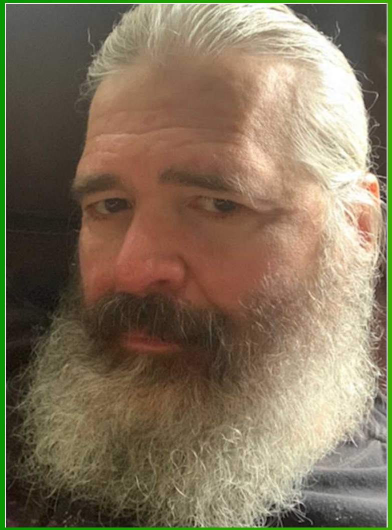
MAT 151 Calculus 1

Prof. Porter



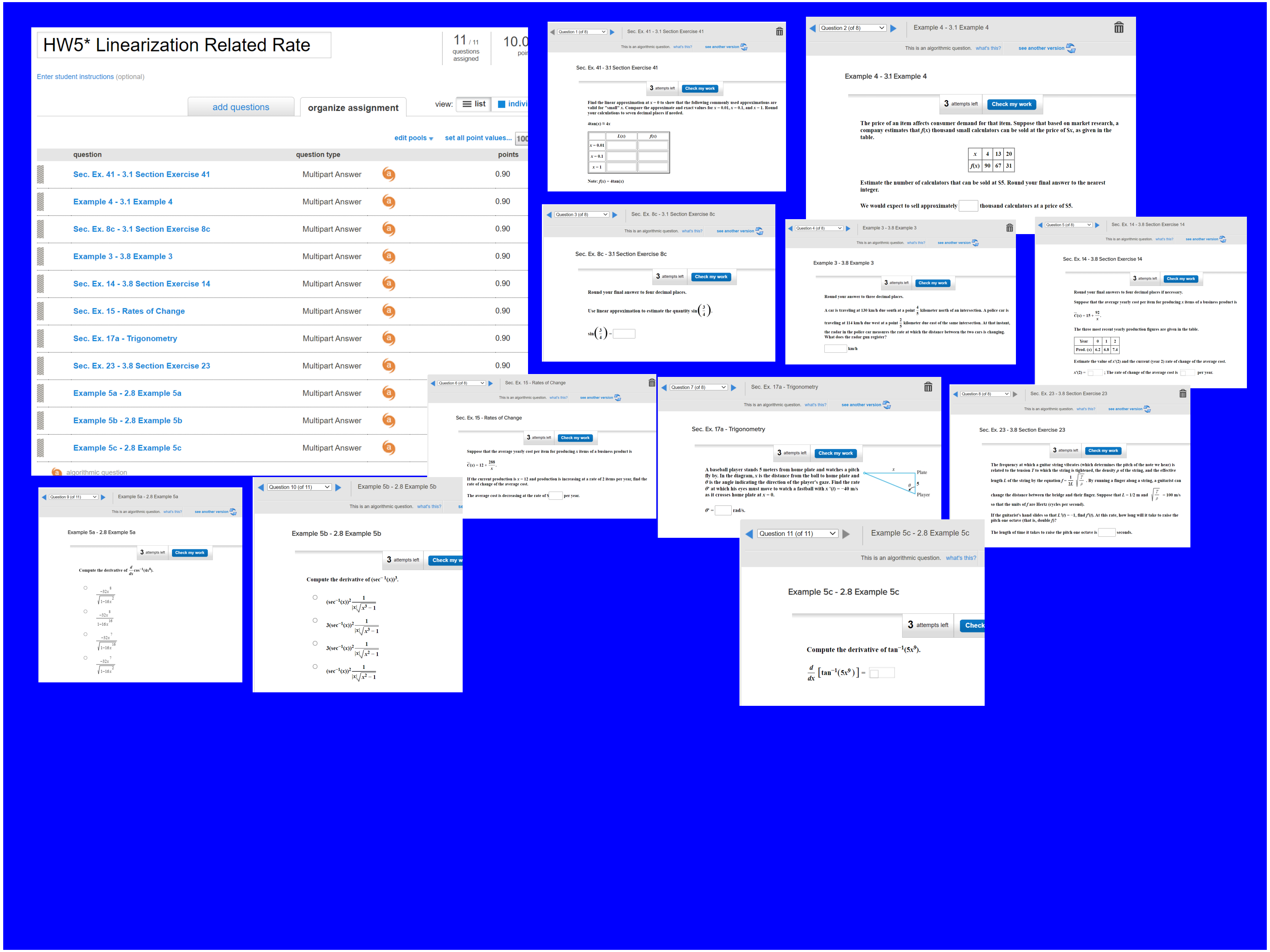
Agenda

Homework Review

Lecture: L’Hôpitals Rule

Groupwork

151d11



HW5* Linearization Related Rate

11 / 11 questions assigned

10.0 poi

Enter student instructions (optional)

question	question type	points
Sec. Ex. 41 - 3.1 Section Exercise 41	Multipart Answer	0.90
Example 4 - 3.1 Example 4	Multipart Answer	0.90
Sec. Ex. 8c - 3.1 Section Exercise 8c	Multipart Answer	0.90
Example 3 - 3.8 Example 3	Multipart Answer	0.90
Sec. Ex. 14 - 3.8 Section Exercise 14	Multipart Answer	0.90
Sec. Ex. 15 - Rates of Change	Multipart Answer	0.90
Sec. Ex. 17a - Trigonometry	Multipart Answer	0.90
Sec. Ex. 23 - 3.8 Section Exercise 23	Multipart Answer	0.90
Example 5a - 2.8 Example 5a	Multipart Answer	
Example 5b - 2.8 Example 5b	Multipart Answer	
Example 5c - 2.8 Example 5c	Multipart Answer	

algorithmic question

Question 1 (of 8) Sec. Ex. 41 - 3.1 Section Exercise 41

This is an algorithmic question.

Sec. Ex. 41 - 3.1 Section Exercise 41

3 attempts left

Find the linear approximation at $x = 0$ to show that the following commonly used approximations are valid for "small" x. Compare the approximate and exact values for $x = 0.01$, $x = 0.1$, and $x = 1$. Round your calculations to seven decimal places if needed.

$4\tan(x) \approx 4x$

	$L(x)$	$f(x)$
$x = 0.01$		
$x = 0.1$		
$x = 1$		

Note: $f(x) = 4\tan(x)$

Question 2 (of 8) Example 4 - 3.1 Example 4

This is an algorithmic question.

Example 4 - 3.1 Example 4

3 attempts left

The price of an item affects consumer demand for that item. Suppose that based on market research, a company estimates that $f(x)$ thousand small calculators can be sold at the price of $\$x$, as given in the table.

x	4	13	20
$f(x)$	90	67	31

Estimate the number of calculators that can be sold at $5. Round your final answer to the nearest integer.

We would expect to sell approximately ☐ thousand calculators at a price of $5.

Question 3 (of 8) Sec. Ex. 8c - 3.1 Section Exercise 8c

This is an algorithmic question.

Sec. Ex. 8c - 3.1 Section Exercise 8c

3 attempts left

Round your final answer to four decimal places.

Use linear approximation to estimate the quantity $\sin\left(\frac{3}{4}\right)$.

$\sin\left(\frac{3}{4}\right) =$ ☐

Question 4 (of 8) Example 3 - 3.8 Example 3

This is an algorithmic question.

Example 3 - 3.8 Example 3

3 attempts left

Round your answer to three decimal places.

A car is traveling at 130 km/h due south at a point $\frac{4}{5}$ kilometer north of an intersection. A police car is traveling at 114 km/h due west at a point $\frac{2}{5}$ kilometer due east of the same intersection. At that instant, the radar in the police car measures the rate at which the distance between the two cars is changing. What does the radar gun register?

☐ km/h

Question 5 (of 8) Sec. Ex. 14 - 3.8 Section Exercise 14

This is an algorithmic question.

Sec. Ex. 14 - 3.8 Section Exercise 14

3 attempts left

Round your final answers to four decimal places if necessary.

Suppose that the average yearly cost per item for producing x items of a business product is

$\overline{C}(x) = 15 + \frac{92}{x}$.

The three most recent yearly production figures are given in the table.

Year	0	1	2
Prod. (x)	6.2	6.8	7.4

Estimate the value of $x'(2)$ and the current (year 2) rate of change of the average cost.

$x'(2) =$ ☐ ; The rate of change of the average cost is ☐ per year.

Question 6 (of 8) Sec. Ex. 15 - Rates of Change

This is an algorithmic question.

Sec. Ex. 15 - Rates of Change

3 attempts left

Suppose that the average yearly cost per item for producing x items of a business product is

$\overline{C}(x) = 12 + \frac{288}{x}$.

If the current production is $x = 12$ and production is increasing at a rate of 2 items per year, find the rate of change of the average cost.

The average cost is decreasing at the rate of $ ☐ per year.

Question 7 (of 8) Sec. Ex. 17a - Trigonometry

This is an algorithmic question.

Sec. Ex. 17a - Trigonometry

3 attempts left

A baseball player stands 5 meters from home plate and watches a pitch fly by. In the diagram, x is the distance from the ball to home plate and θ is the angle indicating the direction of the player's gaze. Find the rate θ' at which his eyes must move to watch a fastball with $x'(t) = -40$ m/s as it crosses home plate at $x = 0$.

$\theta' =$ ☐ rad/s.

Question 8 (of 8) Sec. Ex. 23 - 3.8 Section Exercise 23

This is an algorithmic question.

Sec. Ex. 23 - 3.8 Section Exercise 23

3 attempts left

The frequency at which a guitar string vibrates (which determines the pitch of the note we hear) is related to the tension T to which the string is tightened, the density ρ of the string, and the effective length L of the string by the equation $f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$. By running a finger along a string, a guitarist can change the distance between the bridge and their finger. Suppose that $L = 1/2$ m and $\sqrt{\frac{T}{\rho}} = 100$ m/s so that the units of f are Hertz (cycles per second).

If the guitarist's hand slides so that $L'(t) = -1$, find $f'(t)$. At this rate, how long will it take to raise the pitch one octave (that is, double f)?

The length of time it takes to raise the pitch one octave is ☐ seconds.

Question 9 (of 11) Example 5a - 2.8 Example 5a

This is an algorithmic question.

Example 5a - 2.8 Example 5a

3 attempts left

Compute the derivative of $\frac{d}{dx}\cos^{-1}(4x^8)$.

○ $\frac{-32x^8}{\sqrt{1-16x^2}}$

○ $\frac{-32x^8}{1-16x^{16}}$

○ $\frac{-32x^7}{\sqrt{1-16x^{16}}}$

○ $\frac{-32x^7}{\sqrt{1-16x^2}}$

Question 10 (of 11) Example 5b - 2.8 Example 5b

This is an algorithmic question.

Example 5b - 2.8 Example 5b

3 attempts left

Compute the derivative of $(\sec^{-1}(x))^3$.

○ $(\sec^{-1}(x))^2 \frac{1}{|x|\sqrt{x^3-1}}$

○ $3(\sec^{-1}(x))^2 \frac{1}{|x|\sqrt{x^3-1}}$

○ $3(\sec^{-1}(x))^2 \frac{1}{|x|\sqrt{x^2-1}}$

○ $(\sec^{-1}(x))^2 \frac{1}{|x|\sqrt{x^2-1}}$

Question 11 (of 11) Example 5c - 2.8 Example 5c

This is an algorithmic question.

Example 5c - 2.8 Example 5c

3 attempts left

Compute the derivative of $\tan^{-1}(5x^9)$.

$\frac{d}{dx}\left[\tan^{-1}(5x^9)\right] =$ ☐

Lecture:

What is Math?

What is precalculus?

What is Calculus?

What are the rates of change?

How many points do you need?

Why did we invent limits?

What else is the velocity?

What is needed for continuity?

What is Math? Language

What is precalculus? Functions

What is Calculus? Study of Change

What are the rates of change? Average and Instantaneous

How many points do you need? 2/1

Why did we invent limits? Takes two points to one

What else is the velocity?

slope of the tangent line
is the derivative
is the instantaneous rate of change
is the velocity

What is needed for continuity?-limit exists, function exists, limit=function

Lecture: L'Hôpitals Rule

Using Derivatives to Find Limits!

Used for $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$

But also to find rule breakers like:

0 times ∞ , 1^{∞} , $\infty-\infty$

L'Hôpitals Rule says that if a limit yields 0/0 or ∞/∞ then you can differentiate the nuerator and denominator and the limit of that is the same.

$$\text{IF:} \quad \lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{or} \quad \frac{\infty}{\infty}$$

$$\text{THEN:} \quad \lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

This is NOT the quotient rule, but the derivative of numerator and denominator.

EX: $\lim_{x \to 0} \frac{\sin x}{x}$

EX: $\lim_{x \to \infty} \frac{2x - 5}{3x - 2}$

EX: $\lim_{x \to \infty} \frac{7x^2 - 6x + 9}{3x^2 - 1}$

EX: $\lim\limits_{x\to 0} \frac{\sin x}{x} = \lim\limits_{x\to 0} \frac{\sin x}{x} = \frac{0}{0} = \lim\limits_{x\to 0} \frac{\cos x}{1} = 1$

L'HR

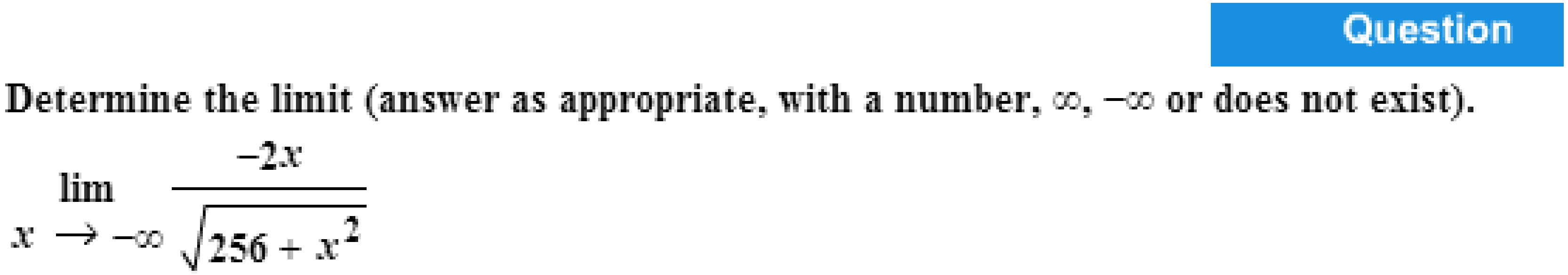
EX: $\lim\limits_{x\to \infty} \frac{2x - 5}{3x - 2} = \lim\limits_{x\to \infty} \frac{2x - 5}{3x - 2} = \frac{\infty}{\infty} \quad \lim\limits_{x\to \infty} \frac{2}{3} = \frac{2}{3}$

L'HR

EX: $\lim\limits_{x\to \infty} \frac{7x^2 - 6x + 9}{3x^2 - 1} = \frac{\infty}{\infty} = \lim\limits_{x\to \infty} \frac{14x - 6}{6x - 1} = \frac{\infty}{\infty} = \lim\limits_{x\to \infty} \frac{14}{6} = \frac{7}{3}$

L'HR L'HR

Sec. Ex. 11 - 1.5 Section Exercise 11



Determine the limit (answer as appropriate, with a number, ∞, −∞ or does not exist).

$$\lim_{x \to -\infty} \frac{-2x}{\sqrt{256 + x^2}}$$

Sec. Ex. 11 - 1.5 Section Exercise 11

Question

Determine the limit (answer as appropriate, with a number, ∞, −∞ or does not exist).

$$\lim_{x \to -\infty} \frac{-2x}{\sqrt{256+x^2}} \qquad \frac{\infty}{\infty}$$

use LHopitals Rule

$$\lim_{x \to -\infty} \frac{-2}{\frac{1}{2}(256+x^2)^{-1/2} \cdot 2x} =$$

Algebra

$$\lim_{x \to -\infty} \frac{-2\sqrt{256+x^2}}{x} = \frac{\infty}{\infty}$$

Use LHopitals Rule AGAIN????

or

$$\lim_{x \to -\infty} \frac{-2\sqrt{256+x^2} \Big/ \sqrt{x^2}}{x \big/ x}$$

$$\lim_{x \to -\infty} -2\sqrt{\cancel{\frac{256}{x^2}}^{\,0} + 1} = -2$$

EX: What about zero times infinity?

Algebra
Change to

$$\lim_{x\to\infty} xe^{-x} = \infty\cdot 0 \longrightarrow \lim_{x\to\infty} \frac{x}{e^{x}} = \frac{\infty}{\infty}$$

Use L'Hopitals Rule

$$= \lim_{x\to\infty} \frac{1}{e^{x}} = \frac{1}{\infty} = 0$$

EX:

$$\lim_{x\to\infty} \frac{1}{x}e^{x} = 0\cdot\infty \longrightarrow \lim_{x\to\infty} \frac{e^{x}}{1} = \infty$$

What about one raised to the infinite power?

$$\begin{cases} .9^{\infty} = 0 \\ 1.^{\infty} = 1 \\ 1.0001^{\infty} = \infty \end{cases}$$

Use the identity: $e^{\ln y} = y$

EX: $\lim_{N\to\infty} (1+R/N)^{NT}$ $= e^{\ln \lim_{N\to\infty} (1+R/N)^{NT}}$

(Compund interest formula)

$$= e^{\lim_{N\to\infty} \ln (1+R/N)^{NT}}$$

$$= e^{\lim_{N\to\infty} NT \ln (1+R/N)} = e^{T \lim_{N\to\infty} \frac{\ln(1+R/N)}{1/N}}$$

use L'Hopitals Rule = $e^{T \lim_{N\to\infty} \frac{\frac{1}{1+R/N} \cdot -R/N^2}{-1/N^2}}$ $= e^{T \cdot R}$

(Compunded Continuously Formula)

Compound Interest Formula $P = Q\,(1 + R/N)^{NT}$

Compounded Continuously $P = Q\,e^{RT}$

$$\lim_{N\text{->}oo} (1 + R/N)^N = e^R$$

$$(1 + 0)^{\infty} = \text{anything}$$

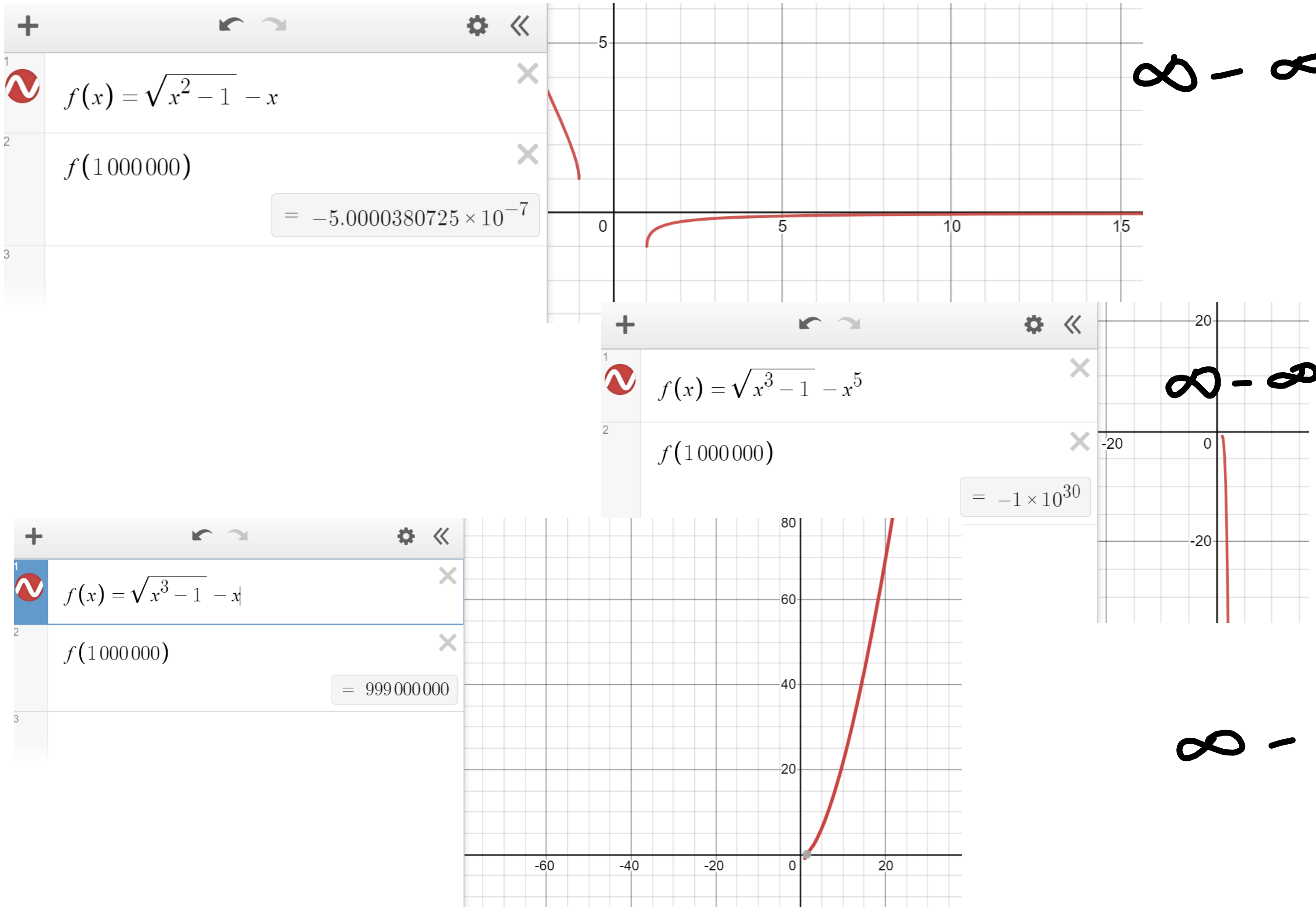
EX: $\infty-\infty$ $\quad \lim_{x\to\infty} \sqrt{x^2-1} - x = \infty - \infty$

(Change Algebraically)

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1} - x}{} \cdot \frac{\sqrt{x^2-1} + x}{\sqrt{x-1} + x} =$$

$$\lim_{x\to\infty} \frac{x^2-1-x^2}{\sqrt{x^2-1}+x} = \lim_{x\to\infty} \frac{-1}{\sqrt{x^2-1}+x}$$

$$= -\frac{1}{\infty} = 0$$



1 $f(x) = \sqrt{x^2 - 1} - x$

2 $f(1\,000\,000)$

$= -5.0000380725 \times 10^{-7}$

1 $f(x) = \sqrt{x^3 - 1} - x^5$

2 $f(1\,000\,000)$

$= -1 \times 10^{30}$

1 $f(x) = \sqrt{x^3 - 1} - x$

2 $f(1\,000\,000)$

$= 999\,000\,000$

$\infty - \infty = 0$

$\infty - \infty = -\infty$

$\infty - \infty = +\infty$

Group Work

Limits at infinity

No L'hoptals Rule in Group work.

No rational regressions

Reminders....