Agenda

Quiz 5 Review

Limits of Trig Functions

Derivatives of Inverse Trigs

Hyperbolic Trig Functions

Derivative of secant

Project Derivatives by Hand

Quiz 5 Review

Find the derivative $y\,'(x)$ implicitly for the equation

$$\frac{2x+3}{y} = 2x + y^3.$$

Find the derivative $y\,'(x)$ implicitly for the equation

$$\frac{2x+3}{y} = 2x + y^3.$$

$$(2x+3)' = (2xy + y^4)'$$

$$2 = 2x\frac{dy}{dx} + 2y + 4y^3\frac{dy}{dx}$$

$$2 - 2y = (2x + 4y^3)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2-2y}{2x+4y^3}$$

Find the derivative of $f(x) = \dfrac{\sin\left(x^5\right)}{x^5}$.

#4

Find the derivative of $f(x) = \dfrac{\sin\left(x^5\right)}{x^5}$.

$$f' = \frac{x^5 \cdot \frac{d}{dx}\sin(x^5) - \sin(x^5)\cdot\frac{d}{dx}x^5}{x^{10}}$$

$$= \frac{x^5 \cos(x^5)\cdot\frac{d}{dx}x^5 - \sin(x^5)\cdot 5x^4}{x^{10}}$$

$$= \boxed{\frac{5x^9\cos(x^5) - 5x^4\sin(x^5)}{x^{10}}}$$

$\underline{\text{Ex}}$

$$\cos\left(\frac{x^2}{\ln x + e^x}\right)$$

$$- \sin\left(\frac{x^2}{\ln x + e^x}\right) \cdot \frac{d}{dx}\left(\frac{x^2}{\ln x + e^x}\right)$$

Lecture

Limits of Trig Functions

Calculus

$$2\text{pt} \xrightarrow{\text{limit}} 1\text{pt}$$

Def of Derivative

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$y = \sin x$$

<u>Def</u>

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} = \frac{d}{dx}(\sin(x))$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h \to 0} \left[\sin x \frac{(\cos h - 1)}{h} + \frac{\sin h}{h} \cos x \right]$$

$$\boxed{\lim_{h \to 0} \frac{(\cos h - 1)}{h} = 0}$$

$$\boxed{\lim_{h \to 0} \frac{\sin h}{h} = 1}$$

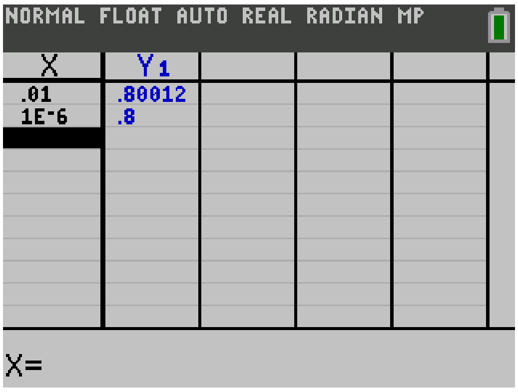
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{k \to 0} \frac{\sin k}{k} = 1 = \lim_{k \to 0} \frac{k}{\sin x}$$

$$\lim_{5k \to 0} \frac{\sin 5k}{5k} = 1$$

Ex

$$\lim_{x \to 0} \frac{\sin(4x)}{\sin(5x)} = \left[\lim_{x \to 0} \frac{\sin(4x)}{4x} \cdot \frac{5x}{\sin(5x)} \, \frac{4}{5}\right]$$



NORMAL FLOAT AUTO REAL RADIAN MP

X	Y_1
.01	.80012
1E-6	.8

X=

$$\lim_{x \to 0} \frac{\sin(4x)}{\sin(5x)} =$$

$$\lim_{x \to 0} \frac{\sin(4x)}{4x} \cdot \frac{5x}{\sin(5x)} \cdot \frac{4}{5} = \frac{4}{5}$$

Derivative of Inverse Functions

$y = f(x)$

$x = f^{-1}(y)$

$$x = \sin^{-1}(y)$$

$$\sin x = y$$

$$\tan^{-1} y = x$$

$$y = \tan x$$

Derivative of

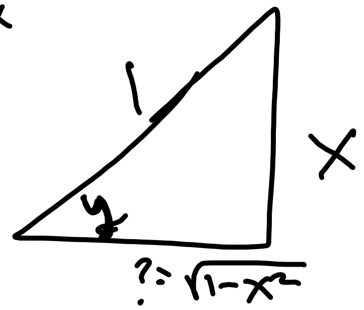
$y = \sin^{-1}(x)$ $\quad \frac{dy}{dx} = ??$

Implicit

$\frac{d}{dx} \quad \boxed{x = \sin y}$

$1 = \cos y \cdot \frac{dy}{dx}$ chain rule

$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$



$\boxed{\sin y = \frac{x}{1}} = \text{sine} = \frac{\text{opp}}{\text{hyp}}$

Pythagorean Theorem

$?^2 + x^2 = 1$

$?^2 = 1 - x^2$

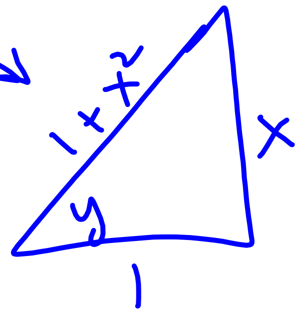
$? = \sqrt{1-x^2}$

CAH

$\cos y = \frac{\text{Adj.}}{\text{hyp}} = \frac{\sqrt{1-x^2}}{1}$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \boxed{\frac{1}{\sqrt{1-x^2}} = \frac{d}{dx}\sin^{-1}(x)}$$

$$\frac{x}{1} = \tan y$$



$$1 = \sec^2 y \frac{dy}{dx}$$

$$\sec y = \frac{H}{A} = \frac{1+x^2}{1}$$

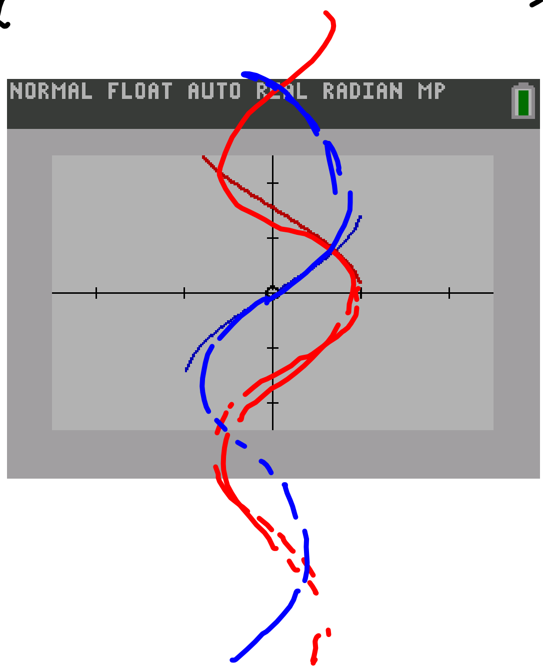
$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$

NOT

$$\frac{d}{dx}\tan x = \sec^2 x$$



$$\frac{d}{dx} e^x = e^x$$

want $\quad y = \ln x \quad \frac{dy}{dx} =$

Implicit Diff $\quad \frac{d}{dx}\left(e^y = x\right)$

$$e^y \cdot \frac{dy}{dx} = 1 \qquad \frac{dy}{dx} = \frac{1}{e^y}$$

$y = \ln x$

$e^y = e^{\ln x} = x$

$$= \frac{1}{x}$$

Also....

$$y = \ln_e x$$

$$\frac{d}{dx} \quad \boxed{x = e^y}$$

$$1 = e^y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

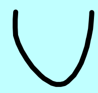
$$\boxed{\frac{d}{dx} \ln x = \frac{1}{x}}$$

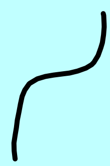
$$\lim_{h \to 0} \frac{e^{x+h} - e^{x}}{h}$$

$$\lim_{h \to 0} e^{x} \left(\frac{e^{h} - 1}{h} \right) = e^{x}$$

1

Hyperbolic Trig Functions

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$$ 

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2}$$ 

cosh(0)=1 sinh(0)=0

$$\frac{d}{dx}\cosh x = \sinh x$$

$$\frac{d}{dx}\sinh x = \cosh x$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{csch} x = \frac{1}{\sinh x}$$

Hyperbolic Trig Functions

Hyperbolic cosine:
$\cosh(x) \equiv \frac{e^{x}+e^{-x}}{2}$

Hyperbolic Sine:
$\sinh(x) \equiv \frac{e^{x}-e^{-x}}{2}$

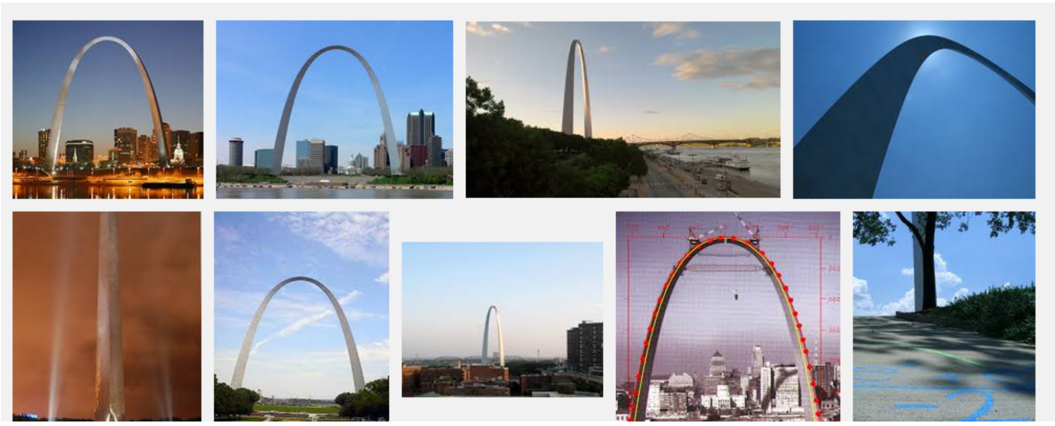
Hyperbolic tangent:
$\tanh = \boxed{\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}}$

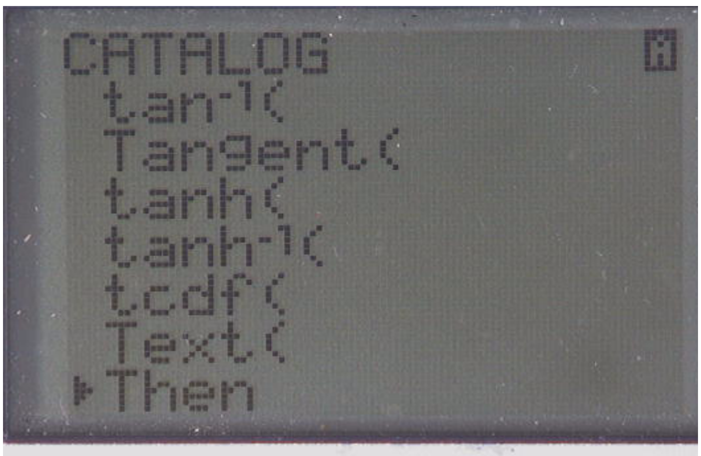
sech
csch
coth

$$\boxed{\begin{array}{l} \frac{d}{dx}\cosh(x) = \sinh(x) \\ \frac{d}{dx}\sinh(x) = \cosh(x) \end{array}}$$

$$\frac{d}{dx}\tanh(x) = \frac{d}{dx}\boxed{\frac{\sinh(x)}{\cosh(x)}}$$

$$\boxed{\cosh^2(x) - \sinh^2(x) = 1}$$





$$\frac{d}{dx}\cosh(x^4)$$

$$= \sinh(x^4)\cdot 4x^3$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

Inverse functions <u>too</u>.

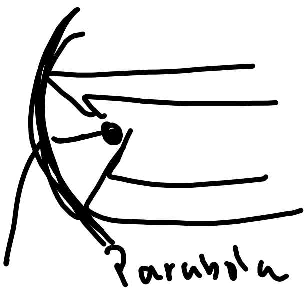
$$y = \cosh^{-1}(x)$$

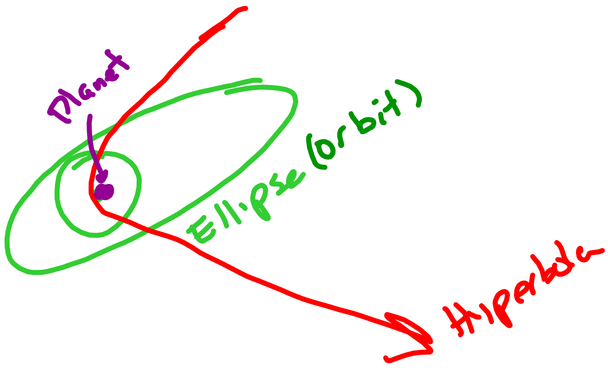
-means-

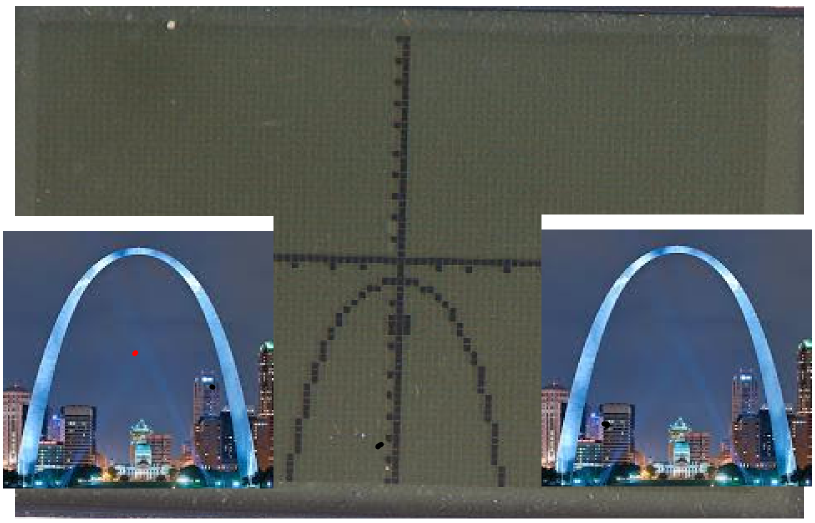
$$\cosh(y) = x$$

NOT / $\frac{1}{\cosh(x)} = \operatorname{sech}(x)$

Real Life







St Louis Arch

More Examples of Derivative

$$y = \sec x$$

$$y' = \underline{\sec x \tan x} = \frac{\sin x}{\cos^2 x}$$

Ex $\frac{d}{dx}\sec(e^x)$

$$\sec(e^x)\tan(e^x)\cdot\frac{d}{dx}e^x$$

Ex

$$\frac{d}{dx} \sin^{-1}(\ln(x^2+7))$$

$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$

Remember

$$\frac{1}{\sqrt{1-(\ln(x^2+7))^2}} \cdot \frac{d}{dx} \ln(x^2+7)$$

$$\frac{1}{\sqrt{1-(\ln(x^2+7))^2}} \cdot \frac{1}{x^2+7} \quad \frac{d}{dx} x^2+7$$

$$\frac{1}{\sqrt{1-(\ln(x^2+7))^2}} \cdot \frac{1}{x^2+7} \cdot 2x$$

Compute the derivative of the given function.

$\sin^{-1}(x^9 + 3)$

Compute the derivative of the given function.

$\frac{d}{dx}\sin^{-1}(x^9 + 3)$

$\frac{d}{dx} \sin x = \frac{1}{\sqrt{1-x^2}}$

$= \frac{1}{\sqrt{1-(x^9+3)^2}} \cdot \frac{d}{dx}(x^9+3)$

$= \frac{9x^8}{\sqrt{1-(x^9+3)^2}}$

Determine whether the families of curves are orthogonal or not.

$y = cx^3$ and $6x^2 + 12y^2 = k$

- A. No, the families of curves are not orthogonal.
- B. Yes, the families of curves are orthogonal.

Determine whether the families of curves are orthogonal or not.

$y = cx^3$ and $6x^2 + 12y^2 = k$ $\longrightarrow$

A. No, the families of curves are not orthogonal.

B. Yes, the families of curves are orthogonal.

$y' = 3cx^2$

$12x + 24y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-12x}{24y}$

$\frac{dy}{dx} = \frac{-12x}{24(cx^3)} = \frac{-1}{2cx^2}$

Compute the derivative of $f(x) = \sinh^2(2x)$.

$f'(x) =$ ☐

Compute the derivative of $f(x) = \sinh^2(2x)$.

$f'(x) =$ ☐

$$y = (\sinh(2x))^2$$

$$y' = 2(\sinh(2x))^1 \cdot \frac{d}{dx} \underline{\sinh}(2x)$$

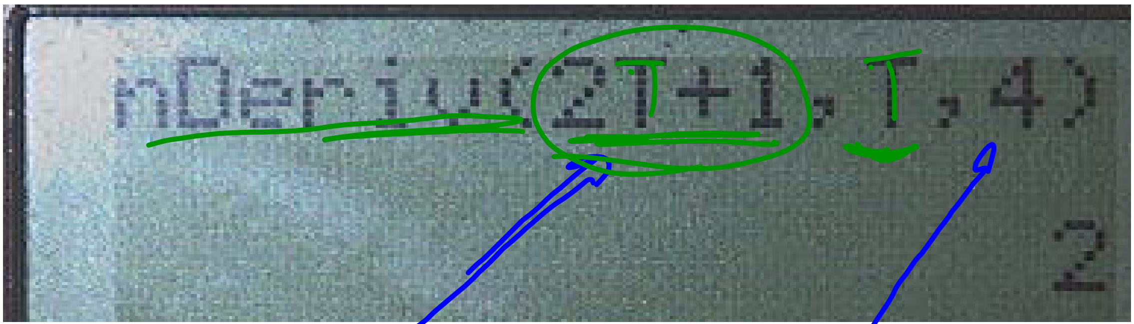
Chain Rule

$$y' = 2\sinh(2x)\cosh(2x) \cdot \frac{d}{dx} 2x$$

$$= 4\sinh(2x)\cosh(2x)$$

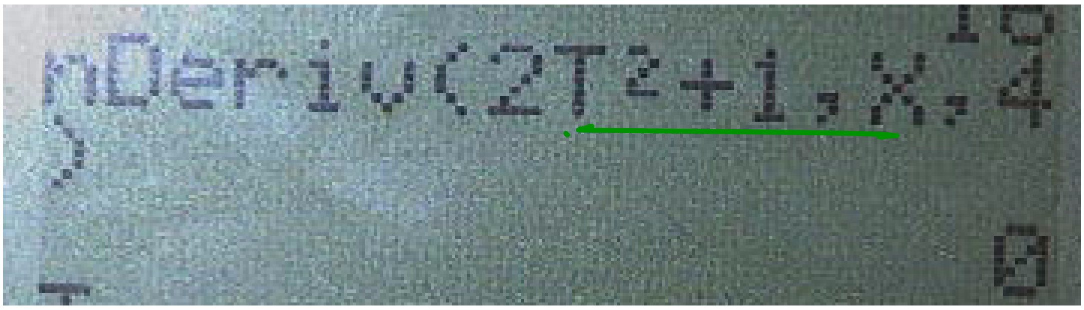
Project

Derivatives of Transcendentals



$y = 2T+1$ evaluat at $T = 4$

$$\frac{d}{dT} y = \frac{d}{dT} 2T+1 = 2$$



$$\frac{d}{dx}\,2T^2+1 = 0$$

because T is a constant here.

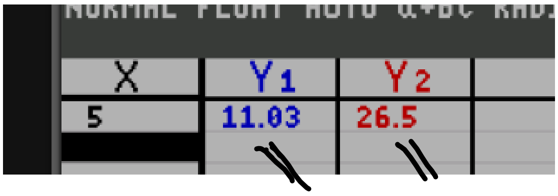
$$\frac{d}{dx}\,2T^2+1 = 4T\frac{dT}{dx}$$

Derivatives by Hand

$$y = Ax^4 + Bx^3 + Cx^2 + Dx + E$$

$$y' = 4Ax^3 + 3Bx^2 + 2Cx + D$$

$$Y_1 = A * B^x$$



X	Y_1	Y_2
5	11.03	26.5

reg eq — deriv

$$Y_2 = A * B^x \cdot \ln(B)$$

$$Y_1 = A + B \ln X$$

$$y_2' = B/x$$

$$y_3'' = -B/x^2$$

Exp. $\quad y = a \cdot b^x$

$y' = a \cdot b^x \cdot \ln b$

ln $\quad y = a + b\ln x$

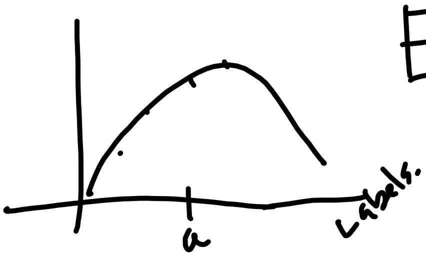
$y' = b/\ln x$

sin $\quad y = a\sin(bx+c)+d$

$y' = a\cos(bx+c)\cdot b$

$y'' = -a\sin(bx+c)b^2$

Discussion- Same as poly except use transcendental function (sine preferred)



x	y_1	y_2	y_3
a	b	c	d

Math-

$y(a) = b$ units

$y'(a) = c$ units

$y''(a) = d$ units

Words

According to the sine regresh,
At $x=a$, changing by b
accelerating by d