Agenda:

Review of Quiz #5

Lecture on Mean Value Theroem

Project: Mean Value Theorem

Review

Your response:

Find the derivative of $f(x) = \dfrac{\sin\left(x^9\right)}{x^9}$.

Your response: ⊗

Find the derivative of $f(x) = \dfrac{\sin\left(x^9\right)}{x^9}$. $\dfrac{f}{g}$

Quotient Rule $\rightarrow$ $\dfrac{g \cdot f' - f g'}{g^2} = \dfrac{x^9 \cdot \frac{d}{dx}\sin(x^9) - \sin(x^9)\, 9x^8}{x^{18}}$

$$\frac{x^9 \cdot \cos(x^9) \cdot \frac{d}{dx} x^9 - \sin(x^9) \cdot 9x^8}{x^{18}}$$

$$\frac{x^9 \cdot \cos(x^9) \cdot 9x^8 - \sin(x^9) \cdot 9x^8}{x^{18}}$$

Use the position function $s(t) = 2t^2 - 8 \sin 8t$ to find the velocity at time $t = 0$. Assume units of feet and seconds.

$v(0) =$ 0 **feet/second**

Use the position function $s(t) = 2t^2 - 8 \sin 8t$ to find the velocity at time $t = 0$. Assume units of feet and seconds.

$v(0) =$ 0 **feet/second**

chain

$s' = 4t' - 8\cos(8t)\frac{d}{dt}(8t)$

$\to s' = 4t - 8\cos(8t) \cdot 8$

$s'(0) = 0 - 8\cos(0) \cdot 8$

$= -64$

Find the derivative of the function $f(x) = 7e^{2x+1}$.

$$\boxed{f'(x) = e^{2x+1}(2)}$$

Find the derivative of the function $f(x) = 7e^{2x+1}$.

$f\,'(x) = 7e^{2x+1}(2)$ ✗

$y = 7e^{\text{stuff}}$

$y' = 7e^{\text{stuff}} \cdot \frac{d}{dx}\,\text{stuff}$

$7e^{2x+1} \cdot 2$

$14e^{2x+1}$

Find the slope of the tangent line at the point (1, 2) for the ellipse $2x^2 + 7y^2 = 30$.

Find the slope of the tangent line at the point (1, 2) for the ellipse $2x^2 + 7y^2 = 30$.

$$\frac{d}{dx}\left(2x^2 + 7y^2 = 0\right)$$

$$4x + 14yy' = 0$$

$$y' = \frac{-4x}{14y}$$

$$y'(1,2) = \frac{-4(1)}{14(2)} = -\frac{1}{7}$$

Find the derivative $y\,'(x)$ implicitly for the equation

$$\frac{x+8}{y} = 3x + 2y^2.$$

Find the derivative $y'(x)$ implicitly for the equation

$\frac{x+8}{y} = 3x + 2y^2.$

$$\frac{y - (x+8)y'}{y^2} = 3 + 4yy'$$

$$y - (x+8)y' = 3y^2 + 4y^3y'$$

$$y - 3y^2 = \left((x+8) + 4y^3\right)y'$$

$$y' = \boxed{\frac{y - 3y^2}{x+8+4y^3}} = \frac{1-3y}{\frac{x+8}{y} + 4y^2}$$

$$= \frac{1-3y}{3x+2y^2+4y^2}$$

$$= \boxed{\frac{1-3y}{3x+6y^2}}$$

Find the derivative $y\,'(x)$ implicitly for the equation

$$\frac{x+8}{y} = 3x + 2y^2.$$

$$\frac{d}{dx}\, x + 8 = \frac{d}{dx}\, 3xy + 2y^3$$

$$1 = \underbrace{(3x)\left(\frac{dy}{dx}\right) + y(3)}_{\text{Product}} + \underbrace{6y^2\left(\frac{dy}{dx}\right)}_{\text{Chain Rule}}$$

$$\frac{1-3y}{3x+6y^2} = \frac{(3x+6y^2)}{3x+6y^2}\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1-3y}{3x+6y^2}$$

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$$\sqrt{xy} - 6y^2 = 57$$

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$$\frac{d}{dx}\left((xy)^{1/2} - 6y^2\right) = \frac{d}{dx}57$$

Power Chain

$$\frac{1}{2}(xy)^{-1/2} \cdot \underbrace{\frac{d}{dx}(xy)}_{\text{Product}} - 12y\frac{dy}{dx} = 0$$

$$\frac{1}{2\sqrt{xy}}\left(x\frac{dy}{dx} + y(1)\right) - 12y\frac{dy}{dx} = 0$$

$$\frac{x}{2\sqrt{xy}}\frac{dy}{dx} - 12y\frac{dy}{dx} = \frac{-y}{2\sqrt{xy}}$$

$$\frac{dy}{dx}\left(\frac{x}{2\sqrt{xy}} - 12y\right) = \frac{-y}{2\sqrt{xy}}$$

$$\frac{dy}{dx} = \frac{-y/2\sqrt{xy}}{\frac{x}{2\sqrt{xy}} - 12y}$$

or

$$\frac{dy}{dx} = \frac{-y}{x - 24y\sqrt{xy}}$$

Find the derivative $y'(x)$ implicitly.

$$\sqrt{xy} - 6y^2 = 57$$

$$\frac{d}{dx}\left((xy)^{1/2} - 6y^2\right) = \frac{d}{dx}57$$

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Find the derivative of $g(x) = \ln x^4$.

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$$\frac{1}{x^4} \cdot 4x^3 = \frac{4}{x}$$

Find the derivative of $g(x) = \ln x^4$.

$= 4 \ln x$

$$\frac{d}{dx} \ln(x^4)$$

$$\frac{1}{(x^4)} \cdot \frac{d}{dx}(x^4)$$

$$\frac{4x^3}{x^4} = \frac{4}{x} = \frac{4}{x}$$

Find the derivative of $g(x) = \ln x^4$.

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$\frac{d}{dx}$

$$4x + 18y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-4x}{18y}$$

At POINT $(1,1)$ $x=1$ $y=1$

$$\frac{dy}{dx} = -\frac{4}{18} \text{ or } -\frac{2}{9}$$

Find the derivative of $f(x) = \dfrac{\sin\left(x^4\right)}{x^4}$.

Find the derivative of $f(x) = \dfrac{\sin\left(x^4\right)}{x^4}$.

$$\frac{x^4 \cdot \frac{d}{dx}\sin(x^4) - \frac{d}{dx}x^4 \cdot \sin(x^4)}{(x^4)^2}$$

$$\boxed{\frac{x^4\cos(x^4)\cdot\underbrace{4x^3}_{\text{chain}} - 4x^3\sin(x^4)}{x^8}}$$

or

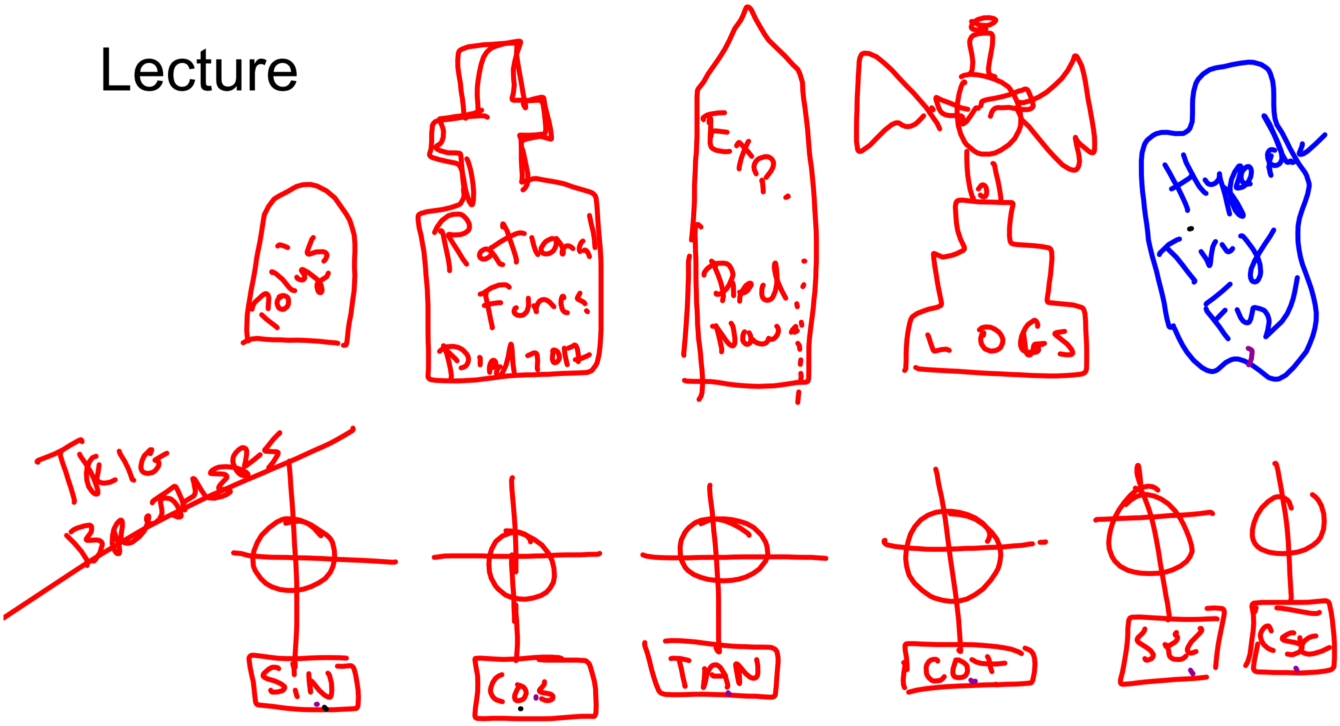
$$\frac{4x^4\cos(x^4) - 4\sin(x^4)}{x^5}$$

Lecture : MVT

Mean Value Theorem

- Squeeze Theorem.
- Intermediate value.
- MVT
- Fundamental Thr. of calc

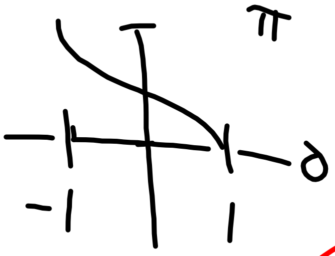
Lecture

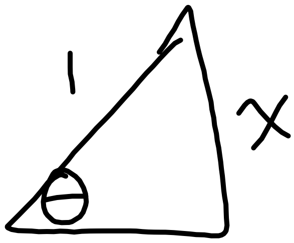

Polys
Rational Funcs
Exp.
LOGS
Hyp Trig Fu
TRIG
SIN
COS
TAN
COT
SEC
CSC

Inverse Trig Functions

$\boxed{y = \sin^{-1}(x)} \rightarrow \boxed{x = \sin y}$

$= \arcsin(x)$





$x = \sin\theta = \frac{\text{opp}}{\text{hyp.}}$

$\sin^{-1}(x) = \theta$

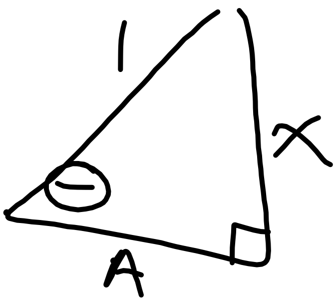
$\frac{d}{dx}$

$\frac{d}{dx} x = \frac{d}{dx} \sin\theta$

$\frac{d}{dx} \sin^{-1}(x) = \frac{d\theta}{dx}$

$1 = \cos\theta \frac{d\theta}{dx}$

$\frac{1}{\cos\theta} = \frac{d\theta}{dx}$



$\frac{1}{\sqrt{1-x^2}}$

$\sqrt{1-x^2} = \cos\theta = \frac{A}{H}$

$A^2 + x^2 = 1$

$A^2 = 1 - x^2$

$A = \sqrt{1-x^2}$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

Compute the derivative of $f(x) = \sinh^2(6x)$.

Compute the derivative of $f(x) = \sinh^2(6x)$.

$$\frac{d}{dx}\left(\sinh(6x)\right)^2$$

$$2\left(\sinh(6x)\right)^1 \cdot \frac{d}{dx}\cdot \sinh(6x)$$

$$2\sinh(6x)\cosh(6x)\cdot 6$$

Find all functions that have a derivative equal to $8x^7 + 6$.

Find all functions that have a derivative equal to $8x^7 + 6$.

x^0

$7+1$

$x^8 + 6x +$ constants

C

$\frac{d}{dx}$

$\boxed{8x^7 + 6} + c$

Compute the derivative of the given function.

$\sin^{-1}(x^9 + 5)$

Compute the derivative of the given function.

$\frac{d}{dx}\sin^{-1}(x^9+5)$

$\frac{d}{dx}\sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$= \frac{1}{\sqrt{1-(x^9+5)^2}} \cdot (9x^8 \ldots)$

Remember

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\sec(y) = \sec(y)\tan(y)\cdot\underbrace{\frac{dy}{dx}}_{\text{Chain Rule.}}$$

$$= \frac{1}{\cos y}\cdot\frac{\sin y}{\cos y}$$

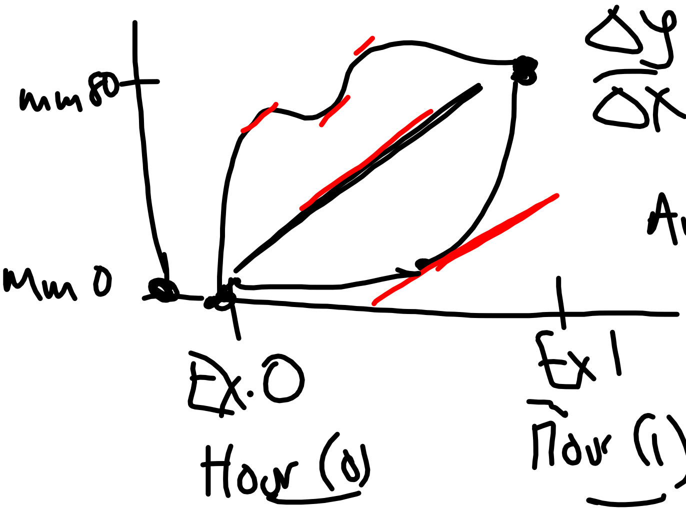
Mean Value Theorem

Given a continious function on an interval [a,b]

There is a moment c on [a,b] where the instantaneous rate of

$f'(c)$ = Average rate of change $\boxed{\frac{f(b)-f(a)}{b-a}}$

Ex

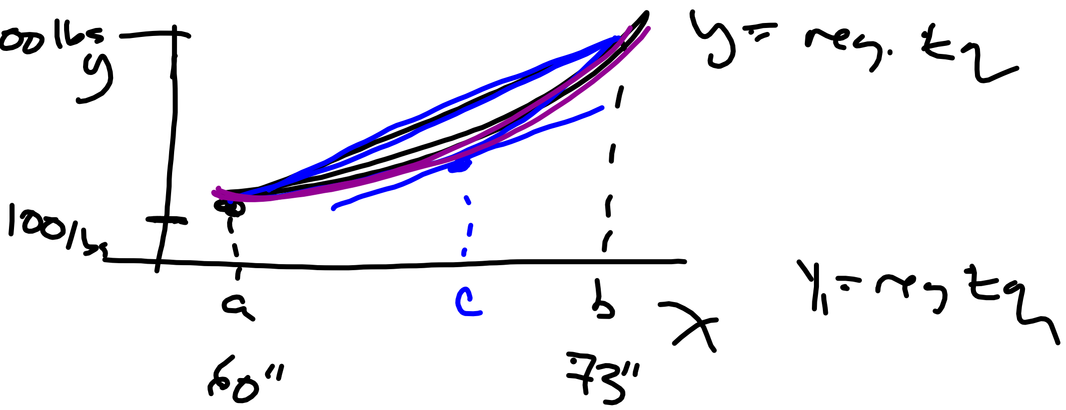


$\frac{\Delta y}{\Delta x} = \frac{80}{1}$

Ave. Rate 80mph

Group Work

MVT



Y_2 Ave Rate / slope = $(Y_1(60) - Y_1(73))/(60-73)$

Y_3 = nderiv(Y_1, X, X)

Calc 5: Intersect

1st Curve: Y2

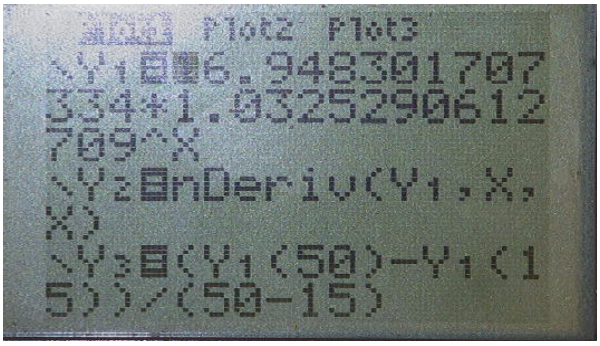
2nd curve: Y3

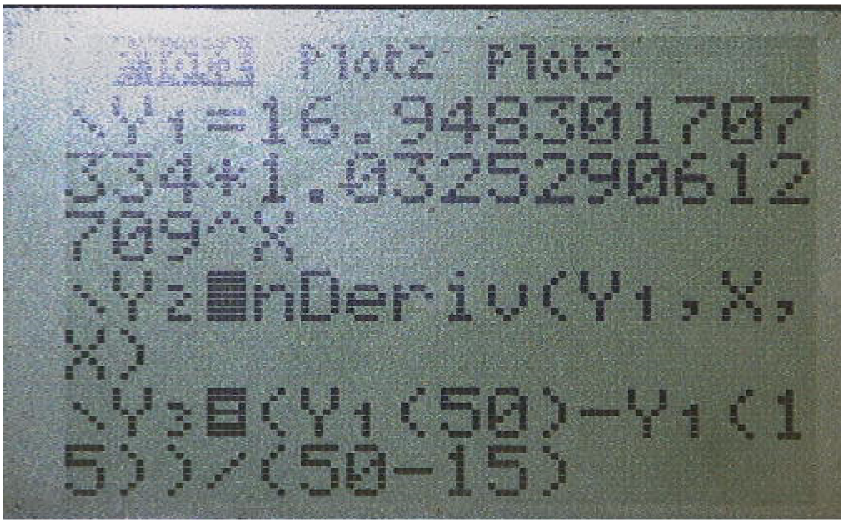
X = 68.3

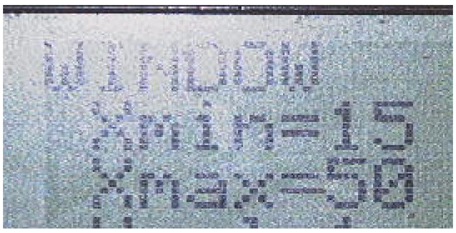
Y_1

A person who is 68" is characteristic of the people between 60" & 73"

And it's 100/113 lbs/in

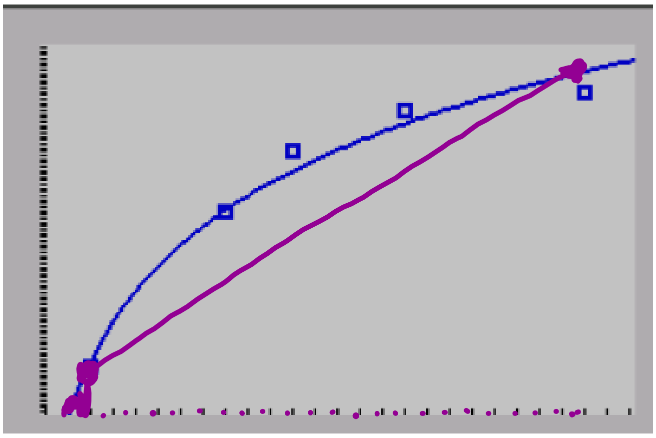




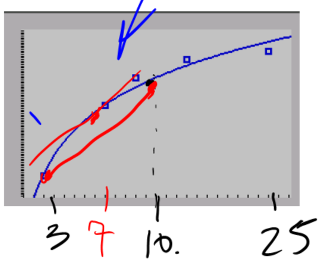


zoom 0: zoomfit

Picture



Picture



Math

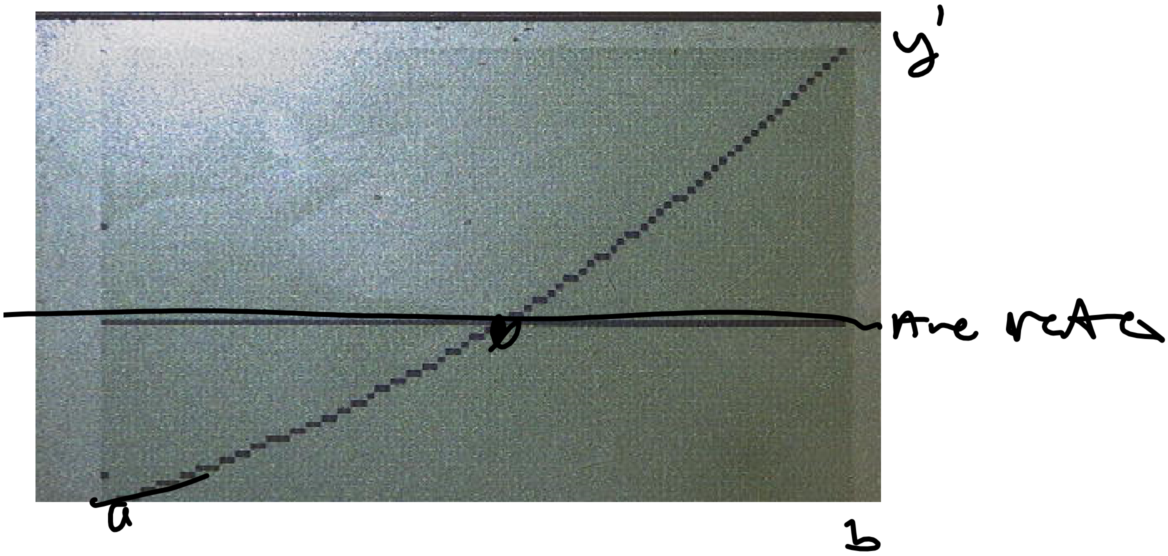
$$f'(\cancel{1}7) = \frac{f(10) - f(3)}{10 - 3} =$$

Instant = Ave

Words

According to the natural log regression

Between 3 & ~~25~~ 10 hours of studying the average growth rate is $\boxed{3.5\%}$ per hour which happens at exactly 10 hrs or 7 ~~10hrs~~ hrs has characteristic growth of the interval



calc 5: intersect

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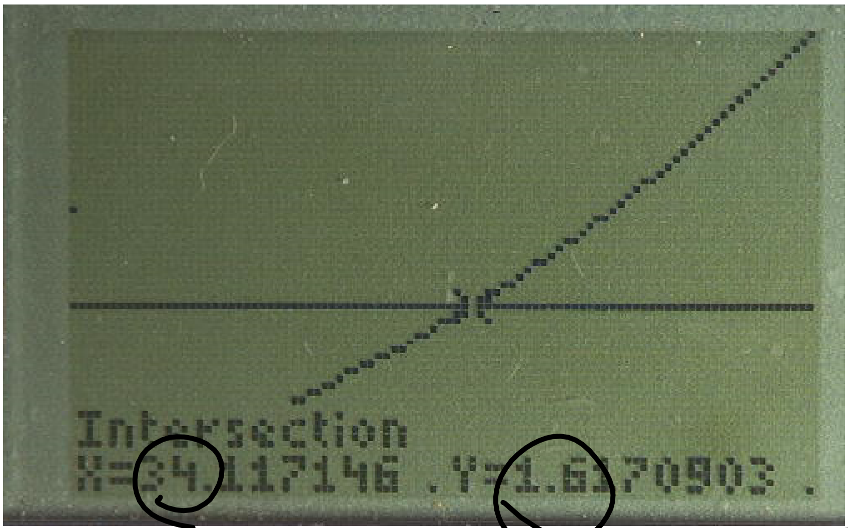


15 50

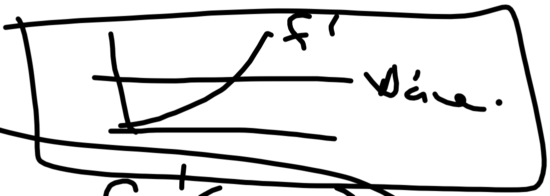
Over the prices from \$15 to \$50, The average sales rate was 1.6 people/\$

At \$34.12, the instantaneous rate of change is also 1.6.people per \$

\$34.12 is representative of the averager rate of sales trends over the prices between \$15 and \$50

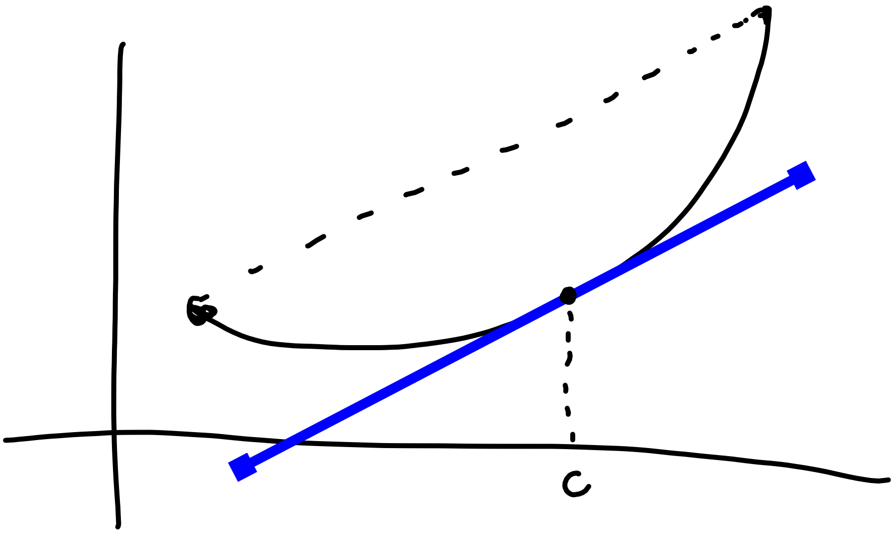
Writers

$$\frac{f(15)-f(50)}{15-50} = f'(34)$$



Speaker

Avg rate of change between 15" and 50" is the same as the instantaneous rate of change at 34" using exponential regression


c

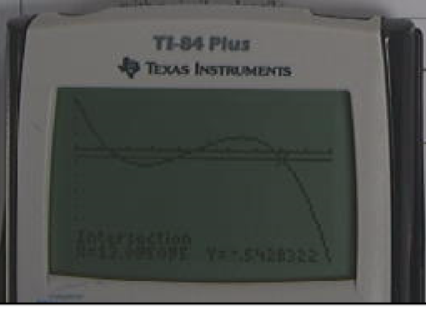
12. Use the mean value theorem on the two end points OF a regression and identify a point on the graph with a similar slope?

Y1=regEq Y2=nderiv(y1,x,x) ~~Y3="average rate of change"~~ Calc 5:intersect	Regression used:	exponential
	Ave Rate of change:	1.61people/$
	Point(s) of intersection:	$34.12

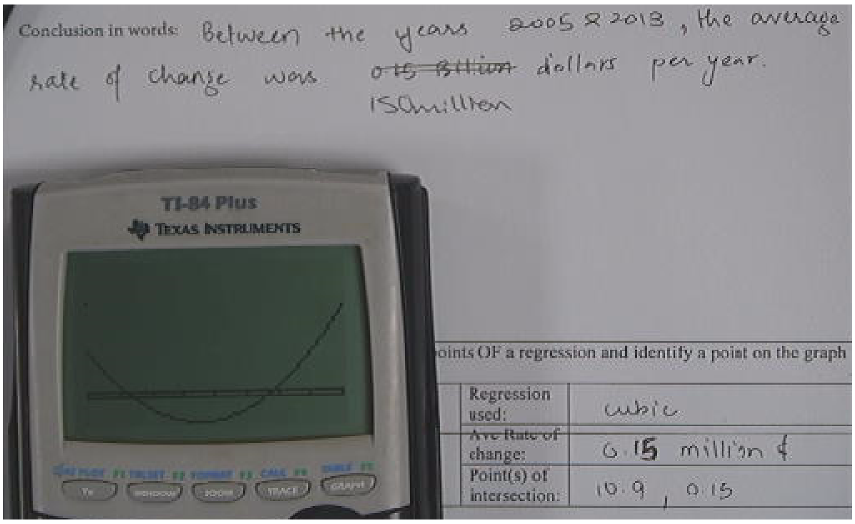
Between the years 2005 and 2015, the average rate of change was ≈ −.5 billion dollars per year. The year 2013 represents the entire decade trend of a decreasing rate and ROI.

Instantaneous rate of change in 2013: −.5428 billions of $/year

12. Use the mean value theorem on the two end points OF a regression and identify a point on the graph



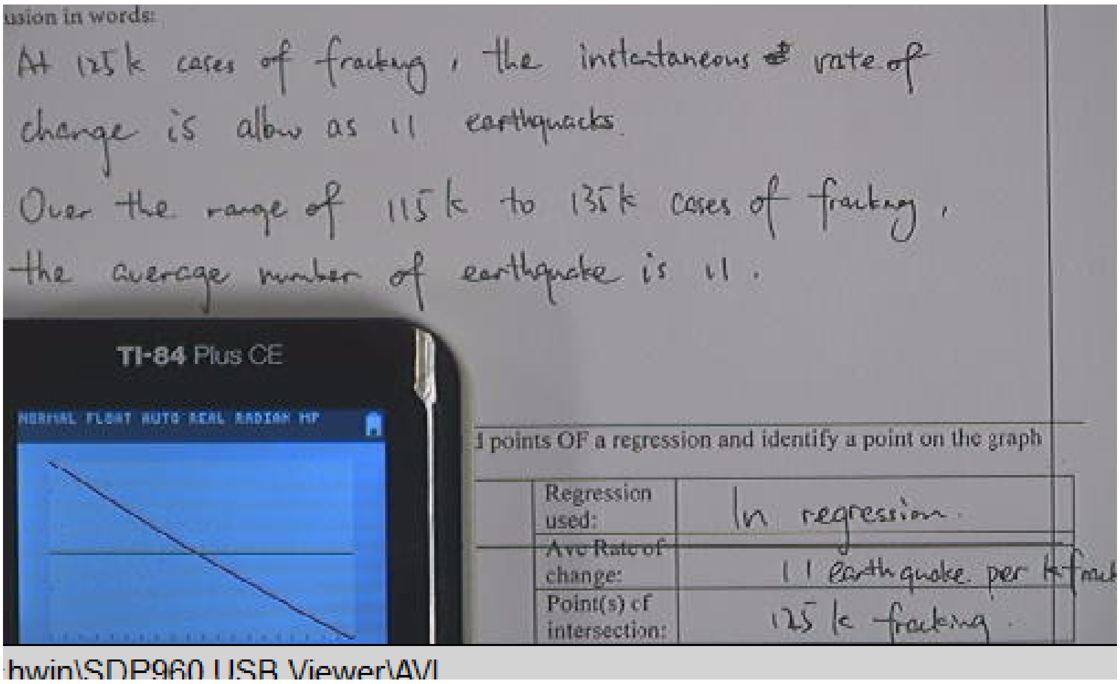
Regression used:	Quartic
Ave Rate of change:	−.5428
Point(s) of intersection:	(13.095095, −.548322)



Conclusion in words: Between the years 2005 & 2013, the average rate of change was ~~0.15 Billion~~ dollars per year.
150million

oints OF a regression and identify a point on the graph

Regression used:	cubic
Ave Rate of change:	0.15 million $
Point(s) of intersection:	10.9 , 0.15



usion in words:
At 125k cases of fracking, the instantaneous rate of change is also as 11 earthquacks.
Over the range of 115k to 135k cases of fracking, the average number of earthquake is 11.
TI-84 Plus CE
NORMAL FLOAT AUTO REAL RADIAN MP
d points OF a regression and identify a point on the graph
Regression used: ln regression.
Ave Rate of change: 11 earthquake per k frack
Point(s) of intersection: 125 k fracking.
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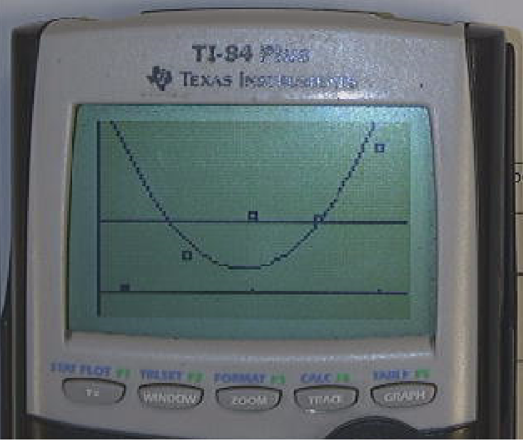
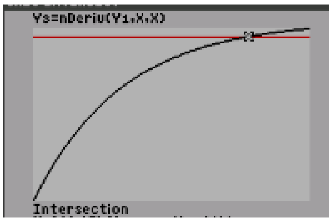
variable):______________________

Conclusion in words: After 2.51 years of legalization, weed revenue in taxes in Colorado's Average rate of change is 49 million dollars per year.



points OF a regression and identify a point on the grap

Regression used:	Cubic
Ave Rate of change:	$(Y_1(3) - Y_1(1)) / (3 - 1)$
Point(s) of intersection:	(2.51, 49)



$$f'(2.13) = -.0144$$

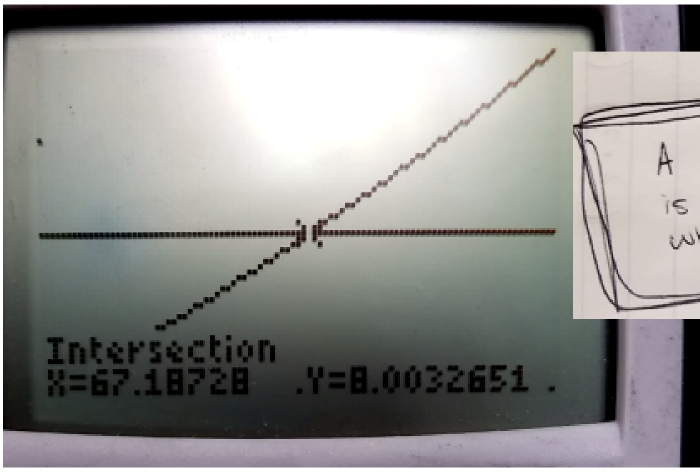
$$= V_{Ave.}$$

$$= \frac{f(25)-f(250)}{25-250}$$

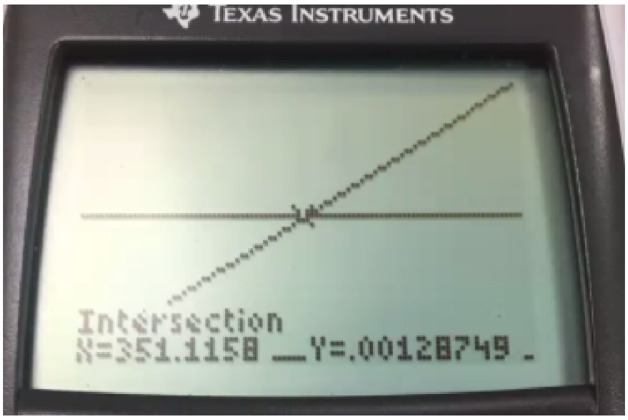
Speaker:

Between 25¢ and $2.50,
The average loss of sales
is represented at the instant of $2.13
according to exponential regression

$-.0144$ cups/¢ent



A person who is 67 inches is characteristic of people who are between 62 and 72 inches

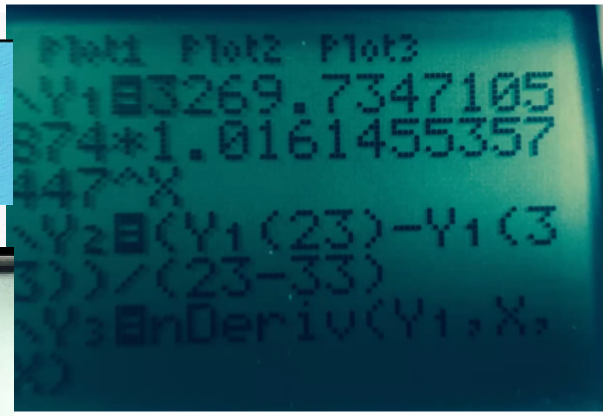


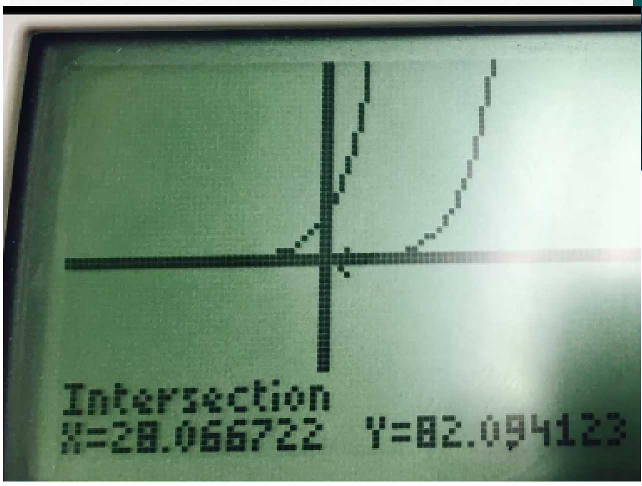
A house that is sold at $351,000, is the average price of houses between $200,000 an
$500,000.

At the point in time 2008.1

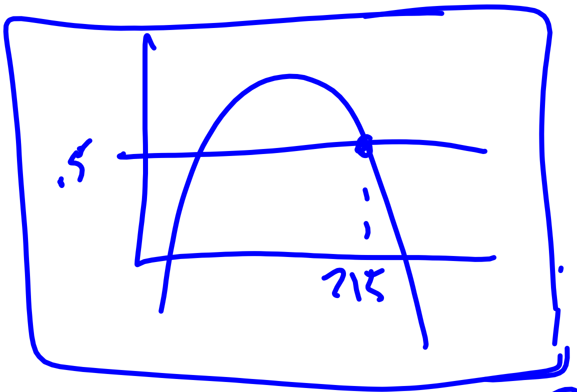
In the year 2008.0667..., the rate of change of individuals being diagnosed with morbid obesity was equal to the average rate of change from 2003-2013.

82.09 morbidly obese people per year





Picture



math

$$P_3'(215) = .5 = \frac{P_3(244) - P_3(124)}{244 - 124}$$

Word

According to the cubic regression a person weighing 215 lbs grows in ability at the same rate on average for people between 124 and 244 lbs.