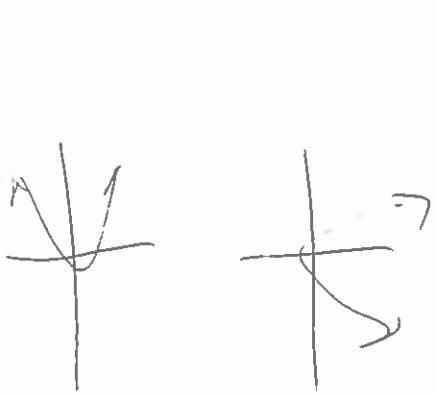
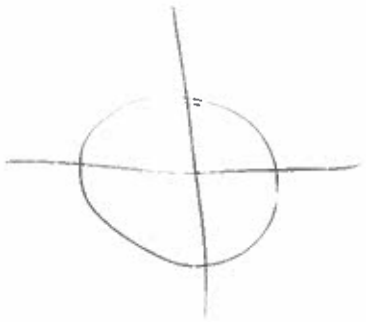
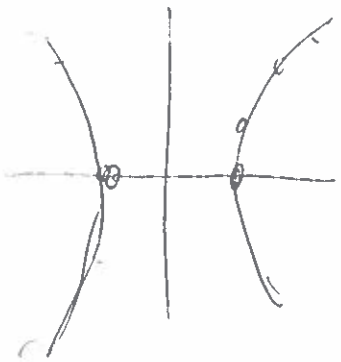
CONIC SECTIONS

$y = x^2$ $x = y^2$ $x^2 + y^2 = 1$ $x^2 - y^2 = 1$

Parabolas Circle. (Ellipse) Hyperbola





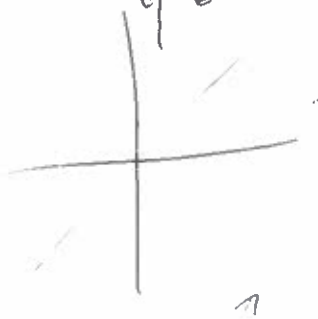


Line

$$\frac{y}{4} = \frac{x}{3}$$

4 → Stretch

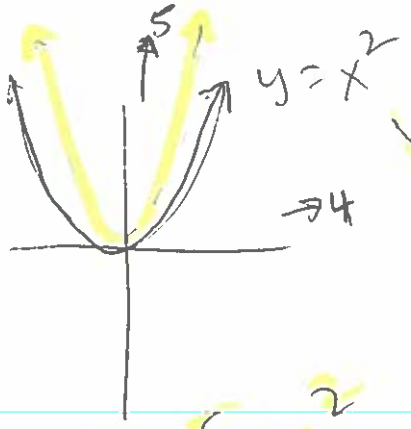
3 → Pull



$y = \frac{4}{3}x$

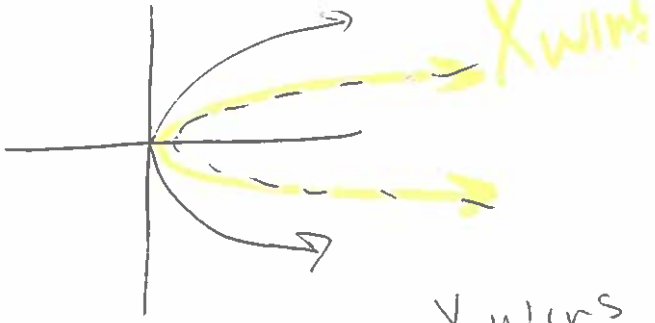
Parabolas

$$\frac{y}{5} = \frac{x^2}{4}$$



$$y = \frac{5}{4}x^2$$

$$\frac{x}{7} = \frac{y^2}{2}$$



X wins

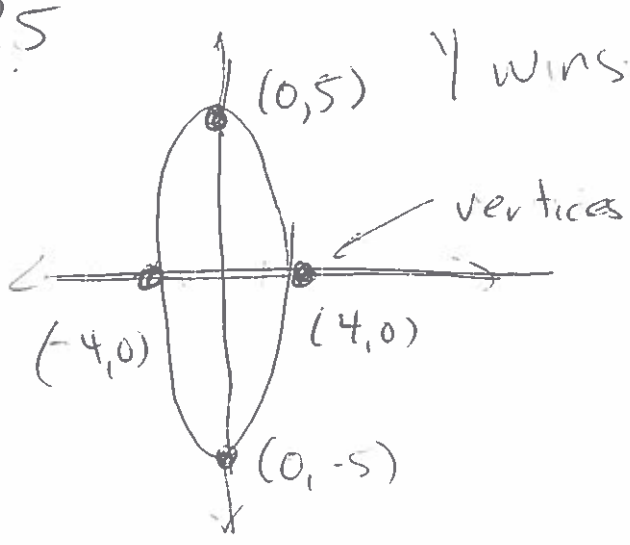
$$x = \frac{7}{2}y^2$$

Ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$

$$\frac{y^2}{25} = 1$$

$$y^2 = 25$$

$$y = \pm 5$$



$y = (x - 1)^2 + 1$ x-axis

$y + 2 = x^2$ -2 y-axis

EQUATION OF A CIRCLE

$$(x+2)^2 + (y-3)^2 = 1$$

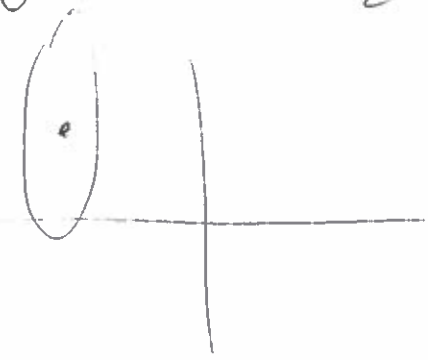
-2 x-axis

$+3$ y-axis

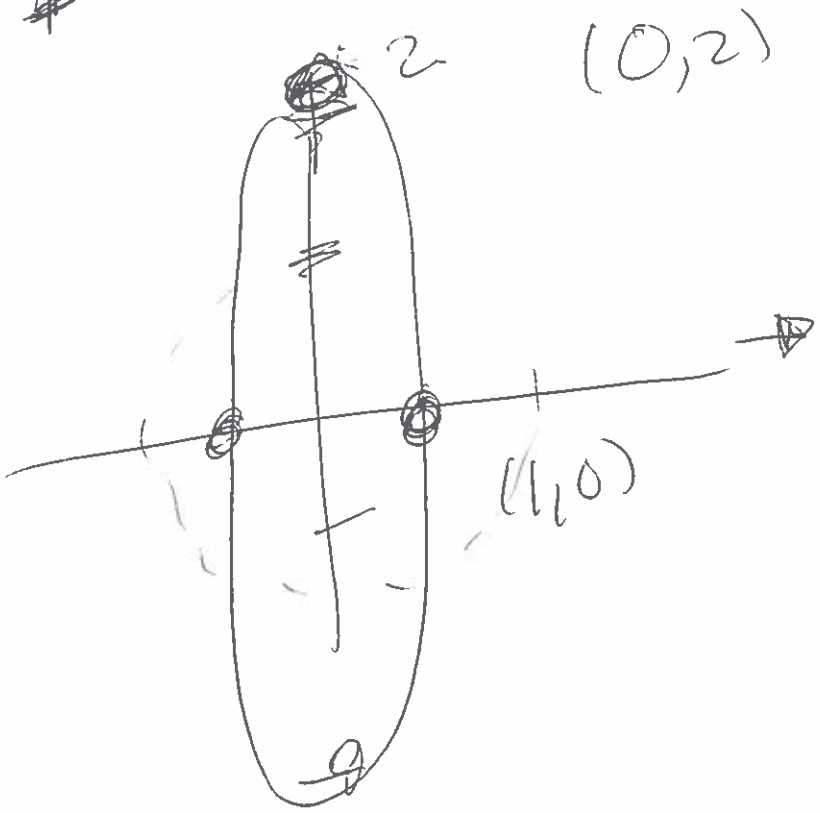
ELLIPSE

$$\frac{(x+2)^2}{16} + \frac{(y-3)^2}{25} = 1$$

y wins



$$\frac{x^2}{3} + \frac{y^2}{4} = 1$$

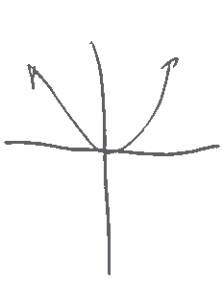


$$\frac{4x^2}{3} + \frac{4y^2}{4}$$

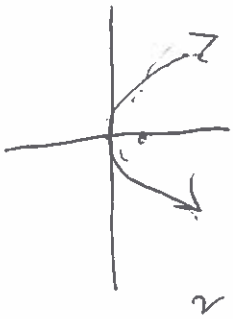
CONIC SECTIONS

NOT FUNCTIONS

Parabolas

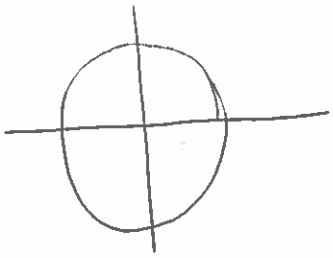


$y = x^2$



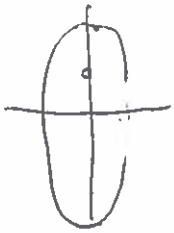
$x = y^2$

Circle

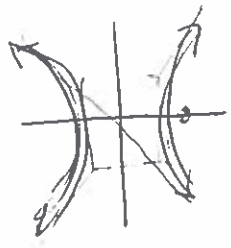


$x^2 + y^2 = 1$

Ellipse



Hyperbolas

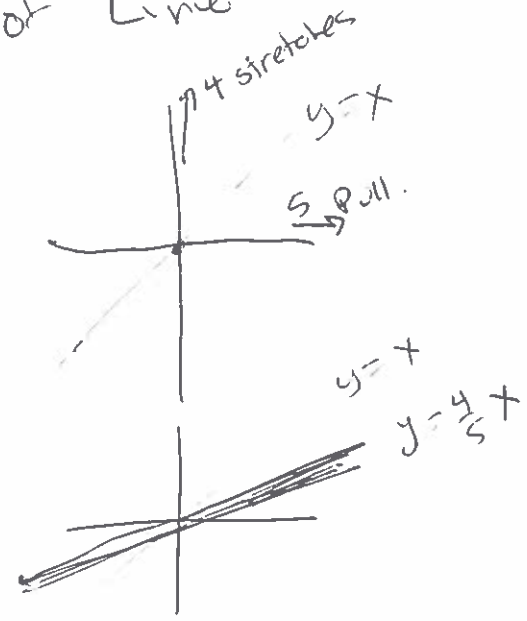


$x^2 - y^2 = 1$

Revisit Equation of Line

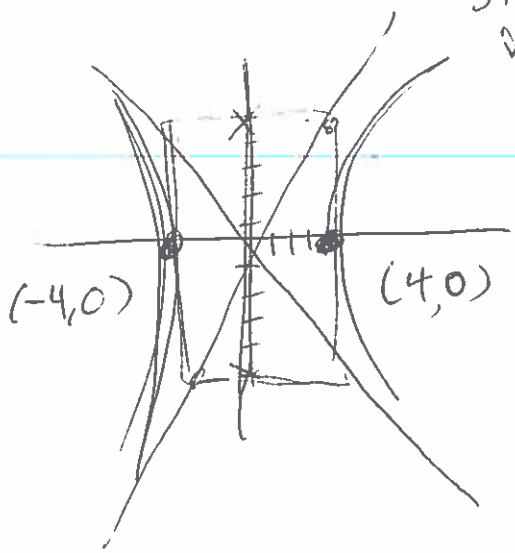
$$\frac{y}{4} = \frac{x}{5}$$

$$y = \frac{4}{5}x$$



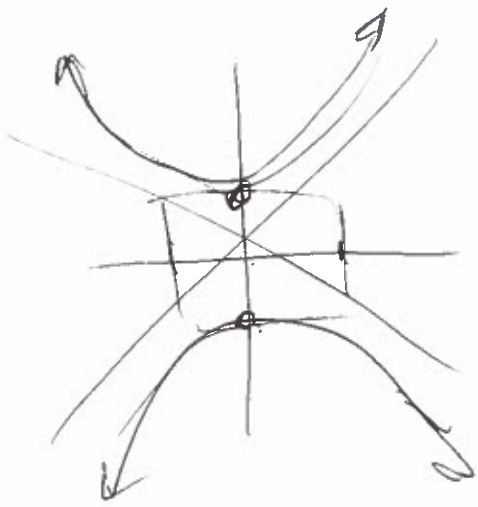
Hyperbola $\frac{x^2}{16} - \frac{y^2}{25} = 1$

Y wins (stretch ↕)



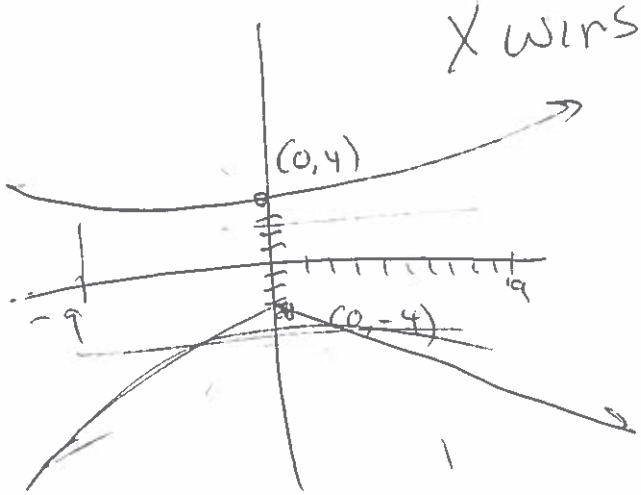
$\frac{y^2}{} - x^2 = \underline{1}$

(0, 1) (NS, 0)
(0, -1) (NS, 0)



$\frac{y^2}{16} - \frac{x^2}{81} = 1$

X wins



Conic Sections

Parabolas	Circles/Ellipse	Hyperbolas
$y = x^2$	$x^2 + y^2 = 1$	$x^2 - y^2 =$
$x = y^2$	(circle of radius 1)	$y^2 - x^2 =$

$y = (x+2)^2$ (Left by 2)

$(x+2) = y^2$

$(x+2)^2 + y^2 = 1$

$(x+2)^2 - y^2 = 1$

$y^2 - (x+2)^2 = 1$

Right by 1, Down by 3

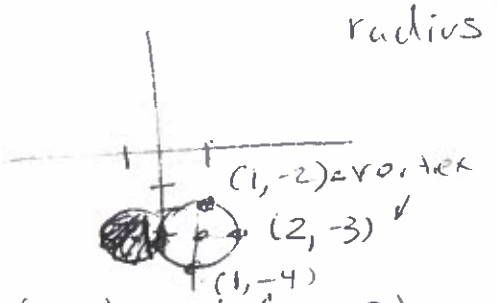
$(y+3) = (x-1)^2$

$(x-1) = (y+3)^2$

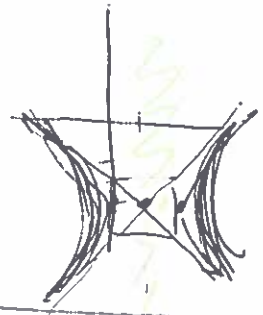
$(x-1)^2 + (y+3)^2 = 1$

$(x-1)^2 - (y+3$

radius 1



center: (1,-3)



$-(y+3)^2 = 1$

NO SOLN

Forms of Conics

Lines

- Point Slope Form
$$y - y_0 = m(x - x_0)$$
Example:
$$y - 1 = 2(x - 3)$$

- General Form
$$Ax + By = C$$
$$2x - y = 5$$

- Slope Intercept
$$y = mx + b$$
$$y = 2x - 5$$

Parabolas

$$y - y_0 = A(x - x_0)^2$$

STANDARD FORM

$$y - k = A(x - h)^2$$

vertex: (h, k)

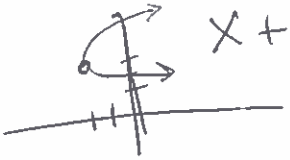
$A > 0$ up parabola
$A < 0$ Down Parabola

$$y - 3 = 2(x + 4)^2$$

vertex: $(-4, 3)$

$$(x - h) = A(y - k)^2$$

vertex (h, k)
$A > 0$ opens right
$A < 0$ opens left



$$x + 2 = \frac{1}{2}(y - 3)^2$$

Vertex: $(-2, 3)$

General Form

$y = Ax^2 + Bx + C$ → Complete Square. Graph in Calculator.

$A > 0$ UP
$A < 0$ Down

Vertex $-B/2A = h$ ↑ Evaluate $= y$

$y(-B/2A)$ → gives K

$X = Ay^2 + By + C$

$A > 0$ opens Right
$A < 0$ opens left

vertex: (h, k)

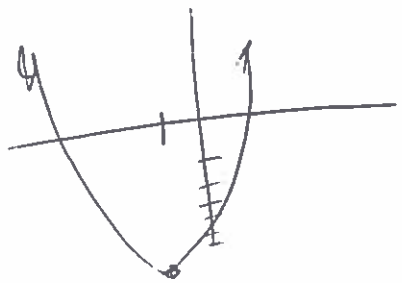
Ex $y = 2x^2 + 4x - 5$

vertex: $h = -\frac{B}{2A} = \frac{-4}{2(2)} = -1$

$K = y(-1) = 2(-1)^2 + 4(-1) - 5$
$= 2 - 4 - 5 = -7$

Vertex $(-1, -7)$

$A = 2$
opens <u>UP</u>



Ex $x = -\frac{1}{2}y^2 + 4y + 10$

$k = -\frac{B}{2A} = \frac{-4}{-1}$

$= 4$

vertex (h, k)

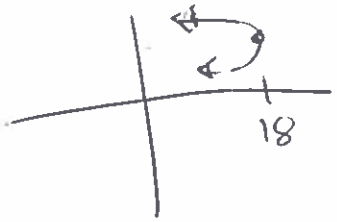
Vertex $(18, 4)$

$h(x(4)) = -\frac{1}{2}4^2 + 4\cdot 4 + 10$

$= -8 + 16 + 10$

$= 18$

Opens: Left



Circle ~~Ellipse~~ in Standard Form

$$\frac{(x-h)^2}{A^2} + \frac{(y-k)^2}{B^2} = 1$$

center (h, k)

Vertices $(h+A, k)$

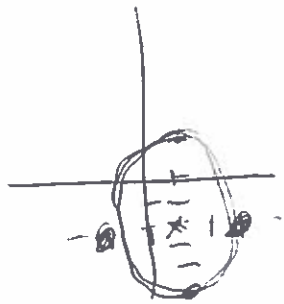
$(h-A, k)$

$(h, k+B)$

$(h, k-B)$

Ex

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} =$$



Center: $(1, -2)$

If $x = 1$

$\frac{(y+2)^2}{9} = 1$

$(y+2)^2 = 9$

$y + 2 = \pm 3$

$y = -2 \pm 3$

If $y = -2$

$\frac{(x-1)^2}{4} = 1$

$(x-1)^2 = 4$

$$3x^2 + 6x \quad -2y^2 - 5y \quad = 0$$

$$3(x^2 + 2x + \textcircled{1}) - 2\left(y^2 + \frac{5}{2}y + \boxed{\frac{25}{16}}\right) = 3\textcircled{1} + 2\boxed{\quad}$$

-2, SQ

$$3(x+1)^2 \quad -2 \quad \left(y + \frac{5}{4}\right)^2$$

$$3(x+1)^2 - 2\left(y + \frac{5}{4}\right)^2 = 3(1) - 2\left(\frac{25}{16}\right)$$

$$\frac{3(x+1)^2}{(-8)} - \frac{2\left(y + 5/4\right)^2}{(-8)} = 3 - \frac{25}{8} = \frac{-1}{8} \quad (\cdot -8)$$

$$-24(x+1)^2 + 16\left(y + 5/4\right)^2 \equiv 1$$

$$\downarrow$$

$$\frac{-(x+1)^2}{1/24} + \frac{\left(y + 5/4\right)^2}{1/16} = 1$$

center $(-1, -5/4)$

$x-1 = \pm 2$

$x = 1 \pm 2$

General Form of Circle.

$Ax^2 + Ay^2 + Bx + Cy + D = 0$ $\quad A = A$

General Form of Ellipse A, B same sign.

$Ax^2 + By^2 + Cx + Dy + E = 0$

Ex $3x^2 + 2y^2 + 6x - 5y = 0$ Ellipse

$-2x^2 - 4y^2 - 7x - 7y + 10 = 0$ Ellipse

$5x^2 + 5y^2 + 7x = 0$ Circle

Hyperbola General Form

$3x^2 - 2y^2 + 6x - 5y = 0$ Hyperbola

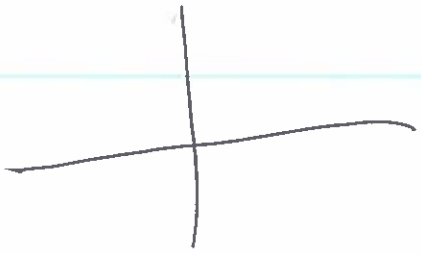
$Ax^2 - By^2 + Cy + Dy + E = 0$

or

$Ay^2 - Bx^2 + Cx + Dy + E = 0$

$$x^2 + y^2 = -1$$

radius $\sqrt{-1} = i$



$$x^2 + y^2 = 0$$