$$\int_0^{\pi} \underbrace{e^{-st}}_{u} \underbrace{\cos t\, dt}_{dv} =$$

$$\underbrace{-se^{-st}}_{du} \quad \underbrace{\sin t}_{v} \qquad \text{or}$$

$$\int e^{at}\cos t\, dt = \frac{e^{at}}{a^2+1}(a\cos t + \sin t)$$

$a=-s$

$$\int e^{-st}\cos t\, dt = \frac{e^{-st}}{s^2+1}(-s\cos t + \sin t)\Big|_0^{\pi}$$

$$= \frac{e^{-s\pi}}{s^2+1}(-s) - \frac{1}{s^2+1}(-s)$$

#2 $$\int_0^{\pi} e^{-st}\sin t\, dt$$

#53 Table of Integrals (Back cover)

$$\int e^{au}\sin bu\, du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu)$$

$a=-s \quad b=1 \quad u=t$

$$= \frac{e^{-st}}{s^2+1}(-s\sin t - \cos t)\Big|_0^{\pi}$$

Review Calc 2:

$$\int e^x \sin x\,dx = -e^x\cos x + \int e^x \cos x\,dx$$

$u = e^x \quad dv = \sin x$

$du = e^x \quad v = -\cos x$

$u = e^x \quad du = \cos x$

$du = e^x \quad v = \sin x$

$$\int e^x \sin x\,dx = -e^x\cos x + e^x \sin x - \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = -e^x\cos x + e^x \sin x$$

$$\int e^x \sin x\,dx = \frac{-e^x\cos x + e^x \sin x}{2}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2-4s+5}\right)$$ No Factor

$$\mathcal{L}^{-1}\left(\frac{1}{(s-2)^2+1}\right)$$

complete the square

$$s^2-4s+\boxed{4}+5-\boxed{4}$$

$$(s-2)^2+1$$

$$\frac{s^2-4s+4}{s^2-6s+9+350-9}$$

$$(s-3)^2+341$$

$$\mathcal{L}^{-1}\left(\frac{1}{s(s+6)}\right)=\mathcal{L}^{-1}\left(\frac{1/6}{s}\right)+\left(\frac{-1/6}{s+6}\right)$$

$$\frac{1}{6}(1)-\frac{1}{6}e^{-6t}$$

partial fract decomosition ☺

$$\frac{1}{s(s+6)}=\frac{A}{s}+\frac{B}{s+6}$$

$$1=A(s+6)+Bs$$

$s=-6$ $B=-1/6$

$s=0$ $A=1/6$

$$\frac{1}{s^2+6s+\boxed{9}-\boxed{9}}$$

$$(s+3)^2-9$$

$$\frac{1}{3}\mathcal{L}^{-1}\left(\frac{3}{(s+3)^2-9}\right)$$

$$\frac{1}{3}e^{3t}\sin 3t$$

#120 $a=-3$ $k=3$

$$\frac{1}{3}\mathcal{L}^{-1}\left(\frac{3}{(s+3)^2-9}\right)=$$

$$e^{-3t}\sinh 3t$$

$$\frac{1}{3}e^{-3t}\left(\frac{e^{3t}-e^{-3t}}{2}\right)$$

$$\frac{1}{6}-\frac{e^{-6t}}{6}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2-6s+10}\right)$$

complete sq

$$\mathcal{L}^{-1}\left(\frac{1}{(s-3)^2+10-9}\right)$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s-3)^2+1}\right)$$

#18 $e^{3t}\sin t$

$$\left(\begin{array}{ccc|ccc} e^t & 2e^{2t} & e^{3t} & 1 & 0 & 0 \\ e^t & e^{2t} & -2e^{3t} & 0 & 1 & 0 \\ e^t & -e^{2t} & e^{3t} & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} e^t & 2e^{2t} & e^{3t} & 1 & 0 & 0 \\ 0 & -e^{2t} & -3e^{3t} & -1 & 1 & 0 \\ 0 & -3e^{2t} & 0 & -1 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} e^t & 2e^{2t} & e^{3t} & 1 & 0 & 0 \\ 0 & -e^{2t} & -3e^{3t} & -1 & 1 & 0 \\ 0 & 0 & 9e^{3t} & 2 & -3 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2e^{t} & e^{2t} & e^{-t} & 0 & 0 \\ 0 & -1 & -3e^{t} & -e^{-2t} & e^{2t} & 0 \\ 0 & 0 & 9 & 2e^{-3t} & -3e^{-3t} & e^{-3t} \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 2e^{t} & e^{2t} & e^{-t} & 0 & 0 \\ 0 & 1 & 3e^{t} & e^{-2t} & -e^{-2t} & 0 \\ 0 & 0 & 1 & 2/9e^{-3t} & -e^{-3t}/3 & e^{-3t}/9 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & -3\frac{2}{9}e^{-2t} & 3\frac{e^{-2t}}{3} & -3\frac{e^{-3t}}{9} \end{array}\right)$$

Use the method of undetermined coefficients to solve the given system.

$$\mathbf{X}' = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{pmatrix}\mathbf{X} + \begin{pmatrix} 1 \\ -1 \\ 6 \end{pmatrix}e^{4t}$$

$\det\begin{pmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 3 \\ 0 & 0 & 5-\lambda \end{pmatrix}$

$\lambda = 1, 2, 5$

$X = c_1\begin{pmatrix} \\ \\ \end{pmatrix}e^{t} + c_2\begin{pmatrix} \\ \\ \end{pmatrix}e^{2t} + c_3\begin{pmatrix} \\ \\ \end{pmatrix}e^{5t} + \begin{pmatrix} A \\ B \\ C \end{pmatrix}e^{4t}$

$$\mathbf{X}' = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{pmatrix}\mathbf{X} + \begin{pmatrix} 1 \\ -1 \\ 6 \end{pmatrix}e^{4t}$$

$X_p = \begin{pmatrix} A \\ B \\ C \end{pmatrix}e^{4t}$

$X' = \begin{pmatrix} 4A \\ 4B \\ 4C \end{pmatrix}e^{4t}$

$\begin{pmatrix} 4A \\ 4B \\ 4C \end{pmatrix}e^{4t} = \begin{pmatrix} A+B+C \\ 2B+3C \\ 5C \end{pmatrix}e^{4t} + \begin{pmatrix} 1 \\ -1 \\ 6 \end{pmatrix}e^{4t}$

$4A = A + B + C + 1$

$4B = 2B + 3C - 1$

$4C = 5C + 6 \longrightarrow C = -6$

$4B = 2B + (-18) - 1 \longrightarrow$

$2B = -19 \longrightarrow B = -19/2$

$3A = B + C + 1$

$3A = -\frac{19}{2} + -6 + 1 = -14.5 \quad A = \frac{-29}{6}$

$X = c_1\begin{pmatrix} \\ \\ \end{pmatrix}e^{t} + c_2\begin{pmatrix} \\ \\ \end{pmatrix}e^{2t} + c_3\begin{pmatrix} \\ \\ \end{pmatrix}e^{5t} + \begin{pmatrix} -29/6 \\ -19/2 \\ -6 \end{pmatrix}e^{4t}$

$$\mathcal{L}^{-1}\left(\frac{.9s}{(s-.1)(s+.2)}\right)$$

$$\frac{.9s}{(s-.1)(s+.2)} = \frac{A}{s-.1} + \frac{B}{s+.2}$$

$$.9s = A(s+.2) + B(s-.1)$$

$s = -.2$

$-.18 = 0 + -.3B \quad B = .6$

$s = .1$

$.09 = A(.3) \quad A = .3$

$$\mathcal{L}^{-1}\left(\frac{.3}{s-.1} + \frac{.6}{s+.2}\right)$$

$$.3\,\mathcal{L}^{-1}\left(\frac{1}{s-.1}\right) + .6\,\mathcal{L}^{-1}\left(\frac{1}{s+.2}\right)$$

$$.3e^{.1t} + .6e^{-.2t}$$

Use the Laplace transform to solve the given initial-value problem.

$y' + 3y = e^{-3t}, \quad y(0) = 2$

$y(t) =$ 10a $e^{-3t}t + 2e^{-3t}$

$\mathcal{L}(y) = Y$

$\mathcal{L}(e^{-3t})$

$$sY - y(0) + 3Y = \frac{1}{s+3}$$

$$Y(s+3) - 2 = \frac{1}{s+3}$$

$$Y(s+3) = 2 + \frac{1}{s+3} = \frac{2s+6+1}{s+3} = \frac{2s+7}{s+3}$$

$s+3$ $s+3$ LCD

$$\mathcal{L}^{-1}\,Y = \frac{2s+7}{(s+3)^2}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{2s+7}{(s+3)^2}\right)$$

$$\frac{2s+7}{(s+3)^2} = \frac{A}{s+3} + \frac{B}{(s+3)^2}$$

$$2s+7 = A(s+3) + B$$

$s = -3 \rightarrow 1 = B$

$s = 0 \rightarrow 7 = 3A + 1 \rightarrow A = 2$

$$\mathcal{L}^{-1}\left(\frac{2}{s+3} + \frac{1}{(s+3)^2}\right)$$

$$2e^{-3t} + te^{-3t}$$

Use the Laplace transform to solve the given initial-value problem.

$y'' + 2y' + y = 0,\ y(0) = 1,\ y'(0) = 1$

$y(t) =$ 11a $2e^{-t}t + e^{-t}$

$$s^2Y - sy(0) - y'(0) + 2sY - 2y(0) + Y$$

$$s^2Y - s - 1 + 2sY - 2 + Y = 0$$

$$(s^2+2s+1)Y - s - 3 = 0$$

$$Y = \frac{s+3}{(s+1)^2}$$

PFD

$$\mathcal{L}^{-1}\{Y\} = \mathcal{L}^{-1}\frac{2}{(s+1)^2} + \mathcal{L}^{-1}\frac{1}{s+1}$$

$$2te^{-t} + e^{-t}$$

$$\frac{s+7}{(s+3)^2} = \frac{s+3}{(s+3)^2} + \frac{4}{(s+3)^2}$$

$$= \frac{1}{s+3} + \frac{4}{(s+3)^2}$$