$$y = e^{-2x}$$

$$Dy = -2e^{-2x}$$

$$+2y \quad +2e^{-2x}$$

$$(D+2)y = 0$$

Solve the given differential equation by undetermined coefficients.

$y''' + y'' = 8x^2$

$(D^3 + D^2)y = 0$

$D^3 D^2 (D+1) y = D^3 8x^2$ ← Annih D^3

$D^3 D^2 (D+1) y = 0$ $\quad C_2 e^{0x}$

$m=0 \quad m=-1$

$y = \boxed{C_1 e^{-1x}} + \underbrace{C_2 + C_3 x + C_4 x^2 + C_5 x^3 + C_6 x^4}_{y_p}$

Homogeneous

$y_p' = C_3 + 2C_4 x + 3C_5 x^2 + 4C_6 x^3$

$y_p'' = \boxed{2C_4} + \boxed{6C_5 x} + \boxed{12C_6 x^2}$

$y_p''' = \boxed{6C_5} + \boxed{24C_6 x}$

$0 + 0x + \boxed{8x^2}$

$+2C_4 + 6C_5 = 0$

$6C_5 + 24C_6 = 0$

$24(\frac{8}{3})$

$6C_5 + 16 = 0$

$C_5 = -16/6 = -8/3$

$12C_6 = 8$

$C_6 = 8/12 = 2/3$

$+2C_4 + 6(-8/3) = 0$

$+2C_4 = 48/3$

$C_4 = +24/3 = +8$

$y = C_1 e^{-x} + C_2 + C_3 x + 8x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4$

$$y_p'' = \boxed{2c_4} + \boxed{6c_5 x} + \boxed{12c_6 x^2}$$

$$y_p''' = \boxed{6c_5} + \boxed{24c_6 x}$$

$$0 + 0x + 8x^2$$

3 rows
$$\begin{cases} 2c_4 + 6c_5 + 0c_6 = 0 \\ 0c_4 + 6c_5 + 24c_6 = 0 \\ 0c_4 + 0c_5 + 12c_6 = 8 \end{cases}$$

4 rows

① Matrix Edit [A]
(2nd)(x⁻¹) >> 1: 3×4
② QUIT
③ Matrix > Math 8: rref
Matrix 1: [A]

$$\text{rref}[A] \to \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -2.\ldots \\ 0 & 0 & 1 & .66\ldots \end{bmatrix}$$

Math 1: ▸frac

$$\begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -8/3 \\ 0 & 0 & 1 & 2/3 \end{bmatrix}$$

47 Variation of Parameters

$$\frac{dy}{dx} + P(x)\,y = f(x)$$

1st order Linear

$$y_1 = e^{-\int P(x)\,dx}$$

$$y_p = y_1 \int \frac{e^{-\int P(x)\,dx}}{y_1^2}\,dx$$

$$y_p = y_1 u_1 \longrightarrow u_1 = \int \frac{f(x)}{y_1(x)}\,dx$$

$$y = c_1 y_1 + c_2 y_p = c_1 y_1 + u_1 y_1$$

Ex $y' + \frac{1}{x} y = x^2$

$e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

Times $\boxed{x}$

$x y' + 1 y - x^3 = 0$

$x\,dy + (y - x^3)\,dx = 0$

EDE

$xy \qquad xy - \frac{x^4}{4}$ ← antiderivative

$d\left(xy - \frac{x^4}{4}\right) = 0$

$xy - \frac{x^4}{4} = c \qquad y = \frac{C_1}{x} + \frac{x^3}{4}$

$u = \int \frac{f(x)}{y_1(x)} dx = \int \frac{x^2}{1/x} dx \qquad y_1$

$= x^4/4$

$y_p = u y_1 = \frac{x^4}{4} \cdot \frac{1}{x} = \frac{x^3}{4}$

2nd Order

$y'' + P(x) y' + Q(x) y = f(x)$

$y_p = u_1 y_1 + u_2 y_2$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$

$u_1' = \frac{\begin{vmatrix} 0 & y_2 \\ f & y_2' \end{vmatrix} = W_1}{\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = W}$

$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$

$u_2' = \frac{W_2}{W}$

$$y'' - 4y' + 4y = (x+1)e^{2x}$$

$$(m^2 - 4m + 4) = 0$$

$$(m-2)(m-2)$$

$$y = c_1 e^{2x} + c_2 x e^{2x} \qquad W = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix} = e^{4x}$$

$$W_1 = \begin{vmatrix} 0 & xe^{2x} \\ (x+1)e^{2x} & 2xe^{2x} + e^{2x} \end{vmatrix} = -x(x+1)e^{4x}$$

$$W_2 = \begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & (x+1)e^{2x} \end{vmatrix} = (x+1)e^{4x}$$

$$u_1' = \frac{W_1}{W} = \frac{-x(x+1)e^{4x}}{e^{4x}} = -x(x+1) \qquad -x^2 - x$$

$$u_2' = \frac{W_2}{W} = \frac{(x+1)e^{4x}}{e^{4x}} = x+1$$

$$u_1 = -\frac{x^3}{3} - \frac{x^2}{2} \qquad u_2 = \frac{x^2}{2} + x$$

$$y = \boxed{c_1 y_1 + c_2 y_2} \text{ Homogeneous}$$

$$= c_1 e^{2x} + c_2 x e^{2x} + \underbrace{\left(-\frac{x^3}{3} - \frac{x^2}{2}\right)}_{u_1} e^{2x} + \underbrace{\left(\frac{x^2}{2} + x\right)}_{v} x e^{2x}$$

$$= \underbrace{c_1 y_1 + c_2 y_2}_{} + u_1 y_1 + u_2 y_2$$

$$= c_1 e^{2x} + c_2 x e^{2x} + \frac{x^3}{6} e^{2x} + \frac{x^2}{2} e^{2x}$$

(2)

$$4y'' + 36y = \csc 3x$$

$$y'' + 9y = \tfrac{1}{4}\csc 3x$$

$$(4m^2 + 36) = 0$$

$$m^2 = -9 \qquad m = \pm\sqrt{-9} = \pm\sqrt{-1}\sqrt{9}$$

$$m = \pm 3i \qquad \pm i\,3$$

$$y = c_1 \cos 3x + c_2 \sin 3x$$

$$W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 3$$

$$W_1 = \begin{vmatrix} 0 & \sin 3x \\ \frac{1}{4}\csc 3x & 3\cos 3x \end{vmatrix} = \frac{1}{4}$$

Investigate the following harvesting model both qualitatively and analytically.

If a constant number h of fish are harvested from a fishery per unit time, then a model for the population

P(t)

of the fishery at time t is given by

dP/dt= P(a − bP) − h, P(0) = P0,

$P' = P(7 - P) - 49/4$

where

$\frac{dP}{dt} = 7P - P^2 - \frac{49}{4}$

a, b, h, and P0

are positive constants. Suppose

a = 7, b = 1, and h =

$\frac{dP}{-P^2 + 7P - \frac{49}{4}} = dt$

$\frac{49}{4}$

$-\frac{1}{4}(4P^2 - 28P + 49) = t$

$-\frac{1}{4}(2P - 7)^2$

.

$$-\frac{1}{2}\int \frac{2}{(2P-7)^2}\,dP = -4dt$$

$$\frac{1}{2}\,\frac{u^{-1}}{-1} = -4t + c$$

$$(2P-7)^{-1} = 8t + c$$

$$(2P-7)^{-1} = 8t + c$$

$$(2P_0 - 7)^{-1} = c$$

$$(2P-7)^{-1} = 8t + (2P_0-7)^{-1}$$

$$\frac{1}{2P-7} = 8t + \frac{1}{2P_0-7}$$

$$8t = \frac{1}{2P-7} - \frac{1}{2P_0-7}$$

$$\frac{2P_0 - 7 - 2P + 7}{(2P-7)(2P_0-7)}$$

$$t = 8\,\frac{2(P_0 - P)}{(2P-7)(2P_0-7)}$$

$$(2P-7)^{-1} = 8t + (2P_0-7)^{-1}$$

$$\frac{1}{-7} = 8t + (2P_0-7)^{-1}$$

$$u''' + u'' - 2u = 0$$

$$(m^3 + m^2 - 2) = 0$$

$m = 1$

$$\begin{array}{r} m^2 + 2m + 2 \\ (m-1)\overline{)\,m^3 + m^2 + 0m - 2} \\ -(m^3 - m^2) \\ \hline 2m^2 \\ -(2m^2 - 2m) \\ \hline 2m + 2 \end{array}$$

$$m = \frac{-2 \pm \sqrt{4-4(2)}}{2}$$

$$= -1 \pm i$$

$$c_1 e^{x} + c_2 e^{-x}\sin x + c_3 e^{-x}\cos x$$