



$$\sin(\theta) = -8/17$$

$$\cos(\theta) = 15/17$$

$$\sin(2\theta) = 2(-8/17)(15/17) = -240/289$$

$$\cos(2\theta) = 2(15/17)(15/17) - 1 = 161/289$$

$$\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{-240}{161}$$

$$\begin{aligned} \sin 2x &= 2 \sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2 \sin^2 x \\ &= 2 \cos^2 x - 1 \\ \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x} \end{aligned}$$

$2 * 15 * 15 / 17 / 17 - 1$

Saving

Input:

$$2 \times 15 \times \frac{15}{17} - 1$$

Exact result:

$$\frac{161}{289}$$

Decimal approximation:

0.557093425605536332179930795847750865

$\frac{\sin\left(x + \frac{\pi}{2}\right)}{\sin(\pi - x)}$	
$= \frac{\sin(x) \cos\left(\frac{\pi}{2}\right) + \cos(x) \sin\left(\frac{\pi}{2}\right)}{\sin(\pi - x)}$	Sum and Difference
$= \frac{\sin(x) \cos\left(\frac{\pi}{2}\right) + \cos(x) \sin\left(\frac{\pi}{2}\right)}{\sin(\pi) \cos(x) - \cos(\pi) \sin(x)}$	Sum and Difference
$= \frac{\sin(x)0 + \cos(x)1}{0 \cos(x) - (-1) \sin(x)}$	Evaluation
$= \frac{\cos(x)}{\sin(x)}$	Algebra
$= \cot x$	Quotient

$\frac{1}{\cot x (1 - \cos 2x)}$	
$= \frac{1}{\cot x (2 \sin^2 x)}$	Double-angle
$= \frac{1}{\frac{\cos x}{\sin x} (2 \sin^2 x)}$	Quotient
$= \frac{1}{2 \cos x \sin x}$	Algebra
$= \frac{1}{\sin(2x)}$	Double-angle
$= \csc(2x)$	Reciprocal

tan(105degrees)

Input:
tan(105°)

Exact result:

Decimal approximation:
-3.732050807568877:

tan(105degrees)

Input:
tan(105°)

Exact result:
 $-2 - \sqrt{3}$

Half-angle identities: Problem type 1

Use a half-angle formula to find the exact value of $\tan 105^\circ$.

We have the following trigonometric half-angle formulas.

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{1 + \cos u}} = \frac{\sin u}{1 + \cos u} = \frac{1 - \cos u}{\sin u}$$

$$\tan(210/2) = (1 - \cos(210)) / \sin(210)$$

$$(1 - (-\sqrt{3}/2)) / -0.5$$

$$= -2 - \sqrt{3}$$

