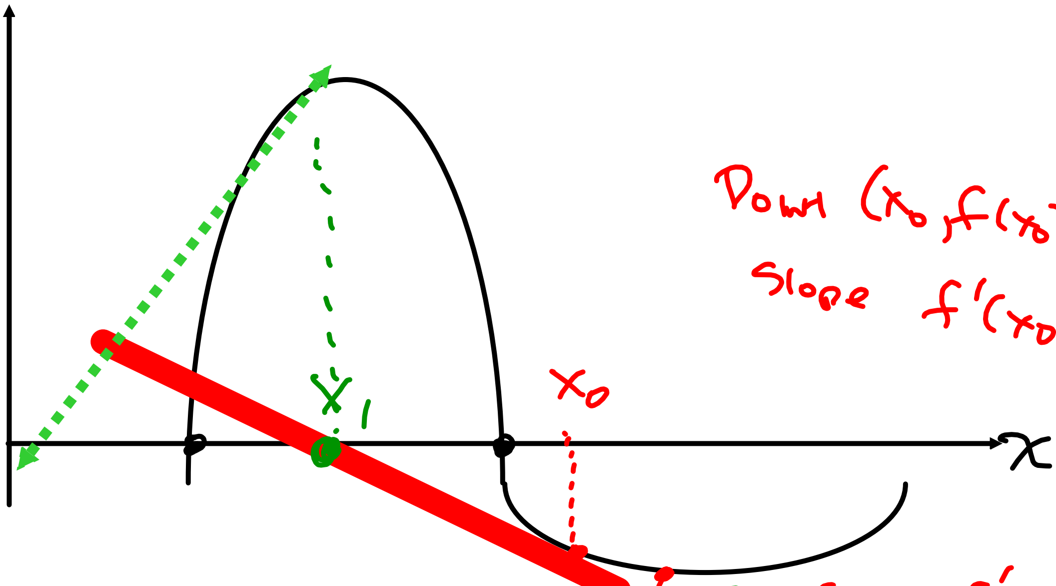
$$X_3 = 2.52$$


Newton's Method used to find zeros
(x,0)
f(x)
initial guess
$x_0$
$x_1$
$x_2$
$x_3$
zero



$$X - \frac{y}{y' = 0}$$ ERROR




Point $(x_0, f(x_0))$

Slope $f'(x_0)$

$$0 - f(x_0) = f'(x_0)(x - x_0)$$

$$+x_0 - \frac{f(x_0)}{f'(x_0)} = x - x_0 \quad +x_0$$

$$x_1 = x_0 - f(x_0)/f'(x_0)$$

$x^2 = 4$

$y = x^2 - 4 = 0$

$y' = \underline{2}x$

Guess : $5 = x_0$

$y(5) = \cancel{x}5^2 - 4 = 21$

$y'(5) = 10$

$x_1 = 5 - 21/10 = 2.9$

$x_2 = 2.9 - \dfrac{(2.9)^2 - 4}{2(2.9)}$

$= 2.9 - 4.41/5.8$

$= 2.9 - .76 = 2.14$

$x_3 = 2.14 - \dfrac{(2.14)^2 - 4}{2(2.14)}$

$\approx 2.07$

EQ. of Line

$$y - y_1 = m(x - x_1)$$

$$y - f(x_0) = f'(x_0)(x - x_0)$$

$$0 - f(x_0) = f'(x_0)(\cancel{x}_1 - x_0)$$

$$\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

Solver

(Math 0: ) B:

$0 = x^2 - 4$ <enter>

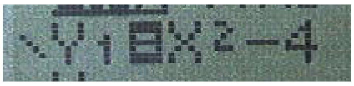
$X = 5$ <alpha><enter>

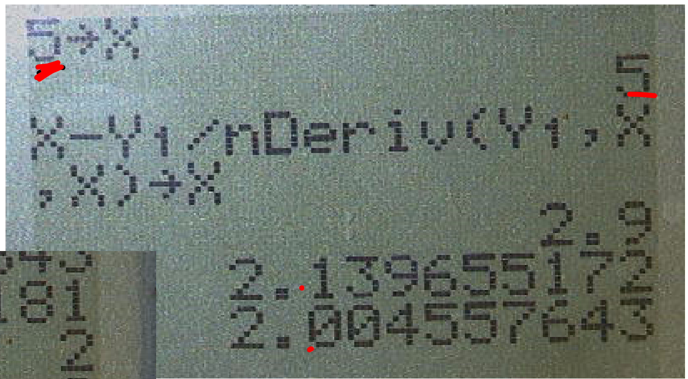
$X = 2$

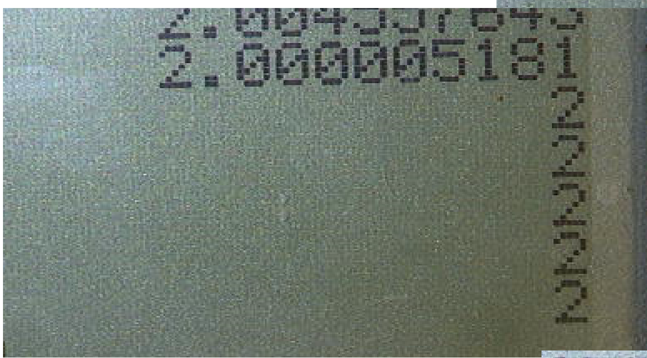
$X = -5$ <alpha><enter>

$X = -2$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$


\Y1=X^2-4


5→X
5
X-Y1/nDeriv(Y1,X
,X)→X
2.9
2.139655172
2.004557643


2.000005181
2
2
2
2
2

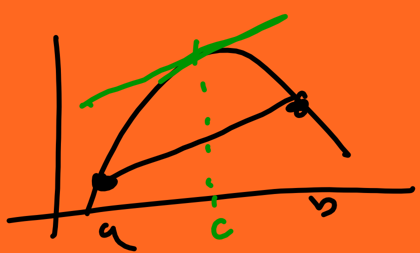

X-Y1/nDeriv(Y1,X
,X)→X

## Project: Solving using Newtons Method

MVT -

Average R o Change [a,b]

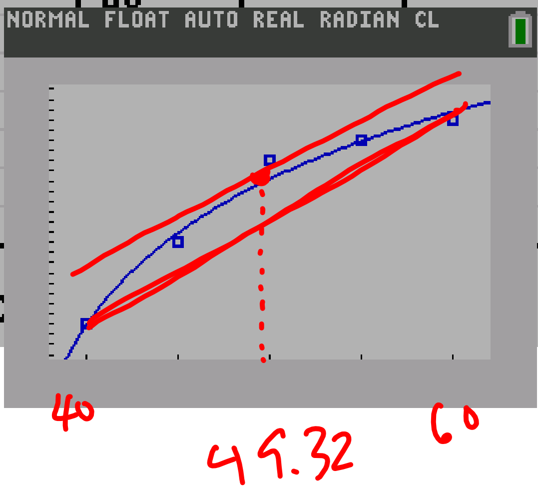
= Instant 'c'



NORMAL FLOAT AUTO REAL RADIAN CL

| L1 | L2 | L3 | L4 | L5 |
|---|---|---|---|---|
| 1 | 40 | ------ | ------ | ----- |
| 2 | 48 | | | |
| 3 | 56 | | | |
| 4 | 58 | | | |
| 5 | 60 | | | |
| --- | | | | |

L2(1)




NORMAL FLOAT AUTO REAL RADIAN CL

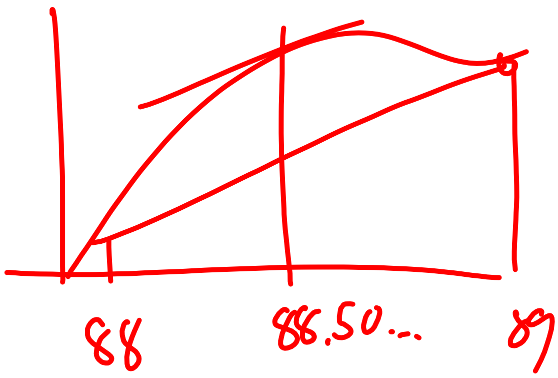
```
Y1(60)-Y1(40)
                         5.245640692
Ans/(60-40)
                         .2622820346
```

NORMAL FLOAT AUTO REAL RADIAN CL

```
Plot1  Plot2  Plot3
Y1=40.012516589835+12.93
7341802042ln(X)
Y2=nDeriv(Y1,X,X)-.26228
20346
Y3=
```

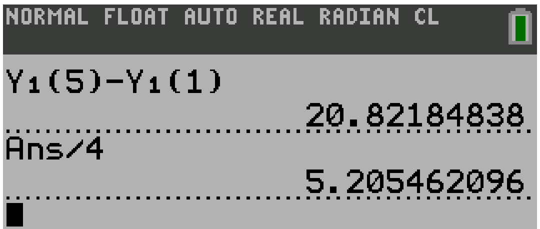
NORMAL FLOAT AUTO REAL RADIAN CL

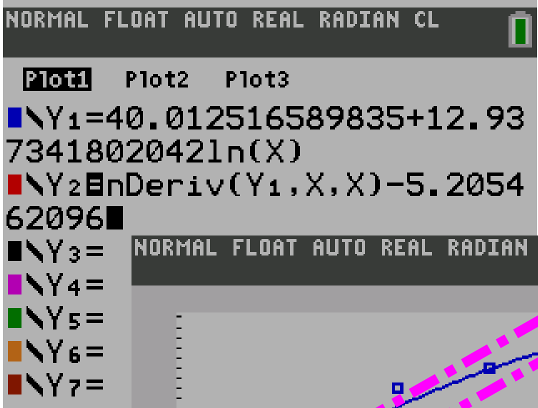
```
                           49.079859
X-Y2/nDeriv(Y2,X,X)→X
                         49.32481947
X-Y2/nDeriv(Y2,X,X)→X
                          49.3260694
X-Y2/nDeriv(Y2,X,X)→X
                         49.32606948
X-Y2/nDeriv(Y2,X,X)→X
                         49.32606946
```

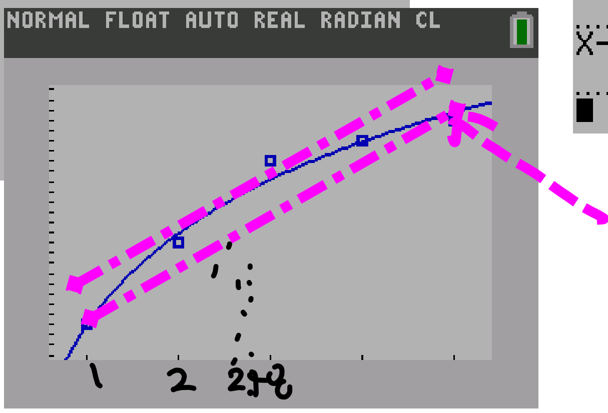



$$\frac{f(88)-f(89)}{88-89} = f'(88.5)$$

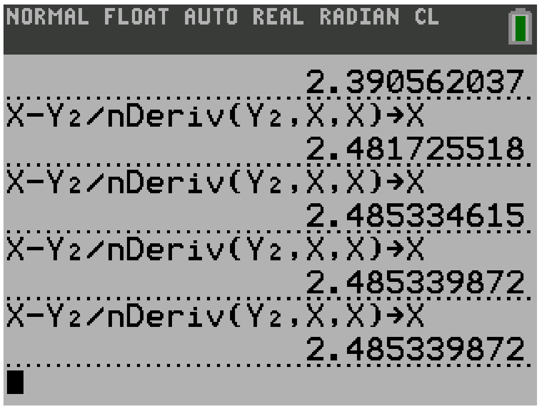
Acca to Quak my Between 88 & 89. Then ARC is equal to the IR of C at 88.5


NORMAL FLOAT AUTO REAL RADIAN CL
Y1(5)-Y1(1)
20.82184838
Ans/4
5.205462096


NORMAL FLOAT AUTO REAL RADIAN CL
Plot1 Plot2 Plot3
Y1=40.012516589835+12.937341802042ln(X)
Y2=nDeriv(Y1,X,X)-5.205462096
Y3=
Y4=
Y5=
Y6=
Y7=


NORMAL FLOAT AUTO REAL RADIAN CL
1
2

See line


NORMAL FLOAT AUTO REAL RADIAN CL
2.390562037
X-Y2/nDeriv(Y2,X,X)→X
2.481725518
X-Y2/nDeriv(Y2,X,X)→X
2.485334615
X-Y2/nDeriv(Y2,X,X)→X
2.485339872
X-Y2/nDeriv(Y2,X,X)→X
2.485339872

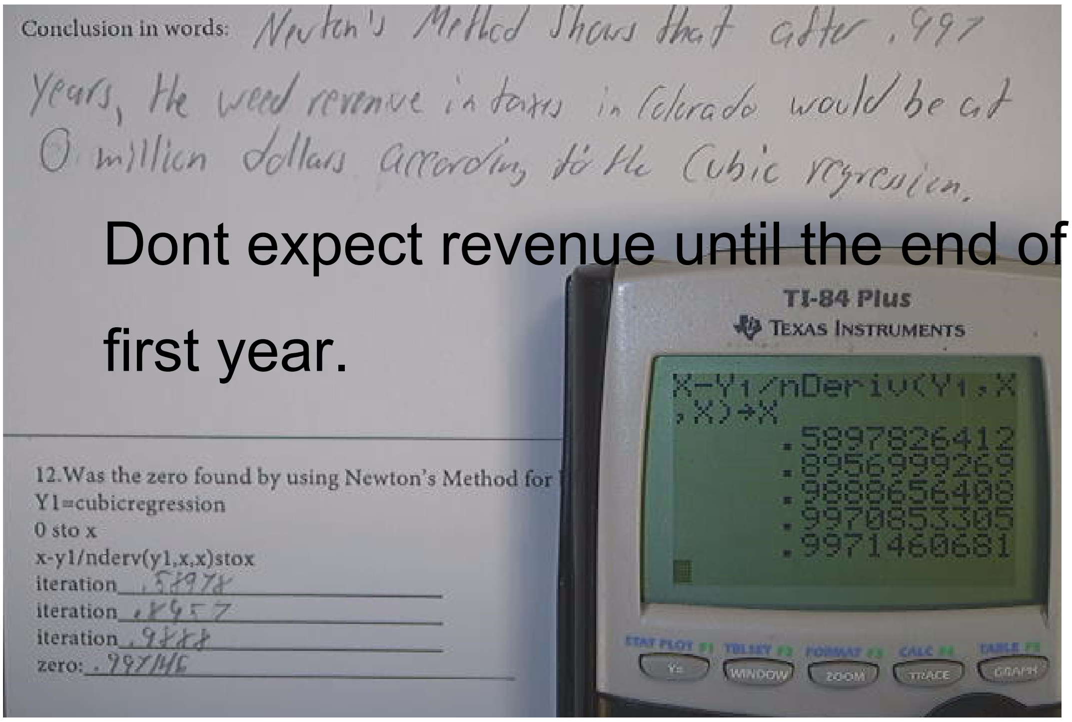
$Y_1$ = Regression = Value

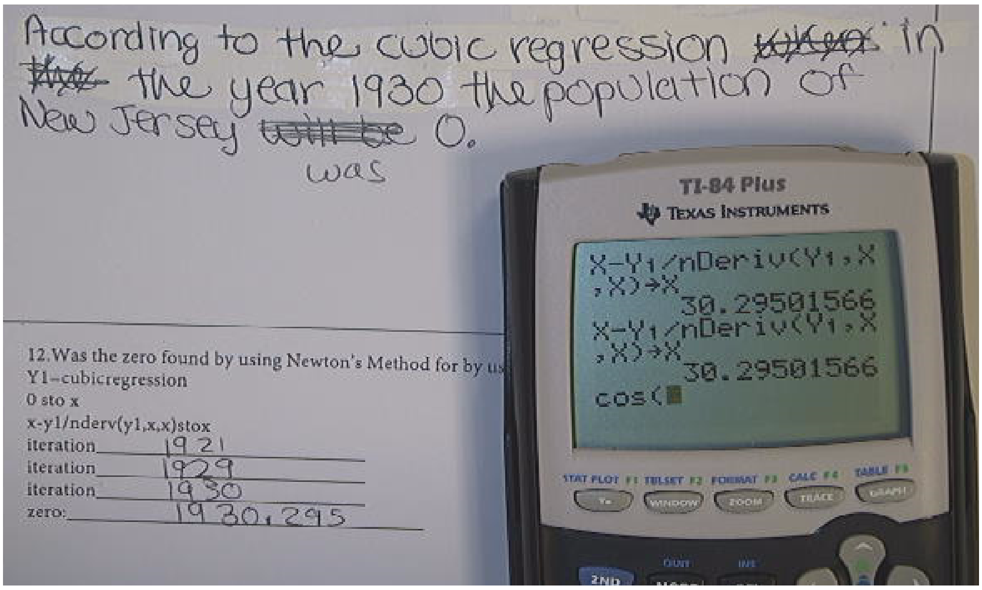
lookin for

same

Regress = value.

Dont expect revenue until the end of first year.




According to the cubic regression ~~when~~ in ~~the~~ the year 1930 the population of New Jersey ~~will be~~ was 0.
12.Was the zero found by using Newton's Method for by us
Y1–cubicregression
0 sto x
x-y1/nderv(y1,x,x)stox
iteration 1921
iteration 1929
iteration 1930
zero: 1930.295
TI-84 Plus
Texas Instruments
X-Y1/nDeriv(Y1,X,X)→X
30.29501566
X-Y1/nDeriv(Y1,X,X)→X
30.29501566
cos(

Newtons Method

y1= function (cubic regression)

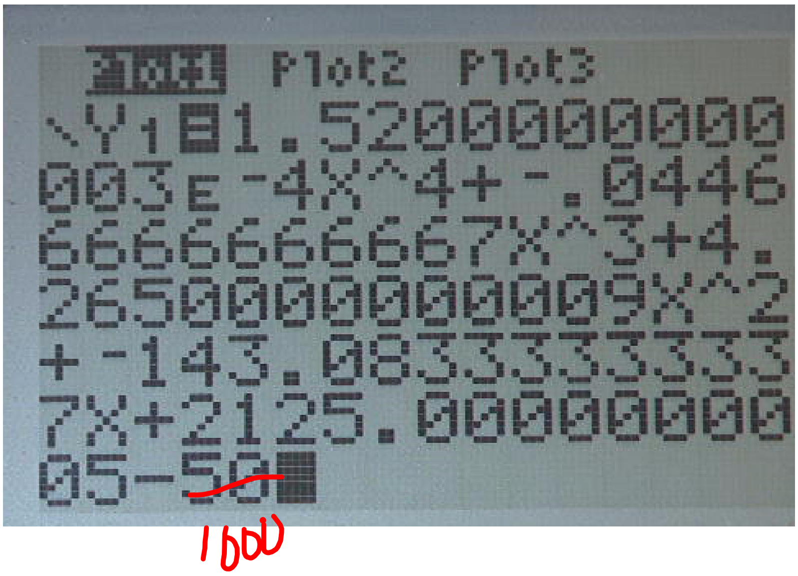
7 sto x  (7 called a guess)

x-y1/nderiv(y1,x,x) sto x

number (enter , enter, ...)

same number every time!

Number is the zero to y1 from guess = 7

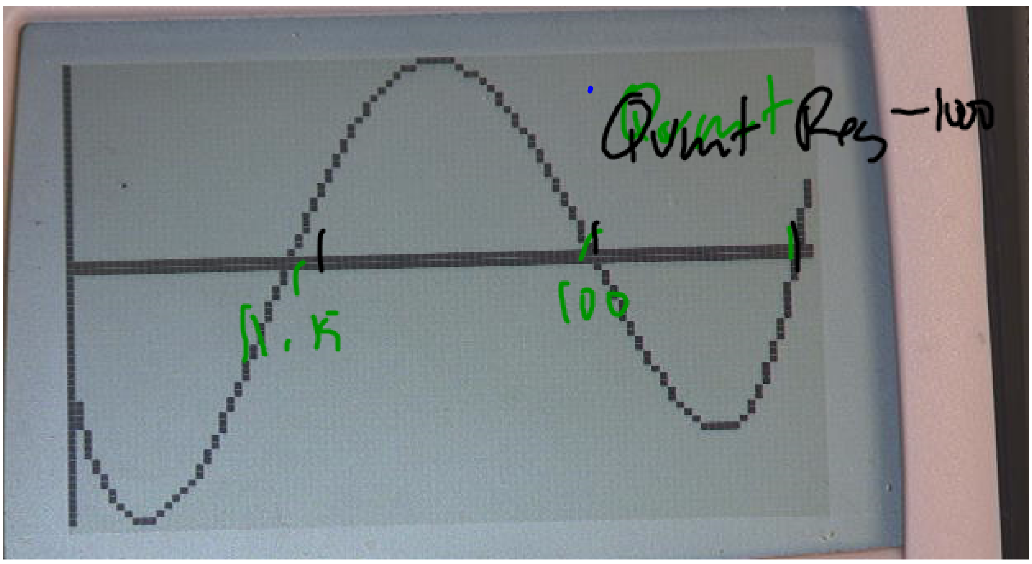



will find when

y1=0

quart regression -50 = 0

quart regression = 50



$y_1 =$ regression

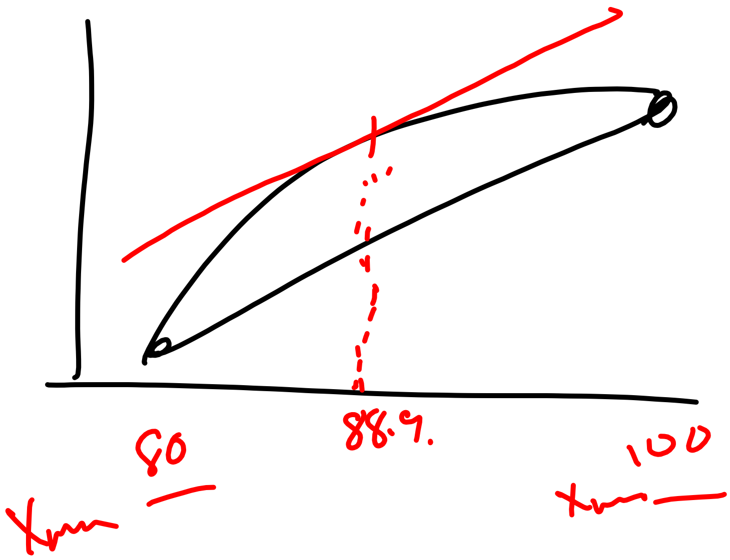
Ave Rat $(y_1(b) - y_1(a)) / (b-a) =$

$y_2 =$ nderiv $(y_1, x, x)$ − Ave. Rate of chang

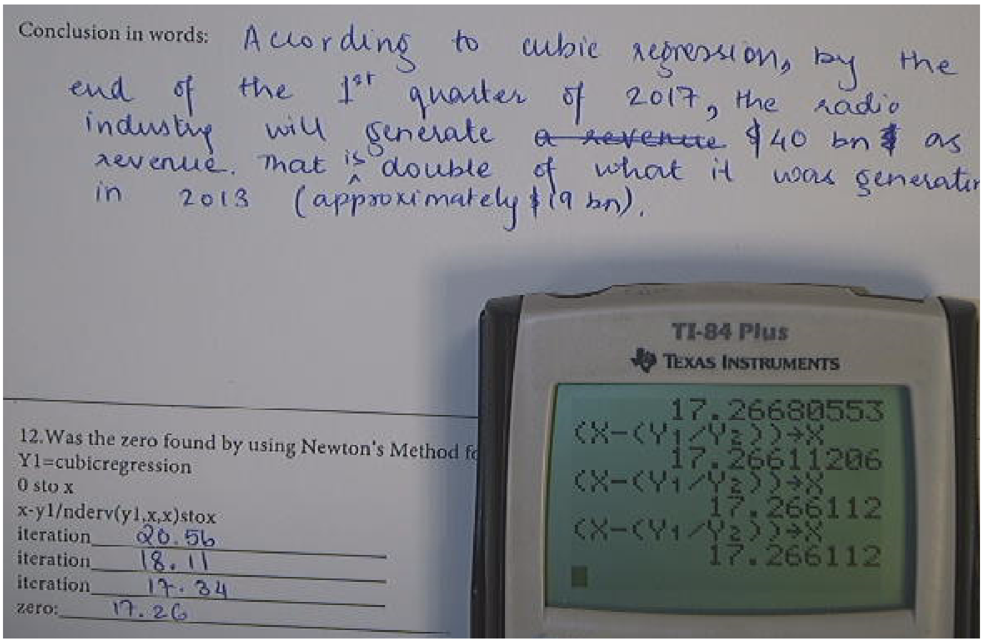
40 ——— 60

50 → x

$x - y_2 /$ nderiv $(y_2, x, x) \rightarrow x$



80
88.9.
100
$X_{rm}$ ___
$X_{rm}$ ___



Conclusion in words: According to cubic regression, by the end of the 1st quarter of 2017, the radio industry will generate ~~a revenue~~ \$40 bn ~~\$~~ as revenue. That is double of what it was generati[ng] in 2013 (approximately \$19 bn).

12.Was the zero found by using Newton's Method f[...]

Y1=cubicregression

0 sto x

x-y1/nderv(y1,x,x)stox

iteration 20.56

iteration 18.11

iteration 17.34

zero: 17.26

# MVT

$y = x^3 + 2x^2$ on $[0, 5]$ — <u>Inst</u> $y'$

<u>AVE</u>

$y(5) = 125 + 50 = 175$ $\qquad 3x^2 + 4x$

$y(0) = 0$

$\dfrac{y(5) - y(0)}{5 - 0} = \dfrac{175}{5} = \boxed{35 = 3x^2 + 4x}$

$f(x)\ \underbrace{3x^2 + 4x}_{y'} - 35 = 0$

$-\text{ARC} = 0$

$0 = 3x^2 + 4x - 35$

$x = \dfrac{-4 \pm \sqrt{16 + 420}}{6}$

436

2 218

2 109

$\square$

$$X = \sqrt[3]{12} \rightarrow X^3 = 12$$

$$f(x) = x^3 - 12 = 0$$

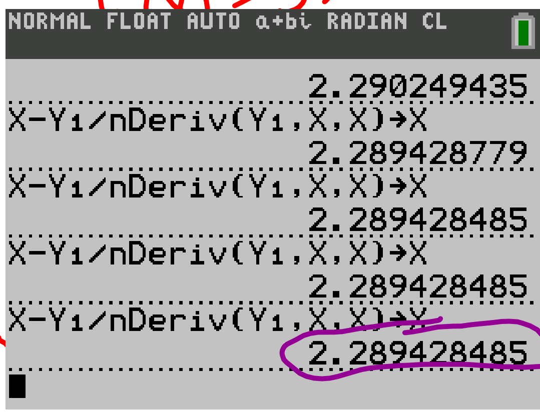
$$f'(x) = 3x^2$$

Guess: $X = 2$

$$X_0 = 2$$

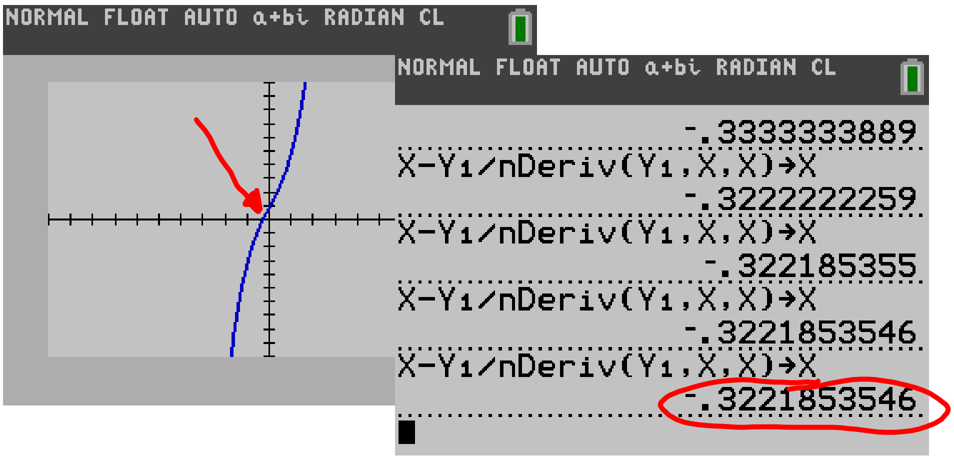
$$X_1 = X_0 - \frac{f(x_0)}{f'(x_0)}$$

$$X_2 = 1\tfrac{2}{3} - \frac{f(1\frac{2}{3})}{f'(1\frac{2}{3})}$$




$+\frac{1}{3} = 2\frac{2}{3}$

$$y = x^3 + 5x^2 - 2x + 1 = 0$$


NORMAL FLOAT AUTO a+bi RADIAN CL
NORMAL FLOAT AUTO a+bi RADIAN CL
-.3333333889
X-Y1/nDeriv(Y1,X,X)→X
-.3222222259
X-Y1/nDeriv(Y1,X,X)→X
-.322185355
X-Y1/nDeriv(Y1,X,X)→X
-.3221853546
X-Y1/nDeriv(Y1,X,X)→X
-.3221853546

$y = x^3 + 8x^2$ on $[0,3]$

Ave. Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{y(3) - y(0)}{3 - 0}$$

$$= \frac{27 + (8)(3)^2 - 0}{3} = \frac{99}{3} = 33$$

Instant

$y' = 3x^2 + 16x$

$3x^2 + 16x$

MVT Instant = Ave Rate

$$3x^2 + 16x = 33$$

$$3x^2 + 16x - 33 = 0$$

$$x = \frac{-16 \pm \sqrt{16^2 - 4 \cdot 3 \cdot (-33)}}{2(3)}$$

Quadratic Formula

on $[0,3]$

$-$ Not $[0,3]$

Find Local Linear Appx for $7\sin x$

near $x = 0$.   $x = a = 0$

POINT $(0,0)$

$f(x) = 7\sin x$

$f(0) = 7\sin 0 = 0$

Slope $= f'(0)$

$f'(x) = 7\cos x$

Slope

$m = f'(0) = 7\cos(0) = 7 \cdot 1 = 7$

$y - y_0 = m(x - x_0)$

$y - 0 = 7(x - 0)$

$y = 7x$

| $x$ | $L(x)$ $7x$ | $f(x)$ $7\sin x$ |
|---|---|---|
| 0 | 0 | 0 |
| .1 | .7 | .6988 |
| 1 | 7 | 5.8903 |