Actual Midterm Exam

View: Student Scores | Per Question Results | Detailed Student Results

All Q#1 Q#2 Q#3 Q#4 Q#5 Q#6 Q#7 Q#8 Q#9 Q#10 Q#11 Q#12 Q#13 Q#14 Q#15 Q#16 Q#17 Q#18 Q#19 Q#20

Question	Answered Correctly	Answered Wrong
1. Finding the maximum or minimum of a quadratic function	80%	20%
2. Graphing a piecewise-defined function: Problem type 1	60%	40%
3. Polynomial long division: Problem type 3	73%	27%
4. Finding a polynomial of a given degree with given zeros: Complex zeros	40%	60%
5. Inferring properties of a polynomial function from its graph	80%	20%
6. Finding x- and y-intercepts given a polynomial function	67%	33%
7. Writing the equation of a rational function given its graph	73%	27%
8. Graphing rational functions with holes	80%	20%
9. Using a graphing calculator to solve an exponential or logarithmic equation	33%	67%
10. The graph, domain, and range of an exponential function	80%	20%
11. Basic properties of logarithms	100%	0%
12. Domain of a logarithmic function: Advanced	73%	27%
13. Finding the time to reach a limit in a word problem on exponential growth or decay	73%	27%
14. Finding a final amount in a word problem on exponential growth or decay	67%	33%
15. Arc length and central angle measure	67%	33%
16. Trigonometric functions and special angles: Problem type 3	73%	27%
17. Sketching the graph of y= *a* sin(*bx*) or y= *a* cos(*bx*)	67%	33%
18. Amplitude, period, and phase shift of sine and cosine functions	53%	47%
19. Writing the equation of a sine or cosine function given its graph: Problem type 2	40%	60%
20. Identifying linear, quadratic, and exponential functions given ordered pairs	87%	13%

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Question	Answered Correctly	Answered Wror
1. Finding the maximum or minimum of a quadratic function	85%	15%
2. Graphing a piecewise-defined function: Problem type 1	69%	31%
3. Polynomial long division: Problem type 3	62%	38%
4. Finding a polynomial of a given degree with given zeros: Complex zeros	62%	38%
5. Inferring properties of a polynomial function from its graph	85%	15%
6. Finding x- and y-intercepts given a polynomial function	85%	15%
7. Writing the equation of a rational function given its graph	62%	38%
8. Graphing rational functions with holes	69%	31%
9. Using a graphing calculator to solve an exponential or logarithmic equation	62%	38%
10. The graph, domain, and range of an exponential function	85%	15%
11. Basic properties of logarithms	92%	8%
12. Domain of a logarithmic function: Advanced	77%	23%
13. Finding the time to reach a limit in a word problem on exponential growth or decay	77%	23%
14. Finding a final amount in a word problem on exponential growth or decay	69%	31%
15. Arc length and central angle measure	62%	38%
16. Trigonometric functions and special angles: Problem type 3	54%	46%
17. Sketching the graph of y= *a* sin(*bx*) or y= *a* cos(*bx*)	85%	15%
18. Amplitude, period, and phase shift of sine and cosine functions	54%	46%
19. Writing the equation of a sine or cosine function given its graph: Problem type 2	46%	54%
20. Identifying linear, quadratic, and exponential functions given ordered pairs	31%	69%

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#1

Finding the maximum or minimum of a quadratic function

Answer the questions below about the quadratic function.

$g(x) = 2x^2 + 8x + 10$

Happy
Up

Does the function have a minimum or maximum value?

Minimum Maximum

What is the function's minimum or maximum value?

2

$g(-2) = 2(-2)^2 + 8(-2) + 10$

$8 - 16 + 10$

Where does the minimum or maximum value occur?

$x =$ -2

The graph of a quadratic function $g(x) = ax^2 + bx + c$ is a parabola.

We are given $g(x) = 2x^2 + 8x + 10$. The leading coefficient $a = 2$ is positive.
So, the parabola opens upward, and g has a minimum value.
(If the parabola opened downward, then the function would have a maximum value.)

The vertex of the parabola tells us about the minimum value. The vertex occurs at $x = -\frac{b}{2a}$.

For our function, $a = 2$, $b = 8$, and $c = 10$, so we get the following.

$$x = -\frac{b}{2a} = -\frac{8}{2(2)} = -2$$

$$\frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

#2

Graphing a piecewise-defined function: Problem type 1

Suppose that the function f is defined on the interval $[-2, 2)$ as follows.

$$f(x) = \begin{cases} -2 & \text{if } -2 \le x < -1 \\ -1 & \text{if } -1 \le x < 0 \\ 0 & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 \le x < 2 \end{cases}$$

Graph the function f.

The function f is defined piecewise. This means that it is defined according to different rules depending on the input x.

More about these rules

- When x is in the interval $[-2, -1)$, we have $f(x) = -2$ (see Figure 1). Note that there is a *closed* circle for the point $(-2, -2)$ because $x = -2$ is included in the interval; thus, the point is part of the graph. Note also that there is an *open* circle for the point $(-1, -2)$ because $x = -1$ is not included in the interval; thus, the point is not part of the graph.

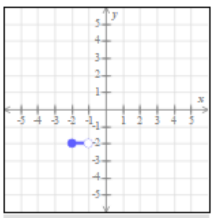


Figure 1

- Similarly, when x is in the interval $[-1, 0)$, we have $f(x) = -1$ (see Figure 2). There is a *closed* circle for the point $(-1, -1)$ because $x = -1$ is included in the interval. Also, there is an *open* circle for the point $(0, -1)$ because $x = 0$ is not included in the interval.

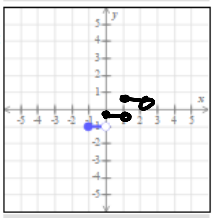
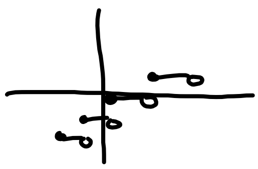


Figure 2



#3

Polynomial long division: Problem type 3

Divide.

$$\left(6x^4+2x^3+8x-13x^2-7\right)\div\left(-2x^2+2x+1\right)$$

Write your answer in the following form: Quotient $+\dfrac{\text{Remainder}}{-2x^2+2x+1}$.

$$\frac{6x^4+2x^3+8x-13x^2-7}{-2x^2+2x+1}=\square+\frac{\square}{-2x^2+2x+1}$$

Note first that the dividend, $6x^4+2x^3+8x-13x^2-7$, is *not* in standard form.
We rewrite it in standard form.

$$-2x^2+2x+1\overline{)6x^4+2x^3-13x^2+8x-7}$$

We start the division with the leading terms and get $\dfrac{6x^4}{-2x^2}=-3x^2$.

Then, we multiply $-3x^2$ by $-2x^2+2x+1$ to get $6x^4-6x^3-3x^2$ and subtract this result as shown below.

$$\begin{array}{r r}
 & -3x^2 \\
-2x^2+2x+1) & 6x^4+2x^3-13x^2+8x-7 \\
 & -(6x^4-6x^3-3x^2) \\
\hline
 & 8x^3-10x^2+8x-7
\end{array}$$

$-2x^2+2x+1 \,\big)\, 6x^4+2x^3-13x^2+8x+7$

$-3x^2 \quad -4x$

$-\left(6x^4-6x^3-3x^2\right)$

$8x^3-10x^2+8x$

$-\left(-8x^3-8x^2-4\right.$

In a similar manner, we continue the division.

$$\begin{array}{r r l}
 & -3x^2-4x+1 & \leftarrow \text{Quotient} \\
-2x^2+2x+1) & 6x^4+2x^3-13x^2+8x-7 & \\
 & -(6x^4-6x^3-3x^2) & \\
\hline
 & 8x^3-10x^2+8x-7 & \\
 & -(8x^3-8x^2-4x) & \\
\hline
 & -2x^2+12x-7 & \\
 & -(-2x^2+2x+1) & \\
\hline
 & 10x-8 & \leftarrow \text{Remainder}
\end{array}$$

#4

Finding a polynomial of a given degree with given zeros: Complex zeros

Find a polynomial $f(x)$ of degree 3 with real coefficients and the following zeros.

$3,\ -2+i$ complex conjugate $-2-i$

The Conjugate Pairs Theorem tells us the following.

If $a+bi$ is a zero of a polynomial with real coefficients, then so is its conjugate, $a-bi$.

For our problem, since $-2+i$ is a zero of $f(x)$, its conjugate $-2-i$ must also be a zero.

Therefore, our polynomial $f(x)$ must have the zeros 3, $-2+i$, and $-2-i$.
These are shown below on the left.
By the Factor Theorem, we get the corresponding factors on the right.

Zero	Factor
3	$x-3$
$-2+i$	$x-(-2+i)$
$-2-i$	$x-(-2-i)$

$x-(-2+i)=x+2-i$

$x-(-2-i)=x+2+i$

These factors give the following polynomial in factored form.

$$f(x)=(x-3)(x-(-2+i))(x-(-2-i))$$

$((x+2)-i)((x+2)+i)$

$(x+2)^2-i^2$

$x^2+4x+4+1$

x^2+4x+5

Note that this polynomial has degree 3.

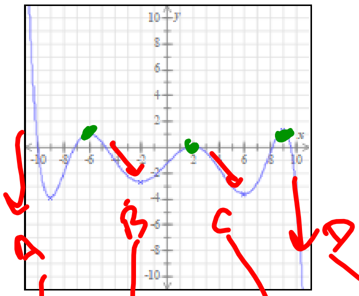
We multiply the factors to get the answer.

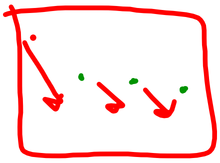
$$(x-3)(x-(-2+i))(x-(-2-i))$$
$$=(x-3)\left(x^2+4x+5\right)$$
More
$$=x^3+4x^2+5x-3x^2-12x-15$$
$$=x^3+x^2-7x-15$$

#5

Inferring properties of a polynomial function from its graph

Below is the graph of a polynomial function f with real coefficients. Use the graph to answer the following questions about f. All local extrema of f are shown in the graph.





(a) The function f is decreasing over which intervals? Choose all that apply.

☐ $(-\infty, -9)$ ☐ $(-6, -2)$ ☐ $(-6, 2)$ ☐ $(2, 6)$ ☐ $(6, 9)$ ☐ $(9, \infty)$

(b) The function f has local maxima at which x-values? If there is more than one value, separate them with commas.

$-6, 2, 7$

(c) What is the sign of the leading coefficient of f?

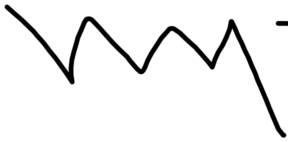
Select One $-$

Disco Left

(d) Which of the following is a possibility for the degree of f? Choose all that apply.

☐ 4 ☐ 5 ☐ 6 ☐ 7 ☐ 8 ☐ 9

NO



7 faces

#6

Finding x- and y-intercepts given a polynomial function

Find all y-intercepts and x-intercepts of the graph of the function.

$f(x) = x^3 - 3x^2 - x + 3$

$f(1) = 1 \quad -3 \quad -1 \quad 3 = 0$

If there is more than one answer, separate them with commas.

Click on "None" if applicable.

zero: 1

Factor: $X-1$

$$x-1\,\overline{)\,x^3-2x^2-x+3}\quad x^2$$

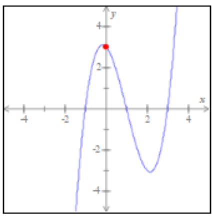
- A y-intercept is the y-coordinate of a point where the graph intersects the y-axis.

A function's graph has at most one y-intercept. Why?

To find it, we find the value of the function when $x = 0$.

$f(0) = (0)^3 - 3(0)^2 - (0) + 3 = 3$

So, the y-intercept is 3.
(The point $(0, 3)$ lies on the graph.)



- An x-intercept is the x-coordinate of a point where the graph intersects the x-axis.

To find the x-intercepts, we set $f(x) = 0$ and solve for x.

$0 = x^3 - 3x^2 - x + 3$

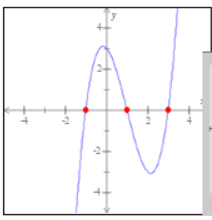
$0 = (x^3 - 3x^2) + (-x + 3)$ Gathering terms

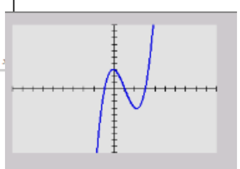
$0 = x^2(x-3) - (x-3)$ Factoring out x^2 and -1

$0 = (x-3)(x^2-1)$ Factoring out $x-3$

$0 = (x-3)(x-1)(x+1)$

$x-3=0$ or $x-1=0$ or $x+1=0$





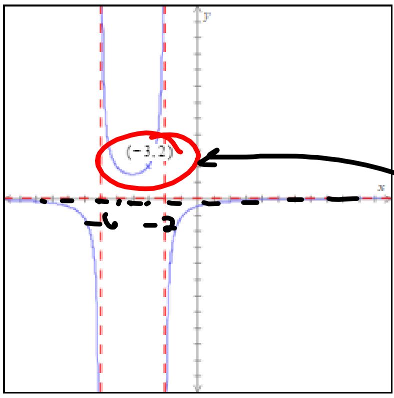
$y_1 = x\wedge 3 - 3x^2 - x + 3$

#7

Writing the equation of a rational function given its graph

The figure below shows the graph of a rational function f.
It has vertical asymptotes $x = -2$ and $x = -6$, and horizontal asymptote $y = 0$.
The graph does not have an x-intercept, and it passes through the point $(-3, 2)$.

The equation for $f(x)$ has one of the five forms shown below.
Choose the appropriate form for $f(x)$, and then write the equation.
You can assume that $f(x)$ is in simplest form.



- $f(x) = \frac{a}{x-b}$
- $f(x) = \frac{a(x-b)}{x-c}$
- $f(x) = \frac{a}{(x-b)(x-c)}$
- $f(x) = \frac{a(x-b)}{(x-c)(x-d)}$
- $f(x) = \frac{a(x-b)(x-c)}{(x-d)(x-e)}$

Vert Asym.

$y = \frac{~~-6~~ \text{ None. Zero}}{(x+6)(x+2)}$

-6

$2 = \frac{a}{(-3+6)(-3+2)}$

$2 = \frac{a}{-3}$

$a = -6$

#8

Graphing rational functions with holes

Graph the rational function $g(x) = \frac{3x+12}{x^2+5x+4}$.

We first factor, if possible, and note any restrictions on the value of x.

$$g(x) = \frac{3x+12}{x^2+5x+4} = \frac{3\cancel{(x+4)}}{(x+1)\cancel{(x+4)}} \quad x \neq -1, x \neq -4$$

We can cancel $x+4$ from the numerator and denominator to write the function in simplest form.

$$\boxed{g(x) = \frac{3}{x+1}} \quad x \neq -1, \boxed{x \neq -4}$$

Finding the hole:

Finally, note that $g(x) = \frac{3}{x+1}$, $x \neq -1$, $x \neq -4$ is not defined at $x = -4$.

Since $x = -4$ is *not* an asymptote, there is a "hole" in the graph at $x = -4$.

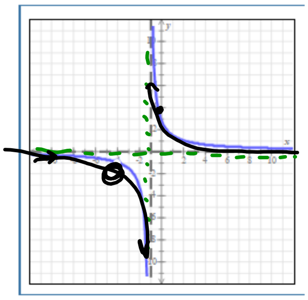
Evaluating $g(x) = \frac{3}{x+1}$ at $x = -4$, we get $g(-4) = \frac{3}{-4+1} = -1$. So the hole is at $(-4, -1)$.

More about this hole

$x = -4 \quad g(-4) = \frac{3}{-4+1} = \frac{3}{-3} = -1$

$(-4, -1)$

Using these facts, we can graph the function and get our answer.



$y = \frac{3}{(x+1)}$ ∴ VA $x = -1$

Headlight

HA: $y = 0$

$y_{int}(x=0) = \frac{3}{1}$

#9

Using a graphing calculator to solve an exponential or logarithmic equation

Use the ALEKS graphing calculator to solve the equation.

$$2\log(3-x)=1-x$$

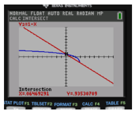
Round to the nearest hundredth.
If there is more than one solution, separate them with commas.

To solve $2\log(3-x)=1-x$, we'll graph the following functions.

$$y=2\log(3-x) \quad \text{and} \quad y=1-x$$

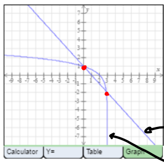
(These are the left and right sides of $2\log(3-x)=1-x$.)
Then we'll see where the graphs intersect.

To get the graphs using the ALEKS graphing calculator, we follow the steps below.



- Click on "**Y=**".
- Enter $Y1=2\log(3-x)$ and $Y2=1-x$.
- Click on "**Graph**".

We see that the graphs intersect at two points.
The x-coordinates of these points are the solutions of $2\log(3-x)=1-x$.



by inspection

$x=0$ $x=3$

$\cancel{2\log(3)=1}$

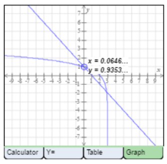
$\cancel{2\log(0)=-2}$

1-x

2log(3-x)

We'll use the ALEKS graphing calculator to find these x-coordinates.

- Click on "**Intersect**".
- Set First Curve Selected: $Y1$
- Set Second Curve Selected: $Y2$
- Set Guess Point $x=0$
- Click on "**Intersect**".

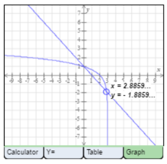


One of the intersection points has x-coordinate $x=0.0646\ldots$.

Rounding to the nearest hundredth gives $x=0.06$.

We find the second x-coordinate as follows.

- Click on "**New**".
- Set First Curve Selected: $Y1$
- Set Second Curve Selected: $Y2$
- Set Guess Point $x=3$
- Click on "**Intersect**".



$y_1=\boxed{2\log(3-x)}$

$y_2=\boxed{1-x}$

Calc 5: Intersect

1st ___ ___

2nd ___ ___

Guess 0 3

x- x-

The second intersection point has x-coordinate $x=2.8859\ldots$.

Rounding to the nearest hundredth gives $x=2.89$.

Here is the answer.

$x=0.06, 2.89$

#10

The graph, domain, and range of an exponential function

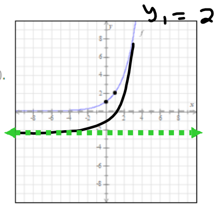
Graph the function $g(x)=2^x-2$ and give its domain and range using interval notation.

The graph of $g(x)=2^x-2$ is a transformation of the graph of $f(x)=2^x$.

So let's first consider the graph of $f(x)=2^x$.
It passes through the points $(0,1)$ and $(1,2)$.
It has a horizontal asymptote $y=0$.
The domain is $(-\infty,\infty)$ and the range is $(0,\infty)$.

More

$y_1=2\wedge x$



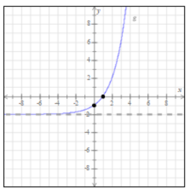
From the graph of $f(x)=2^x$, we can obtain the graph of $g(x)=2^x-2$.

Specifically, we **shift** the graph of $f(x)=2^x$ **downward 2 units** to get the graph of $g(x)=2^x-2$.

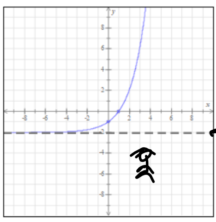
So the points $(0,1)$ and $(1,2)$ get shifted to $(0,-1)$ and $(1,0)$, respectively. The horizontal asymptote is now $y=-2$.

The domain of $g(x)=2^x-2$ is $(-\infty,\infty)$ and the range is $(-2,\infty)$.

More



Here is the answer.



Domain: $(-\infty,\infty)$
Range: $(-2,\infty)$

#11

Basic properties of logarithms

Fill in the missing values to make the equations true.

(a) $\log_5 11 - \log_5 8 = \log_5 \square$ ← $\frac{11}{8}$

(b) $\log_8 7 + \log_8 5 = \log_8 35$ $x \cdot 5 = 35$

(c) $\log_8 25 = 2\log_8 5$ $x^2 = 25$

The following properties of logarithms hold for any base a (with $a > 0$ and $a \neq 1$), any positive numbers M and N, and any number p.

Logarithm of a product: $\log_a(MN) = \log_a M + \log_a N$

Logarithm of a quotient: $\log_a \frac{M}{N} = \log_a M - \log_a N$

Logarithm of a power: $\log_a M^p = p \log_a M$

Proofs

We can use these properties to fill in the missing values.

Here is the answer.

(a) $\log_5 11 - \log_5 8 = \log_5 \frac{11}{8}$

(b) $\log_8 7 + \log_8 5 = \log_8 35$

(c) $\log_8 25 = 2\log_8 5$

#12

Domain of a logarithmic function: Advanced

Find the domain of the function.

$f(x) = \log_4 \sqrt{2x+5}$

Write your answer as an interval or union of intervals.

The function $f(x) = \log_4 \sqrt{2x+5}$ is defined if and only if $\sqrt{2x+5}$ is positive.

Why?

Note that $\sqrt{2x+5}$ is positive exactly when $2x+5$ is positive.

So, the domain of $f(x) = \log_4 \sqrt{2x+5}$ is given by the inequality $2x+5 > 0$.

We solve this inequality for x.

$$\begin{aligned} 2x+5 &> 0 \\ 2x &> -5 \\ x &> -\frac{5}{2} \end{aligned}$$

This means that the domain is the interval $\left(-\frac{5}{2}, \infty\right)$.

Here is the answer.

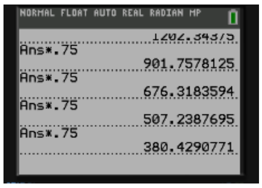
Domain: $\left(-\frac{5}{2}, \infty\right)$

#13

Finding the time to reach a limit in a word problem on exponential growth or decay

A laptop computer is purchased for \$3800. Each year, its value is 75% of its value the year before. After how many years will the laptop computer be worth \$500 or less? (Use the calculator provided if necessary.)

Write the *smallest possible* whole number answer.



In any year, the laptop's value is 75% of its value from the previous year.
So after t years, the laptop's value is 0.75^t times its initial value.

Why?

— 3800

1yr. — 3800 · .75

2yrs. — 3800 $(.75)^2$

n yrs — 3800 $(.75)^n$

We must find the smallest whole number t that satisfies the following inequality.

$$3800 \cdot 0.75^t \le 500$$

This inequality can be rewritten as follows.

$$0.75^t \le \frac{500}{3800}$$

We'll write the right-hand side as a decimal.

$$0.75^t \le 0.1315\ldots$$

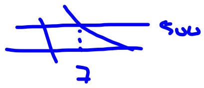
We need to find the smallest whole number t that satisfies this inequality.
One way to do this is to try different values of t until we have the answer.

Intersection

$0.75^5 = 0.2373\ldots$, which is greater than $0.1315\ldots$
$0.75^{10} = 0.0563\ldots$, which is less than $0.1315\ldots$

$Y_1 = (.75)^\wedge X \cdot 3800$

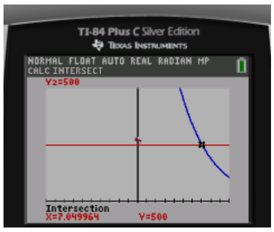
$Y_2 = 500$



So, we can narrow our search to values of t from $t = 5$ to $t = 10$.
Trying such values, we get the following.

$0.75^7 = 0.1334\ldots$, which is greater than $0.1315\ldots$
$0.75^8 = 0.1001\ldots$, which is less than $0.1315\ldots$

So, $t = 8$ is the smallest whole number that satisfies the inequality.



#14

Finding a final amount in a word problem on exponential growth or decay

The half-life of a radioactive isotope is the time it takes for a quantity of the isotope to be reduced to half its initial mass. Starting with 150 grams of a radioactive isotope, how much will be left after 3 half-lives?

Use the calculator provided and round your answer to the nearest gram.

Solver

P :

Q :

After each half-life, the amount left is reduced by half.
So, the amount left after 1 half-life is given by the following.

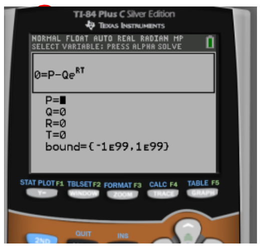
$$150 \cdot 0.5$$

1 half 75

Similarly, we can find the amount left after 2 half-lives.

$$(150 \cdot 0.5) \cdot 0.5 = 150 \cdot 0.5^2$$

2half 37.5



Here is the amount left after 3 half-lives.

$$\left(150 \cdot 0.5^2\right) \cdot 0.5 = 150 \cdot 0.5^3$$

3half 18.75

We see that after t half-lives, the amount left will be 0.5^t times the original value.
So, the amount left after t half-lives is given by the following.

$$150 \cdot 0.5^t$$

P: 75

Q: 150

R =

t =

In this problem, we need to find the amount left after 3 half-lives.

$$150 \cdot 0.5^3 = 18.75$$

Rounded to the nearest gram, we get 19 grams.

Here is the answer.

19 grams

#15

Arc length and central angle measure

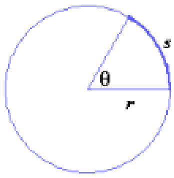
A circle has a radius of 13 ft. Find the degree measure of the central angle θ that intercepts an arc of length 15 ft.

Do not round any intermediate computations, and round your answer to the nearest tenth.

Suppose that a central angle of θ radians intercepts an arc of length s in a circle of radius r.

Then the length s of the arc is given by the following.

$s = r\theta$ Deriving this formula



We can solve this equation for θ.

$\theta = \frac{s}{r}$ radians

We can now find the value of θ.

$\theta = \frac{s}{r}$ radians $= \frac{15}{13}$ radians $= \left(\frac{15}{13}\right) \cdot \left(\frac{180}{\pi}\right)^{\circ} = 66.110^{\circ}\ldots$

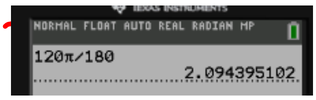
Rounding to the nearest tenth, we get $\theta = 66.1^{\circ}$.

Here is the answer.

$\theta = 66.1^{\circ}$

Ex 40° and $r = 3$

what's s



$S = 3 \cdot 40^{\circ} \cdot \frac{\pi}{180} =$

#16

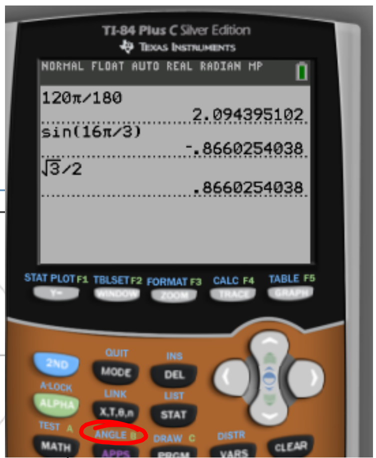
Trigonometric functions and special angles: Problem type 3

Find the exact values below. If applicable, click on "Undefined."

$\sin \frac{16\pi}{3}$

$\cot(-540^{\circ})$

$\frac{540}{360}$
180

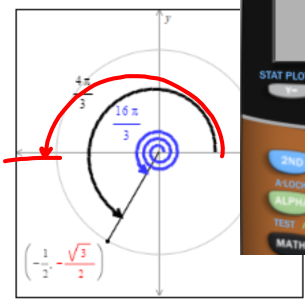


- **Finding** $\sin \frac{16\pi}{3}$

Note that $\frac{16\pi}{3}$ is coterminal with $\frac{4\pi}{3}$.

Why?

So, $\sin \frac{16\pi}{3} = \sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$.



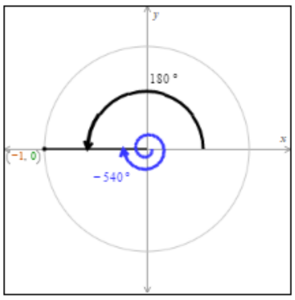
- **Finding** $\cot(-540^{\circ})$

Note that -540° is coterminal with 180°.

Why?

So,

$\cot(-540^{\circ}) = \cot 180^{\circ} = \frac{-1}{0}$, which is undefined.



Here is the answer.

#17

Sketching the graph of y= *a* sin(*bx*) or y= *a* cos(*bx*)

Graph the function $y = 3\cos 3x$.

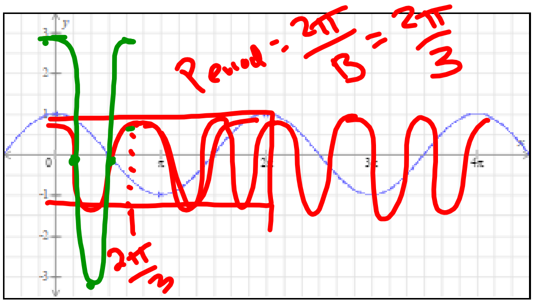
We'll look at the graph of $y = \cos x$, then $y = 3\cos x$, then $y = 3\cos 3x$.

Ampl. = 3

- To graph $y = \cos x$, we'll plot key points as shown below.

How to get the graph

x	$y = \cos x$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1



#18

Amplitude, period, and phase shift of sine and cosine functions

Find the phase shift, amplitude, and period of the function.

$$y = -1 - 4\cos\left(x - \frac{\pi}{3}\right)$$

Give the exact values, not decimal approximations.

✓ Period: $\frac{2\pi}{B} = 2\pi$

Amp: 4 (reflected)

Phase Shift: $-C/B = \pi/3$

Drop Down: −1

Background:

We assume below that a is nonzero and that $b > 0$.

A function of the form $y = a\cos bx$ has an amplitude of $|a|$ and a period of $\frac{2\pi}{b}$.

More

Now consider the function $y = a\cos(bx - c) + d$.

We can rewrite the function as $y = a\cos\left(b\left(x - \frac{c}{b}\right)\right) + d$. How

We note the following.

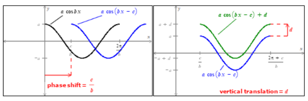
- Starting with the graph of $y = a\cos bx$, the graph of $y = a\cos(bx - c) + d$ is obtained by a horizontal translation of $\left|\frac{c}{b}\right|$ units and a vertical translation of $|d|$ units.
- If $c > 0$, the horizontal translation is to the right, and if $c < 0$, the translation is to the left.
- If $d > 0$, the vertical translation is up, and if $d < 0$, the translation is down.

We call the number $\frac{c}{b}$ the phase shift.

So, a positive phase shift implies a horizontal translation to the right.
And a negative phase shift implies a horizontal translation to the left.

Below we illustrate the effect of the phase shift $\frac{c}{b}$ and vertical translation d on the graph.

In the figures, we assume $a > 0$, $c > 0$, and $d > 0$ and show only one cycle of each function.



Horizontal and vertical translations do not affect the amplitude and period.
So, $y = a\cos bx$ and $y = a\cos(bx - c) + d$ have the same amplitude and period.
We now have the following.

Show this

For the function $y = a\cos(bx - c) + d$, the amplitude is $|a|$, the period is $\frac{2\pi}{b}$, and the phase shift is $\frac{c}{b}$.

The current problem:

We first write the given equation in the form $y = a\cos(bx - c) + d$.

$$y = -1 - 4\cos\left(x - \frac{\pi}{3}\right)$$
$$y = -4\cos\left(x - \frac{\pi}{3}\right) - 1$$

So, we have $a = -4$, $b = 1$, $c = \frac{\pi}{3}$, and $d = -1$.

The phase shift is $\frac{c}{b} = \frac{\frac{\pi}{3}}{1} = \frac{\pi}{3}$, the amplitude is $|a| = |-4| = 4$, and the period is $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$.

The vertical translation is 1 unit downward, because $d = -1$.

Show the graph

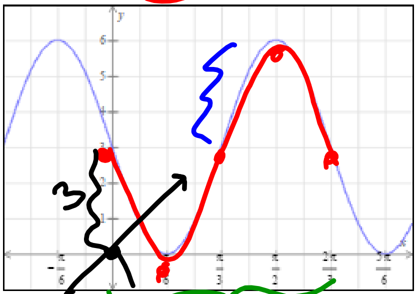
Here is the answer.

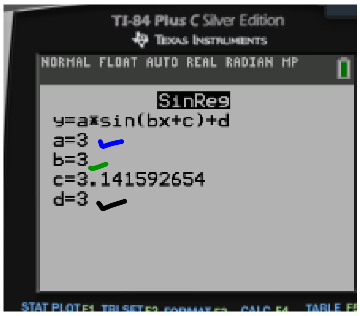
Phase shift: $\frac{\pi}{3}$
Amplitude: 4
Period: 2π

#19

Writing the equation of a sine or cosine function given its graph: Problem type 2

Write the equation of a sine or cosine function to describe the graph.





Sinreg.

$a=3$
$b=3$
$c=\pi$
$d=3$

Period $\frac{2\pi}{3} = \frac{2\pi}{B}$

$y = 3\sin(3x+\pi)+3$

Amp: $A = 3$

Vals

sinreg

P.S $= \frac{\pi}{3} = \frac{-C}{B}$ $\quad c = -\pi$

Lots of Possibilities

#20

Identifying linear, quadratic, and exponential functions given ordered pairs

For each function, state whether it is linear, quadratic, or exponential.

Function 1

x	y
3	10
4	14
5	10
6	-2
7	-22

- Linear
- Quadratic
- Exponential
- None of the above

Function 2

x	y
1	-4
2	3
3	10
4	17
5	24

- Linear
- Quadratic
- Exponential
- None of the above

$m=7$

Function 3

x	y
2	-192
3	-96
4	-48
5	-24
6	-12

- Linear
- Quadratic
- Exponential
- None of the above

$r^2 = 1$

Test 2 (Practice)

#1

Finding values of trigonometric functions given information about an angle: Problem type 2

Let θ be an angle in quadrant IV such that $\cos\theta = \frac{4}{5}$.

CAH

Find the exact values of $\csc\theta$ and $\cot\theta$.

We will use the following definition.

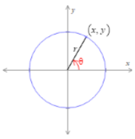
Suppose that (x, y) is on the terminal side of θ and let $r = \sqrt{x^2+y^2}$. More on r

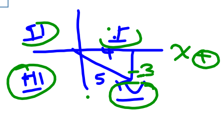
Then the trigonometric functions are given as follows.

$\sin\theta = \frac{y}{r}$ $\quad\csc\theta = \frac{r}{y},\ y \neq 0$

$\cos\theta = \frac{x}{r}$ $\quad\sec\theta = \frac{r}{x},\ x \neq 0$

$\tan\theta = \frac{y}{x},\ x \neq 0$ $\quad\cot\theta = \frac{x}{y},\ y \neq 0$





y (−)

$\sin\theta = \frac{O}{H} = \frac{-3}{5}$

$\csc = \frac{5}{-3}$

$\tan\theta = \frac{O}{a} = -\frac{3}{4}$

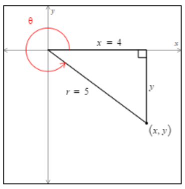
$\cot = \frac{4}{-3}$

We're given that $\cos\theta = \frac{4}{5}$ and that θ is in quadrant IV.

So we can draw the triangle shown, where (x, y) is on the terminal side of θ.

Note that $x = 4$ and $r = 5$. We can use that information along with the Pythagorean Theorem to find y.

$$x^2+y^2 = r^2$$
$$4^2+y^2 = 5^2$$
$$16+y^2 = 25$$
$$y^2 = 9$$
$$y = \pm\sqrt{9}$$



Since the point (x, y) is in quadrant IV, the y-coordinate must be negative.

$$y = -\sqrt{9} = -3$$

Then by the definition of the trigonometric functions, we get the following.

$$\csc\theta = \frac{r}{y} = \frac{5}{-3} = -\frac{5}{3}$$
$$\cot\theta = \frac{x}{y} = -\frac{4}{3}$$

Here is the answer.

$$\boxed{\csc\theta = -\frac{5}{3} \qquad \cot\theta = -\frac{4}{3}}$$

#2

Simplify.

$\cos x \csc x$

Use algebra and the fundamental trigonometric identities.
Your answer should be a number or use a single trigonometric function.

In simplifying trigonometric expressions, we often use methods similar to those used when simplifying algebraic expressions.

We can also use identities such as the following fundamental trigonometric identities.

Reciprocal identities:

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u} \quad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u} \quad \cot u = \frac{1}{\tan u}$$

Must Know

Quotient identities:

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

Must Know

Pythagorean identities:

$$\sin^2 u + \cos^2 u = 1$$
$$\tan^2 u + 1 = \sec^2 u$$
$$\cot^2 u + 1 = \csc^2 u$$

Must Know

The current problem:

To simplify $\cos x \csc x$, we can work as follows.

$$\cos x \csc x = \cos x \cdot \frac{1}{\sin x} \quad \text{Using a reciprocal identity}$$
$$= \frac{\cos x}{\sin x}$$
$$= \cot x \quad \text{Using a quotient identity}$$

Odd

$\sin(-x) = -\sin x$
$\tan(-x) = -\tan x$

Even

$\cos(-x) = \cos(x)$

#3

The current problem:

We'll start with the left side.
We'll transform it step by step until it is identical to the right side.

$(1+\tan^2 x)\cot x = \sec^2 x \cot x$	Pythagorean identity : $\sec^2 x = 1+\tan^2 x$
$= \left(\frac{1}{\cos^2 x}\right)\cot x$	Reciprocal identity for secant : $\sec^2 x = \frac{1}{\cos^2 x}$
$= \left(\frac{1}{\cos^2 x}\right)\left(\frac{\cos x}{\sin x}\right)$	Quotient identity for cotangent : $\cot x = \frac{\cos x}{\sin x}$
$= \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)$	Algebra : Cancel common factor(s)
$= \sec x \csc x$	Reciprocal identity : $\sec x = \frac{1}{\cos x}$, $\csc x = \frac{1}{\sin x}$

Here is the answer.

Statement	Rule
$(1+\tan^2 x)\cot x$	
$= \sec^2 x \cot x$	Pythagorean
$= \left(\frac{1}{\cos^2 x}\right)\cot x$	Reciprocal
$= \left(\frac{1}{\cos^2 x}\right)\left(\frac{\cos x}{\sin x}\right)$	Quotient
$= \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)$	Algebra
$= \sec x \csc x$	Reciprocal

#4

Proving trigonometric identities: Problem type 2

Prove the identity.

$$(\tan^2 x+1)(\cos^2 x-1) = -\tan^2 x$$

We'll start with the left side.
We'll transform it step by step until it is identical to the right side.

$(\tan^2 x+1)(\cos^2 x-1) = (\tan^2 x+1)(-1)(1-\cos^2 x)$	Algebra : Factor out -1
$= -\sec^2 x \sin^2 x$	Pythagorean identity : $\sec^2 x = \tan^2 x+1$, $\sin^2 x = 1-\cos^2 x$
$= -\frac{1}{\cos^2 x}\sin^2 x$	Reciprocal identity for secant
$= -\tan^2 x$	Quotient identity for tangent

#5

Double-angle identities: Problem type 1

Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ if $\tan x = -\frac{1}{2}$ and x terminates in quadrant IV.

Since $\tan x = -\frac{1}{2}$ and x terminates in quadrant IV, we first draw the reference triangle for x and find the unknown side c.

$$c = \sqrt{2^2 + 1^2} = \sqrt{5}$$

Therefore, $\cos x = \frac{2}{\sqrt{5}}$ and $\sin x = -\frac{1}{\sqrt{5}}$.

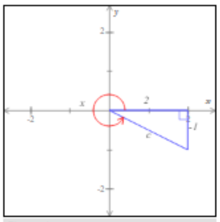
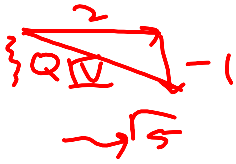


Figure 1



$\cos x = \frac{2}{\sqrt{5}}$ $\sin x = \frac{-1}{\sqrt{5}}$

Now we can use the double-angle identities to find $\sin 2x$, $\cos 2x$, and $\tan 2x$.

$$\sin 2x = 2\sin x \cos x = 2\cdot\left(-\frac{1}{\sqrt{5}}\right)\cdot\frac{2}{\sqrt{5}} = -\frac{4}{5}$$

$$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{2}{\sqrt{5}}\right)^2 - \left(-\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\cdot\left(-\frac{1}{2}\right)}{1-\left(-\frac{1}{2}\right)^2} = -\frac{4}{3}$$

Alternatively, we could have found $\cos 2x$ by using one of the other formulas.

$$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{2}{\sqrt{5}}\right)^2 - 1 = \frac{3}{5}$$

$$\cos 2x = 1 - 2\sin^2 x = 1 - 2\left(-\frac{1}{\sqrt{5}}\right)^2 = \frac{3}{5}$$

Also, we could have found $\tan 2x$ by dividing $\sin 2x$ and $\cos 2x$.

$$\tan 2x = \frac{\sin 2x}{\cos 2x} = \frac{-\frac{4}{5}}{\frac{3}{5}} = -\frac{4}{3}$$

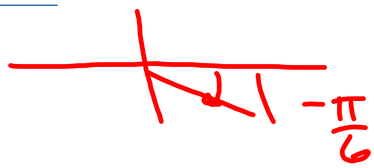
#6

Values of inverse trigonometric functions

Find the exact value of $\sin^{-1}\left(-\frac{1}{2}\right)$. $= \theta$

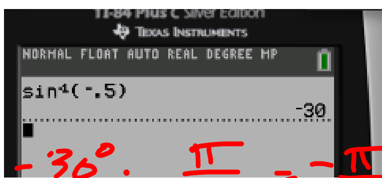
$\sin\theta = -\frac{1}{2}$

Write your answer in radians in terms of π.



The function $\sin^{-1}$ is an inverse trigonometric function.
Here are some facts about such functions.

Function	Meaning	
Inverse Sine Function	$y = \sin^{-1}x$ means that $\sin y = x$ and y is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	More
Inverse Cosine Function	$y = \cos^{-1}x$ means that $\cos y = x$ and y is in $[0, \pi]$	More
Inverse Tangent Function	$y = \tan^{-1}x$ means that $\tan y = x$ and y is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	More



$-30^\circ \cdot \frac{\pi}{180^\circ} = -\frac{\pi}{6}$

We also write $\sin^{-1}x$, $\cos^{-1}x$, and $\tan^{-1}x$ as $\arcsin x$, $\arccos x$, and $\arctan x$.

In this problem, we need to find $\sin^{-1}\left(-\frac{1}{2}\right)$.

This means we need to find the angle y in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin y = -\frac{1}{2}$.

This angle is $-\frac{\pi}{6}$. More

Here is the answer.

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

#7

Using a graphing calculator to solve a trigonometric equation

Use the ALEKS graphing calculator to solve the equation.

$$\cos 3x = 0.6x - 1$$

Round to the nearest hundredth if necessary.
If there is more than one solution, separate them with commas.

We first graph the functions $y = \cos 3x$ and $y = 0.6x - 1$ using the ALEKS graphing calculator.

- Click on "**Y=**".
- Enter $Y1 = \cos(3x)$ and $Y2 = 0.6x - 1$.
- Click on "**Graph**".

The graphs are displayed in Figure 1.

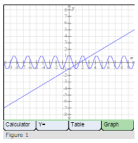


Figure 1

Using x-values from -1 to 3 and y-values from -2 to 2, we can take a closer look at the intersection points.

- Click on "**Window**".
- Enter Xmin = -1, Xmax = 3, Ymin = -2, Ymax = 2.
  (You do not need to adjust Xscale and Yscale.)
- Click on "**Ok**".

The results are shown in Figure 2.

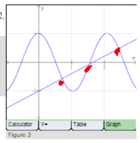
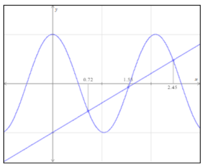


Figure 2

The solutions to $\cos 3x = 0.6x - 1$ are given by the x-coordinates of the three intersection points.
We can use the "**Intersect**" tool of the ALEKS graphing calculator to find these values.

- Click on "**Intersect**".
- For Guess Point, enter an x-value that is near an intersection point of the two graphs.
  The first intersection point of the two graphs is near $x = 0.7$.
- Click on "**Intersect**". The first intersection point has x-coordinate $x = 0.7239\ldots$
- To find the second intersection point, click on "**New**".
  Enter a new Guess Point near the second intersection point, and then click on "**Intersect**".
  The second intersection point has x-coordinate $x = 1.5468\ldots$
- To find the third intersection point, repeat this process.
  The third intersection point has x-coordinate $x = 2.4539\ldots$

Rounding these values to the nearest hundredth, we get $x = 0.72$, $x = 1.55$, and $x = 2.45$.



Here is the answer.

$$\boxed{x = 0.72, 1.55, 2.45}$$

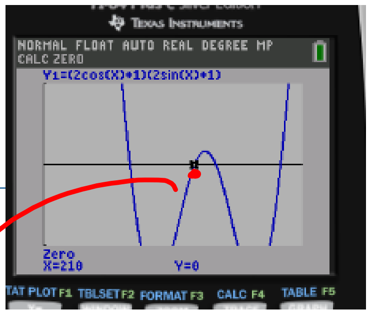
#8

Finding solutions in an interval for a trigonometric equation in factored form

Find all solutions of the equation in the interval $[0, 2\pi)$.

$$(2\cos x + 1)(2\sin x + 1) = 0$$

Write your answer in radians in terms of π.
If there is more than one solution, separate them with commas.



If a product is zero, then at least one of the factors must be zero.
So, we have the following.

$$2\cos x + 1 = 0 \quad \text{or} \quad 2\sin x + 1 = 0$$
$$\cos x = -\frac{1}{2} \qquad\qquad \sin x = -\frac{1}{2}$$

Cosine has the value $-\frac{1}{2}$ at $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$.

Sine has the value $-\frac{1}{2}$ at $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

$$210^\circ \cdot \boxed{\frac{\pi}{180^\circ}} = \frac{7\pi}{6}$$

$2 \cdot 30$

$6 \cdot 30$

CONVERT

Writing these solutions in a single list, we get our answer.

$$\boxed{x = \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6}}$$

#9

Finding solutions in an interval for a basic equation involving sine or cosine

Find all solutions of the equation in the interval $[0, 2\pi)$.

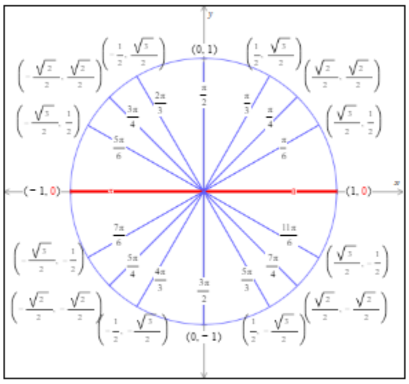
$$\sin\theta + 1 = 1$$

Write your answer in radians in terms of π.
If there is more than one solution, separate them with commas.

We must find all the values of θ in $[0, 2\pi)$ for which $\sin\theta + 1 = 1$.
We first solve for $\sin\theta$.

$$\sin\theta + 1 = 1$$
$$\sin\theta = 0$$

The sine of θ is the y-coordinate of its terminal point on the unit circle.
The two points on the unit circle with a y-coordinate of 0 are $(1, 0)$ and $(-1, 0)$.
The first point corresponds to $\theta = 0$ and the second to $\theta = \pi$.



Here is the answer.

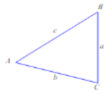
$\boxed{\theta = 0, \pi}$

#10

Solving a triangle with the law of sines: Problem type 1

Consider a triangle ABC like the one below. Suppose that $A = 42°$, $C = 82°$, and $b = 68$. (The figure is not drawn to scale.) Solve the triangle.

Round your answers to the nearest tenth.
If there is more than one solution, use the button labeled "or".



We want to find all the remaining side lengths and angle measures.

We are given two angles and the side they share.
This is the ASA case (Angle-Side-Angle).

We first find angle B using the fact that the sum of the angle measures of a triangle is $180°$.
Since $A = 42°$ and $C = 82°$, we get the following.

$$42° + B + 82° = 180°$$
$$B = 180° - 42° - 82°$$
$$B = 56°$$

$$\frac{68}{\sin(56°)} = \frac{a}{\sin(42°)}$$

We can find the remaining side lengths using the law of sines, which is written as follows.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{64}{\sin(56°)} = \frac{c}{\sin(82°)}$$

Using the other given fact that $b = 68$, we find a as follows.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{68}{\sin 56°} = \frac{a}{\sin 42°}$$
$$a = \frac{68 \cdot \sin 42°}{\sin 56°}$$
$$a = 54.884\ldots$$
$$a \approx 54.9$$

Similarly, we find c as follows.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{68}{\sin 56°} = \frac{c}{\sin 82°}$$
$$c = \frac{68 \cdot \sin 82°}{\sin 56°}$$
$$c = 81.2246\ldots$$
$$c \approx 81.2$$

Here is the answer.

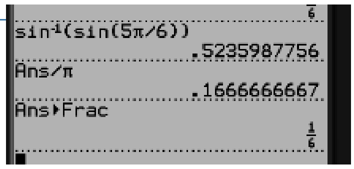
$\boxed{B = 56°, a = 54.9, c = 81.2}$

#11

Composition of a trigonometric function with its inverse trigonometric function: Problem type 1

Find the exact value of $\arcsin\left(\sin\frac{5\pi}{6}\right)$.

Write your answer in radians in terms of π.



Since $\sin\frac{5\pi}{6}=\frac{1}{2}$, we have the following.

$$\arcsin\left(\sin\frac{5\pi}{6}\right)=\arcsin\left(\frac{1}{2}\right)=\frac{\pi}{6}$$ More

Here is the answer.

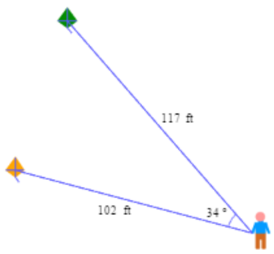
The answer is $\frac{\pi}{6}$.

Note that the answer is different from the input $\frac{5\pi}{6}$. More

#12

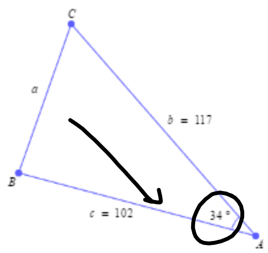
Solving a word problem using the law of cosines

Felipe is flying two kites. He has 102 feet of string out to one kite and 117 feet out to the other kite. The angle between the strings is $34°$ as shown in the figure below. Find the distance between the kites.



Carry your intermediate computations to at least four decimal places.
Round your answer to the nearest tenth of a foot.

We have the following figure.



SAS

$a^2 = 117^2 + 102^2 - 2(117)(102)\cdot\cos 34°$

Using the Law of Cosines, we can find a, the distance between the kites.

$$\begin{aligned} a^2 &= b^2+c^2-2bc\cos A \\ a &= \sqrt{b^2+c^2-2bc\cos A} \\ &= \sqrt{117^2+102^2-2(117)(102)\cos 34°} \\ &\approx 65.6 \end{aligned}$$

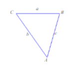
#13

Solving a triangle with the law of sines: Problem type 2

Consider a triangle ABC like the one below. Suppose that $a = 31$, $c = 39$, and $A = 37°$. (The figure is not drawn to scale.) Solve the triangle.

Carry your intermediate computations to at least four decimal places, and round your answers to the nearest tenth.

If no such triangle exists, enter "No solution." If there is more than one solution, use the "or" button.



ASS

$a^2 = b^2 + c^2 - 2bc \cos A$

$31^2 = b^2 + 39^2 - \underbrace{2b \cdot 39 \cdot \cos 37°}$

$0 = b^2 - \underbrace{78\cos 37°}\, b + 39^2 - 31^2$

$0 = \underbrace{b^2 - 62.29b + 560}$

Quadrat.

$b = \dfrac{+62.29 \pm \sqrt{62.29^2 - 4(560)}}{2}$

We want to find all the remaining side lengths and angle measures.

We are given two sides and an angle *not* included between them.
This is the SSA case (Side-Side-Angle).

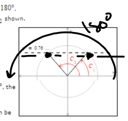
We know a and its opposite angle A.
We also know c, so we can use the law of sines to find $\sin C$.

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin C}{39} = \frac{\sin 37°}{31}$$
$$\sin C = 39 \cdot \frac{\sin 37°}{31}$$
$$\sin C \approx 0.7571$$

Since C is an angle in a triangle, we have that $0° < C < 180°$.
The only two possibilities for C are the angles C_1 and C_2 shown.

$$C_1 = \arcsin 0.7571 \approx 49.2092°$$ ✓

$$C_2 = 180° - \arcsin 0.7571 \approx 130.7908°$$



y_1: $\sin C = .75$

y_2:

Since the sum of the angle measures of a triangle is $180°$, the sum of any two angle measures must be less than $180°$.

So, using $A = 37°$, we check C_1 and C_2 to see if they can be solutions for C.

$$C_1 + A = 49.2092° + 37° = 86.2092° < 180°$$
$$C_2 + A = 130.7908° + 37° = 167.7908° < 180°$$

We see that both C_1 and C_2 are solutions for C.

Solution using C_1:

We solve for B by again using the fact that the sum of the angle measures of a triangle is $180°$.
Since $A = 37°$ and $C = C_1 \approx 49.2092°$, we get the following.

$$37° + B + 49.2092° = 180°$$
$$B = 180° - (37° + 49.2092°) = 93.7908°$$

Using the law of sines, we then find b.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 93.7908°} = \frac{31}{\sin 37°}$$
$$b = \sin 93.7908° \cdot \frac{31}{\sin 37°}$$
$$b \approx 51.3981$$

Solution using C_2:

We solve for B using $A = 37°$ and $C = C_2 \approx 130.7908°$.

$$37° + B + 130.7908° = 180°$$
$$B = 180° - (37° + 130.7908°) = 12.2092°$$

Using the law of sines, we then find b.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 12.2092°} = \frac{31}{\sin 37°}$$
$$b = \sin 12.2092° \cdot \frac{31}{\sin 37°}$$
$$b \approx 10.8936$$

To minimize rounding error, we found b using a and A (which were given in the question) rather than c and C.

Here is the answer, with values rounded to the nearest tenth.

$C = 49.2°$, $B = 93.8°$, $b = 51.4$,
or $C = 130.8°$, $B = 12.2°$, $b = 10.9$

#14

Amplitude, period, and phase shift of sine and cosine functions

Find the amplitude, phase shift, and period of the function.

$$y = 4\cos\left(x - \frac{\pi}{4}\right) - 3$$

Give the exact values, not decimal approximations.

Period $= \dfrac{2\pi}{B} = 2\pi$

Amp $= A = 4$

P.S $= -C/B = \pi/4$

down by 3

Background:

We assume below that a is nonzero and that $b > 0$.

A function of the form $y = a\cos bx$ has an amplitude of $|a|$ and a period of $\frac{2\pi}{b}$.

More

Now consider the function $y = a\cos(bx - c) + d$.

We can rewrite the function as $y = a\cos\left(b\left(x - \frac{c}{b}\right)\right) + d$.

How

We note the following.

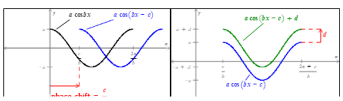
- Starting with the graph of $y = a\cos bx$, the graph of $y = a\cos(bx - c) + d$ is obtained by a horizontal translation of $\left|\frac{c}{b}\right|$ units and a vertical translation of $|d|$ units.
- If $c > 0$, the horizontal translation is to the right, and if $c < 0$, the translation is to the left.
- If $d > 0$, the vertical translation is up, and if $d < 0$, the translation is down.

We call the number $\frac{c}{b}$ the phase shift.

So, a positive phase shift implies a horizontal translation to the right.
And a negative phase shift implies a horizontal translation to the left.

Below we illustrate the effect of the phase shift $\frac{c}{b}$ and vertical translation d on the graph.

In the figures, we assume $a > 0$, $c > 0$, and $d > 0$ and show only one cycle of each function.



Horizontal and vertical translations do not affect the amplitude and period.
So, $y = a\cos bx$ and $y = a\cos(bx - c) + d$ have the same amplitude and period.
We now have the following.

Show this

For the function $y = a\cos(bx - c) + d$, the amplitude is $|a|$, the period is $\frac{2\pi}{b}$, and the phase shift is $\frac{c}{b}$.

The current problem:

Note that the given equation is in the form $y = a\cos(bx - c) + d$.

$$y = 4\cos\left(x - \frac{\pi}{4}\right) - 3$$

So, we have $a = 4$, $b = 1$, $c = \frac{\pi}{4}$, and $d = -3$.

The amplitude is $|a| = |4| = 4$, the phase shift is $\frac{c}{b} = \frac{\frac{\pi}{4}}{1} = \frac{\pi}{4}$, and the period is $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$.

The vertical translation is 3 units downward, because $d = -3$.

Show the graph

Here is the answer.

Amplitude: 4

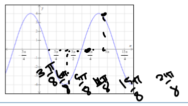
Phase shift: $\frac{\pi}{4}$

Period: 2π

#15

Writing the equation of a sine or cosine function given its graph: Problem type 2

Write the equation of a sine or cosine function to describe the graph.



X	
$-\frac{3\pi}{8}$	4
$\frac{3\pi}{8}$	0
π	-4
$\frac{15\pi}{8}$	0

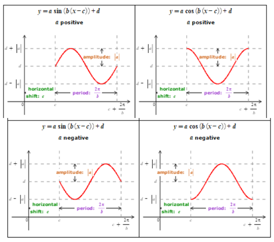
$\frac{3\pi}{8} + \frac{3\pi}{4}$

$\frac{7(3\pi)}{8}$

Background:

Consider $y = a\sin(b(x-c)) + d$ and $y = a\cos(b(x-c)) + d$, where b is positive.
The values a, b, c, and d affect the graphs of these equations, as shown below.
One cycle of each graph is shown.

The graphs shown have positive d. When d is negative, the graphs are shifted down instead of up.
The graphs have positive c. When c is negative, the graphs are shifted left instead of right.

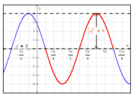


The amplitude and period are as follows.

amplitude $= |a|$ period $= \frac{2\pi}{b}$

The current problem:

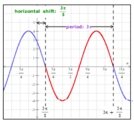
The red portion in the figure to the right shows one cycle of our graph. We'll use this cycle to write the equation in the form $y = a\sin(b(x-c)) + d$.



The vertical shift is $d = 0$ and the amplitude is $|a| = 4$. Because $|a| = 4$, we get that a must be -4 or 4. Comparing to the sine graphs above, we see that a is negative, and so $a = -4$.

The horizontal shift c for our graph is $\frac{1}{2}$ of $\frac{3\pi}{4}$.

$$c = \frac{1}{2}\cdot\frac{3\pi}{4} = \frac{3\pi}{8}$$



The period for our graph is 3π. More

We can use the period to find b as follows.

$$\text{period} = \frac{2\pi}{b}$$
$$3\pi = \frac{2\pi}{b}$$
$$3\pi b = 2\pi$$
$$b = \frac{2\pi}{3\pi} = \frac{2}{3}$$

So, the equation of our graph can be written as $y = -4\sin\left(\frac{2}{3}\left(x - \frac{3\pi}{8}\right)\right)$.

Equivalently, it can be written as $y = -4\sin\left(\frac{2}{3}x - \frac{\pi}{4}\right)$.

It is one possible answer. More

$\boxed{y = -4\sin\left(\frac{2}{3}x - \frac{\pi}{4}\right)}$

Amp: 4

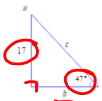
A = -4 (4π)
B = .666... 2/3
C = -.7... $\frac{\pi}{4}$
D = 0

#16

Solving a right triangle

Solve the right triangle.

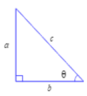
Round your answers to the nearest tenth.



b = adj
17 = opp

To solve a triangle is to find all of its side lengths and angle measures.
The following trigonometric ratios are useful in solving a right triangle.

$\tan 47 = \frac{17}{b}$



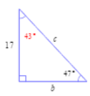
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

Finding the value of B :

The sum of the angle measures of a triangle is $180°$.
Therefore, $90° + 47° + B = 180°$, and so we get that $B = 43°$.



Finding the value of b :

We can use a trigonometric ratio to find b.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 47° = \frac{17}{b}$$
$$b = \frac{17}{\tan 47°} \approx 15.9 \quad \text{Rounding to the nearest tenth}$$

Finding the value of c :

We can use a trigonometric ratio to find c.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 47° = \frac{17}{c}$$
$$c = \frac{17}{\sin 47°} \approx 23.2 \quad \text{Rounding to the nearest tenth}$$

Here is the answer.

$B = 43°$
$b = 15.9$
$c = 23.2$

Test 2 Practice

View: Student Scores | **Per Question Results** | Detailed Student Results

All Q#1 Q#2 Q#3 Q#4 Q#5 Q#6 Q#7 Q#8 Q#9 Q#10 Q#11 Q#12 Q#13 Q#14 Q#15 Q#16

Question	Answered Correctly	Answered Correctly
1. Finding values of trigonometric functions given information about an angle: Problem type 2	30%	64%
2. Simplifying trigonometric expressions	50%	79%
3. Verifying a trigonometric identity	40%	86%
4. Proving trigonometric identities: Problem type 2	50%	64%
5. Double-angle identities: Problem type 1	30%	57%
6. Values of inverse trigonometric functions	30%	86%
7. Using a graphing calculator to solve a trigonometric equation	60%	57%
8. Finding solutions in an interval for a trigonometric equation in factored form	30%	43%
9. Finding solutions in an interval for a basic equation involving sine or cosine	50%	57%
10. Solving a triangle with the law of sines: Problem type 1	40%	64%
11. Composition of a trigonometric function with its inverse trigonometric function: Problem type 1	40%	71%
12. Solving a word problem using the law of cosines	40%	43%
13. Solving a triangle with the law of sines: Problem type 2	10%	29%
14. Amplitude, period, and phase shift of sine and cosine functions	30%	57%
15. Writing the equation of a sine or cosine function given its graph: Problem type 2	10%	50%
16. Solving a right triangle	40%	57%