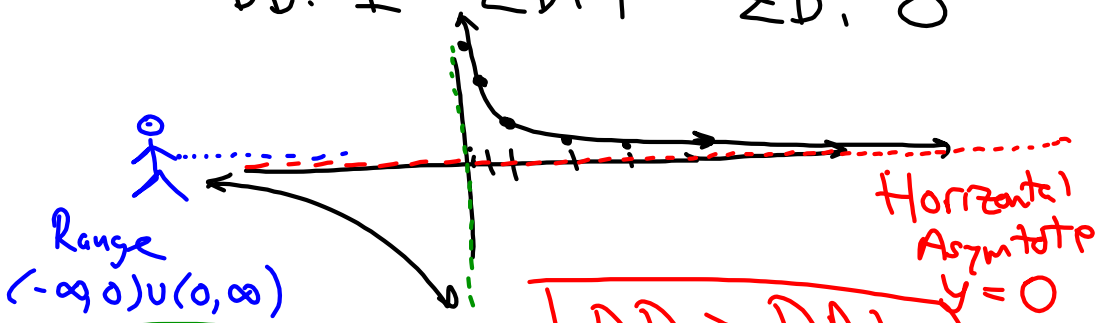
Rational Functions

$r(x) = \frac{p(x)}{q(x)}$ ← Numerator ← Denominator

Degree of Numerator (DN)
" " Denominator (DD)
Lead of Numerator (LN)
" " Denom (LD)
Zeros of Numerator (ZN)
" " Denom. (ZD)

Ex $y = \frac{1}{x}$ $x^0 = 1$

DN: 0 LN: 1 ZN: —
DD: 1 LD: 1 ZD: 0



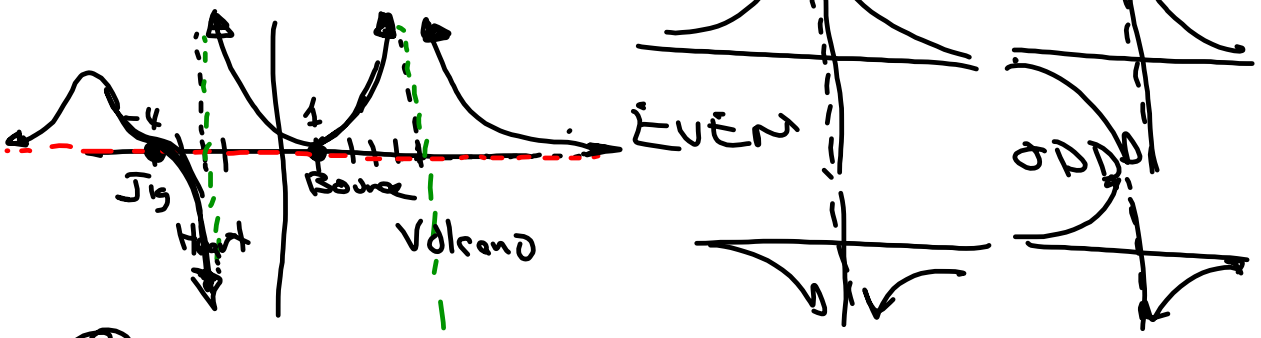
Vertical Asymtote
ZD

DD > DN
HA: y = 0

Zeros
ZN (None)

$$y = \frac{2(x-1)^2(x+4)^3}{3(x-5)^6(x+2)^3}$$

DN: 5 LN: 2 ZN: 1 (bounce), −4 (Jig)
DD: 9 LD: 3 ZD: 5, −2 (Heart beat)
HA: $y=0$ (volcano)



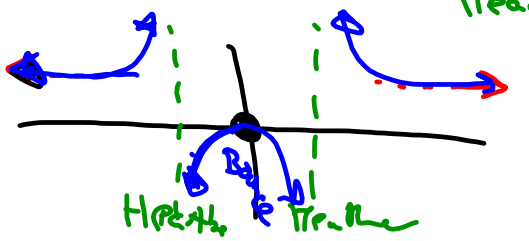
Qualitative
(Not to scale)

What if DN = DD ? HA: $y = \frac{LN}{LD}$

$$y = \frac{3x^2}{x^2-1}$$

DN: 2 LN: 3 ZN: 0, 0
DD: 2 LD: 1 ZD: 1, −1

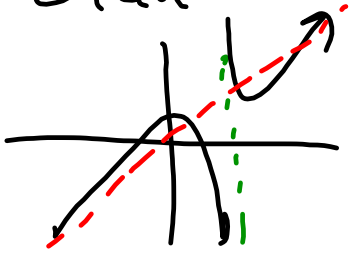
HA $y = \frac{3}{1} = 3$



What if DN > DD

Disco's it happy/sad

or Slant Asymptotes



DN − DD

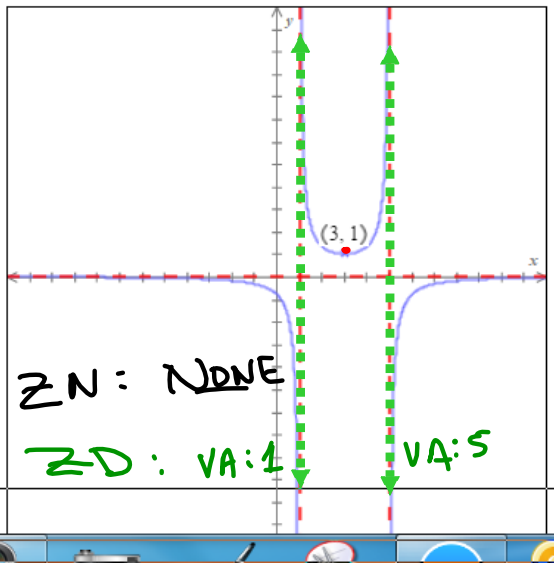
EVEN
Happy/Sad
LN/LD = +, −

ODD
Disco slant
LN/LD = +, −

Right, Left

The figure below shows the graph of a rational function f.
It has vertical asymptotes $x = 1$ and $x = 5$, and horizontal asymptote $y = 0$.
The graph does not have an x-intercept, and it passes through the point $(3, 1)$.

The equation for $f(x)$ has one of the five forms shown below.
Choose the appropriate form for $f(x)$, and then write the equation.
You can assume that $f(x)$ is in simplest form.



- $f(x) = \frac{a}{x - b}$
- $f(x) = \frac{a(x - b)}{x - c}$
- $f(x) = \frac{-4}{(x - 1)(x - 5)}$
- $f(x) = \frac{a(x - b)}{(x - 1)(x - 5)}$
- $f(x) = \frac{a(x - b)(x - c)}{(x - 1)(x - 5)}$

$y = \frac{a}{(x-1)(x-5)}$

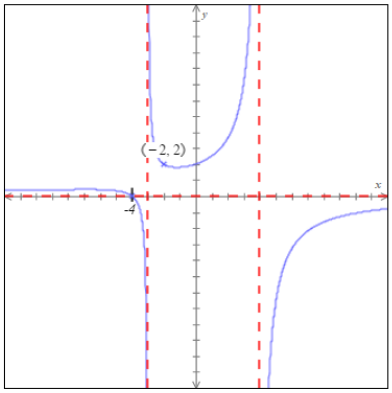
$1 = \frac{a}{(3-1)(3-5)}$

$1 = \frac{a}{-2}$

$-4 = a$

The graph has x-intercept -4, and it passes through the point $(-2, 2)$.

The equation for $f(x)$ has one of the five forms shown below.
Choose the appropriate form for $f(x)$, and then write the equation.
You can assume that $f(x)$ is in simplest form.



- $f(x) = \dfrac{a}{x-b}$
- $f(x) = \dfrac{a(x-b)}{x-c}$
- $f(x) = \dfrac{a}{(x-b)(x-c)}$
- $f(x) = \dfrac{a(x-b)}{(x-c)(x-d)}$
- $f(x) = \dfrac{a(x-b)(x-c)}{(x-d)(x-e)}$

Talking:

$y = \dfrac{a(x+4)}{(x+3)(x-4)}$

$2 = \dfrac{a(-2+4)}{(-2+3)(-2-4)}$

$(x-2)(x-2)(x-2+3i)(x-2-3i)$

CLOSE WINDOW

Using a given zero to write a polynomial as a product of linear factors: Co zeros

Talking:

For the polynomial below, 2 is a zero of multiplicity two.

Factor $(x-2)^2$

x^2-4x+4

$g(x) = x^4 - 8x^3 + 33x^2 - 68x + 52$

Express $g(x)$ as a product of linear factors.

We are given that 2 is a zero of multiplicity two for $g(x) = x^4 - 8x^3 + 33x^2 - 68x + 52$.
So by the factor theorem, there must exist a polynomial $Q(x)$ that satisfies the following equation.

$$x^4 - 8x^3 + 33x^2 - 68x + 52 = (x-2)(x-2)Q(x)$$

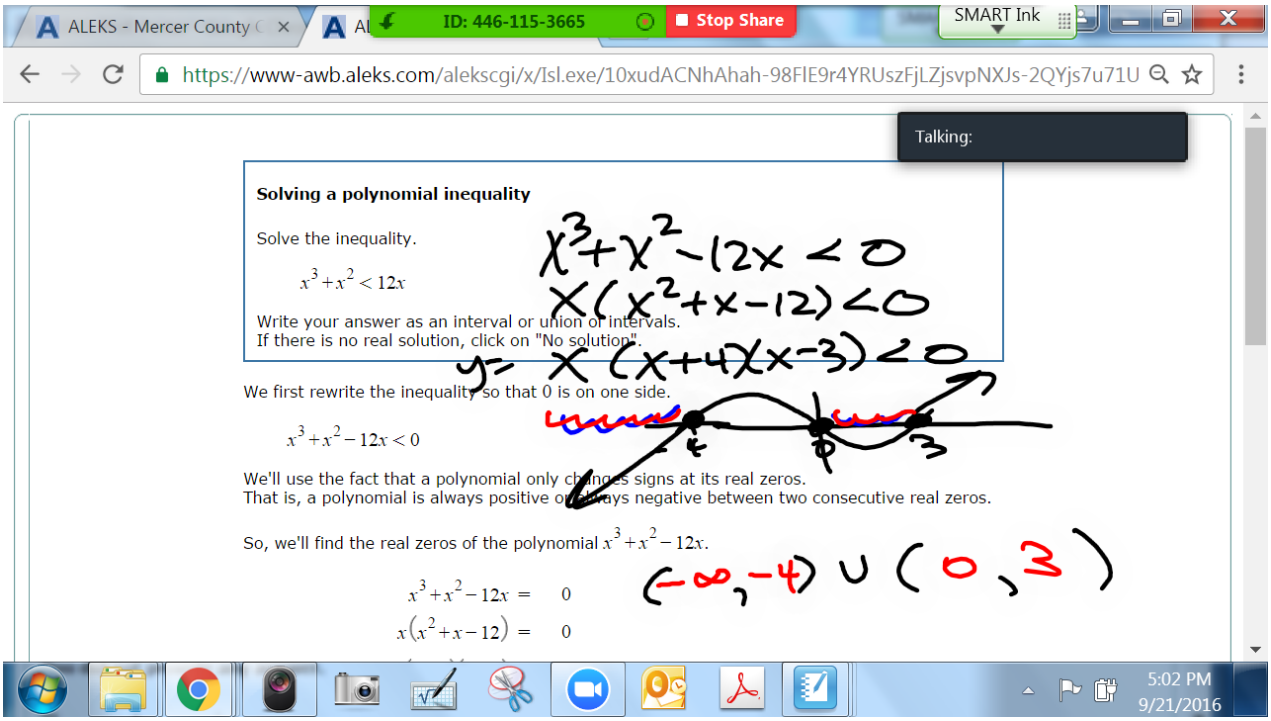
To find $Q(x)$, we divide $x^4 - 8x^3 + 33x^2 - 68x + 52$ by $(x-2)(x-2) = x^2 - 4x + 4$.

$$x^2-4x+4 \overline{)\; x^4-8x^3+33x^2-68x+52} \quad \leftarrow Q(x)$$

$x^2-4x+13$

$2 \pm 3i$

$\dfrac{4 \pm \sqrt{16-52}}{2}$


ALEKS - Mercer County
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Stop Share
SMART Ink
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Talking:
Solving a polynomial inequality
Solve the inequality.
$x^3+x^2<12x$
Write your answer as an interval or union of intervals.
If there is no real solution, click on "No solution".
We first rewrite the inequality so that 0 is on one side.
$x^3+x^2-12x<0$
We'll use the fact that a polynomial only changes signs at its real zeros.
That is, a polynomial is always positive or always negative between two consecutive real zeros.
So, we'll find the real zeros of the polynomial x^3+x^2-12x.
$x^3+x^2-12x = 0$
$x(x^2+x-12) = 0$
$x^3+x^2-12x<0$
$x(x+4)(x-3)<0$
$(-\infty,-4)\cup(0,3)$
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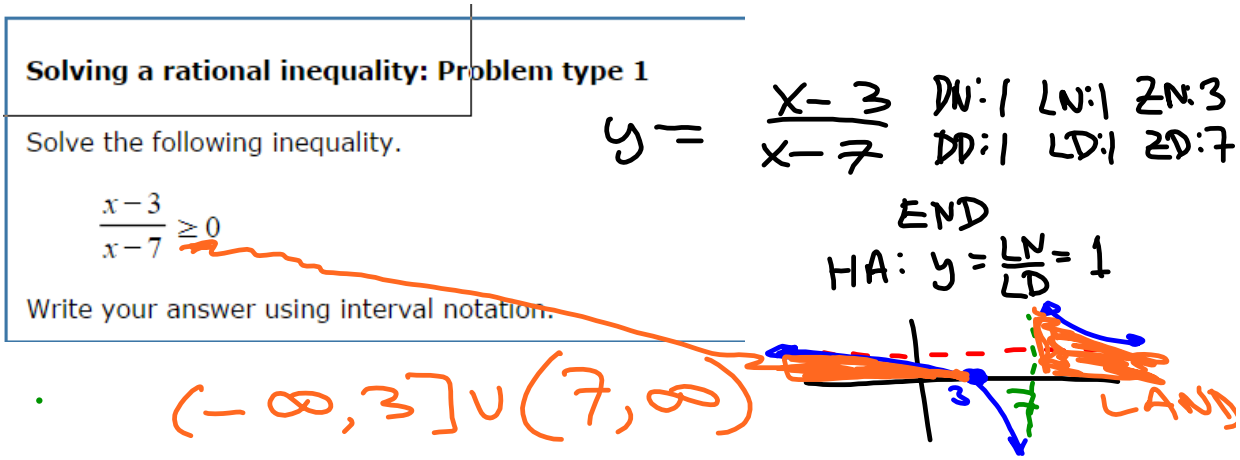

Solving a rational inequality: Problem type 1
Solve the following inequality.
$\frac{x-3}{x-7}\geq 0$
Write your answer using interval notation.
$y=\frac{x-3}{x-7}$
DN:1 LN:1 ZN:3
DD:1 LD:1 ZD:7
END
HA: $y=\frac{LN}{LD}=1$
$(-\infty,3]\cup(7,\infty)$
3
7
LAND

Solving

Regression = value.

- Intersection method

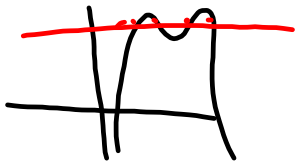
y_1 = regression

y_2 = value

Calc : 5 Intersect

<enter><enter><enter>

Note: you may have to adjust window



Solver.

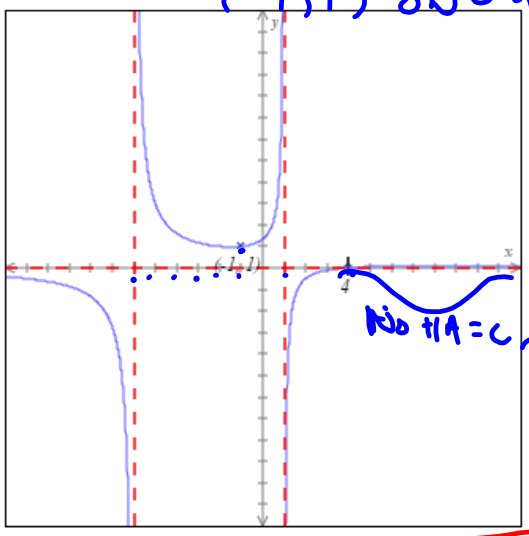
Math 0: Solver

0 = Regression − y

X =

y =

(-1,1) ONGRAPH



HOLE AT x=4

- $f(x) = \frac{a}{x-b}$
- $f(x) = \frac{a(x-b)}{x-c}$
- $f(x) = \frac{a}{(x-b)(x-c)}$
- $f(x) = \frac{a(x-b)}{(x-c)(x-d)}$
- $f(x) = \frac{a(x-b)(x-c)}{(x-d)(x-e)}$

$y = \frac{q(x-4)}{(x-1)(x+6)}$

$y = \frac{2(x-4)}{(x-1)(x+6)}$

$1 = \frac{q(-5)}{-2 \cdot 5}$ $q = 2$

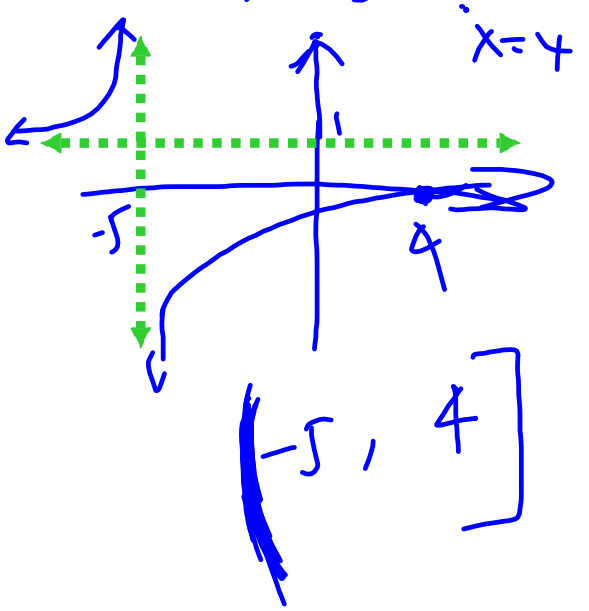
Solving a rational inequality: Problem type 1

Solve the following inequality.

$$\frac{x-4}{x+5} \leq 0$$

Write your answer using interval notation.

$y = \frac{x-4}{x+5} \leq 0$ $x+5 \neq 0$ $x = 4$



$[-5, 4]$

Day 8 - Question #13;
Graphing a rational function: Quadratic over linear

Graph the rational function $f(x) = \dfrac{9x^2 + 12x + 1}{3x + 2}$.

To graph the function, draw the asymptotes (if any) and plot at least t
graph.

$3x+2 \overline{)\,9x^2+12x+1}$

$-(9x^2+6x)$

$6x+1$

$-(6x+4)$

Zeros: $0 = 9x^2+12x+1$

$x = \dfrac{-12 \pm \sqrt{144-4(9)}}{2(9)}$

$\dfrac{-12 \pm \sqrt{108}}{18}$