Match the vector-valued function

\[ f(t) = \langle t \cos t, t \sin t, t \rangle \]

with the corresponding computer-generated graph.

\[ x^2 = t^2 \cos^2 t \]
\[ y^2 = t^2 \sin^2 t \]
\[ x^2 + y^2 = t^2 = z^2 \]

Find the derivative of \( r(t) = \langle \cos 3t, \tan t, 9 \sin t \rangle \).

\[ r'(t) = \langle -3\sin(3t), \sec^2(t), 9\cos(t) \rangle \]
\[ r(t) = \langle t^2 + 1, 2t, t^2 - 1 \rangle, \quad 0 \leq t \leq 2 \]

\[ s = \int_{0}^{2} \sqrt{4t^2 + 4 + 4t^2} \, dt \]

\[ \approx 7.25 \]

\[ r'(t) = \langle 2t, 2, 2t \rangle \]

\[ r' = \langle -\sin t, \cos t, 1 \rangle \]

\[ \sqrt{x^2 + y^2 + z^2} \]

\[ \int_{0}^{2\pi} \sqrt{1 + 1} \, dt \]

\[ \int_{0}^{\sqrt{a \cdot 2\pi}} \]

**Arc length:**

\[ r(0) = \langle 1, 0, 0 \rangle \]

\[ r(2\pi) = \langle 2\pi^2, 2\pi, 2\pi^2 - \pi \rangle \]
s(t) position now $\mathbf{r}(t)$

$s'(t)$ velocity now $\mathbf{v}(t) = \mathbf{r}'(t)$

$s''(t)$ acceleration now $\mathbf{r}''(t)$

Parameterization:

circle $x(t) = \cos t$  \hspace{1cm} $y(t) = \sin t$

$t$ is in radians

$x(s) = \cos (s)$  \hspace{1cm} $y(s) = \sin (s)$
Find an arc length parameterization of the circle of radius 8 centered at the origin.

- A. $C : x = 8 \sin \left( \frac{s}{8} \right), \ y = 8 \cos \left( \frac{s}{8} \right), \ 0 \leq s \leq 16\pi$
- B. $C : x = 8 \cos \left( \frac{s}{8} \right), \ y = 8 \cos \left( \frac{s}{8} \right), \ 0 \leq s \leq 16\pi$
- C. $C : x = 8 \cos \left( \frac{s}{8} \right), \ y = 8 \sin \left( \frac{s}{8} \right), \ 0 \leq s \leq 16\pi$
- D. $C : x = 8 \sin \left( \frac{s}{8} \right), \ y = 8 \sin \left( \frac{s}{8} \right), \ 0 \leq s \leq 16\pi$

$t = \frac{s}{8}$
\[ r = \langle t, 1-t, t+7 \rangle \quad \Rightarrow \quad t = \frac{s}{\sqrt{3}} \]

\[ 0 \leq t \leq 1 \]

\[ r'(s) = \langle \frac{\sqrt{3}}{\sqrt{3}}, -1, 1 \rangle \]

\[ \mathbf{r}(s) = \left\langle \frac{s}{\sqrt{3}}, \frac{s}{\sqrt{3}}, \frac{s}{\sqrt{3}} \right\rangle \]

\[ s = \int_0^t \sqrt{1+1+1} \, dt \]

\[ 0 \leq s \leq 15 \]

\[ s = \sqrt{3} t \]

\[ \frac{s}{7} = t \]

\[ \int_0^T \sqrt{4t^2 + 2} \, dt \]

\[ \frac{1}{2} T \sqrt{4T^2 + 2} + \frac{1}{2} \text{arcsinh}(\sqrt{2} T) = s \]

\[ T = f(s) \]
Radius of Curvature

Find the curvature of the straight line \( r(t) = \langle 8t + 4, 3t + 2, 5t + 7 \rangle \).

First, think about what we're asking. Straight lines are, well, straight, so their curvature should be zero at every point. Let's see. We have

\[ r'(t) = \langle 8, 3, 5 \rangle, \]

so that

\[ \|r'(t)\| = \sqrt{8^2 + 3^2 + 5^2} = \sqrt{98}. \]

The unit tangent vector is then

\[ T(t) = \frac{r'(t)}{\|r'(t)\|} = \frac{\langle 8, 3, 5 \rangle}{\sqrt{98}}, \]

which is a constant vector. This gives us \( T'(t) = 0 \), for all \( t \). We now have

\[ \kappa = \frac{\|T'(t)\|}{\|r'(t)\|} \frac{\|0\|}{\sqrt{98}} = 0, \]

as expected.