1. Related Rates
2. Logarithmic Differentiation
3. L'Hopitals Rule.

Related Rates. (Balloon Problems)
Word Problem

Difference : Vale & Rate.

ft, Sec   y ft "Per" time

y, t $\frac{dy}{dt}$

Main Formula

$$V = \frac{4}{3}\pi r^3 \qquad x^2 + y^2 = z^2 \qquad R = 10p - p^2$$

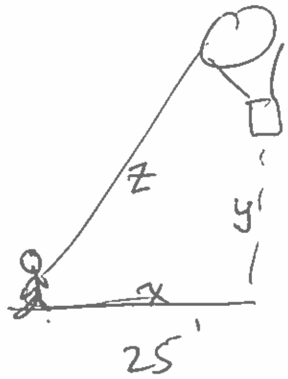
$$\frac{d}{dt} V = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) \qquad \frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(z^2) \qquad \frac{dR}{dt} = (10-2p)\frac{dp}{dt}$$

$$\frac{dV}{dt} = 4\pi \boxed{r}^2 \frac{dr}{dt} \qquad 2\boxed{x}\frac{dx}{dt} + 2\boxed{y}\frac{dy}{dt} = 2\boxed{z}\frac{dz}{dt}$$

Chain Rule

Rising Balloon

2



10' per second

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

After 2 seconds

How fast is the distance between you & balloon changing? Rate $= \frac{dz}{dt} = ?$

$x = 25$ $\quad dx/dt = 0$ Standing

$y = 20 = 10 \cdot 2$ $\quad dy/dt = 10$

$z = 32\ldots$

Note:
$\frac{dx}{dt}$ is $+$ walking away
$\frac{dx}{dt}$ is $-$ walking Towards

$z = \sqrt{20^2 + 25^2}$ Pyth. Theorem

$$2\cancel{(25)(0)} + 2(20)(10) = 2(32)\cdot\frac{dz}{dt}$$

$$\frac{dz}{dt} = 6.2 \text{ ft/sec}$$

After 20 seconds

$$\frac{dz}{dt} = 9.9 \text{ ft/sec}$$

Filling Balloon.

$V = \frac{4}{3}\pi r^3$ ③

$\frac{dV}{dt} = 1000$ cc per sec
rate

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

How fast balloon growing at radius $= 7$?
at radius $= 100$?
values.

$1000 = 4\pi (7)^2 \frac{dr}{dt}$

Small Balloon At $r=7$ $\frac{dr}{dt} = 1.62$ cm/sec

AT $r=100$ $\frac{dr}{dt} = .01$ cm/sec

Big Balloon

Selling a Balloon

4

R = Rev. $\quad P$ = price $\quad S$ = sales

$$S = 10 - P$$

$$R = P \cdot S$$

Main: $\boxed{R = P(10-P)}$

$$R = 10P - P^2$$

$$\frac{d}{dt}R = 10\frac{dP}{dt} - 2P\frac{dP}{dt}$$

Rates $\qquad$ Values $\qquad$ How fast is Revenue changing if Price = \$1 Raise by .05 per week

$\frac{dR}{dt} = ?$ $\qquad P = 1$

$\frac{dP}{dt} = \$.05/\text{wk.}$

$$\frac{dR}{dt} = 10(.05) - 2(1)(.05)$$
$$= \$.4/\text{week}$$

$\frac{dR}{dt} = ?$ $\qquad P = \$8$

$\frac{dP}{dt} = +.05$ $\qquad 10(.05) - 2(8)(.05)$

$-.3/\text{week}$

lower $-.05$ $\qquad$ (chang $+.3$/week)

5

$R = 10p - p^2$

$\frac{dR}{dP} = R' = 10 - 2P$

$\frac{dR}{dt} = R' \cdot \frac{dP}{dt}$

month

$\frac{1}{30}$

$y_1 = f(x)$

$\frac{dy}{dt} = f'(x) \cdot \frac{dx}{dt}$

Months

$M = 30$ days

$\frac{d\cancel{t}}{dt}$

Logarithmic Differentiation

$$y = f(x)^{g(x)}$$

Ex $y = x^{\sin x}$

or $y = e^{x}$ $\quad$ $\cancel{y' = x e^{x-1}}$

1 Log of Both Sides
2 differentiate.

Ex $y = B^{x}$

1. $\ln y = \ln B^{x}$
$\ln y = x \ln B$

2. $\frac{d}{dx} \ln y = \frac{d}{dx} x \ln B$

$$\underbrace{\frac{1}{y}\cdot\frac{dy}{dx}}_{\text{Chain Rule}} = \ln B$$

$$\frac{dy}{dx} = y \ln B$$

$$\boxed{y = B^x \qquad \frac{dy}{dx} = B^x \ln B}$$

Ex $y = x^{\sin x}$

1. $\ln y = \ln x^{\sin x}$

$\ln y = \sin x \cdot \ln x$

Product Rule

2. $\frac{d}{dx}\ln y = \sin x \cdot \frac{d}{dx}\ln x + \ln x \frac{d}{dx}\sin x$

$$\frac{1}{y}\frac{dy}{dx} = \sin x \frac{1}{x} + \ln x \cos x$$

$$dy/dx = y\left(\frac{\sin x}{x} + \ln x \cos x\right)$$

$$\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \ln x \cos x\right)$$

L'Hôpitals Rule

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

$$= \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Ex $\lim_{x \to 0} \frac{\sin x}{x} = \frac{0}{0}$

use LHR

$$\lim_{x \to 0} \frac{\cos x}{1} = \frac{1}{1} = 1$$

Ex $$\lim_{x \to \infty} \frac{x^2 + 2x + 8}{3x^2 + 7} = \frac{\infty}{\infty}$$

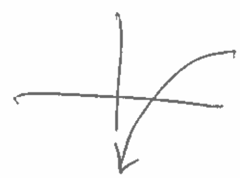
USE LHR

$$\lim_{x \to \infty} \frac{2x + 2}{6x} = \frac{\infty}{\infty}$$

USE LHR

$$\lim_{x \to \infty} \frac{2}{6} = \frac{1}{3}$$

Ex $$\lim_{x \to 0^+} x \ln x = 0 \cdot (-\infty)$$



$$\lim_{x \to 0^+} \frac{\ln x}{x^{-1}} = \frac{-\infty}{\infty}$$

use LHR

$$\lim_{x \to 0^+} \frac{1/\cancel{x}}{-\cancel{x}^{-2}} = \lim_{x \to 0^+} -x = 0$$

$$P = Q\left(1+\frac{R}{N}\right)^{NT}$$

Compound Interest

$$\lim_{N\to\infty}\left(1+\frac{R}{N}\right)^{NT} = e^{RT}$$

$$P = Qe^{RT}$$

Compound continuously

$e^{\ln 1^{\infty}}$ = could be anything

11

$$e^{\ln \lim_{N \to \infty} \left(1+\frac{R}{N}\right)^{NT}}$$

$$\lim_{N \to \infty} NT \ln\left(1+\frac{R}{N}\right)$$

$$\infty \cdot 0$$

$$e$$

$$e^{\lim_{N \to \infty} \frac{\ln\left(1+\frac{R}{N}\right)}{(NT)^{-1}}} \quad \frac{0}{0}$$

USE L'HR

$$0 \cdot \infty$$

$$\frac{\infty}{\infty} \quad \frac{0}{0}$$

$$1^{\infty}$$

$$\infty - \infty$$

Ex $\lim_{x \to \infty} x - \sqrt{x^2 - 7} = \infty - \infty$

$$\lim_{x \to \infty} \frac{x - \sqrt{x^2-7}}{1} \cdot \frac{x + \sqrt{x^2-7}}{x + \sqrt{x^2-7}}$$

$$\lim_{x \to \infty} \frac{\cancel{x^2} - \cancel{x^2} + 7}{x + \sqrt{x^2-7}}$$

$$\lim_{x \to \infty} \frac{7}{x + \sqrt{x^2-7}} = \frac{7}{\infty} = 0$$

END QUIZ 2, 3