Match the vector–valued function

$$f(t) = \langle t \cos t,\ t \sin t,\ t \rangle$$

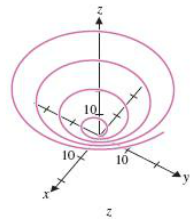
with the corresponding computer–generated graph.

$x^2 = t^2 \cos^2 t$

$y^2 = t^2 \sin^2 t$

$x^2 + y^2 = t^2 = z^2$

C.

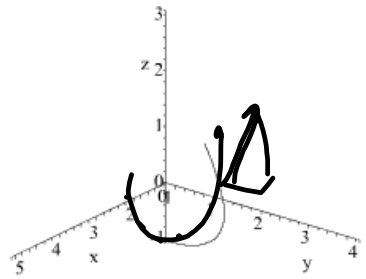


Find the derivative of $\mathbf{r}(t) = \langle \cos 3t,\ \tan t,\ 9 \sin t \rangle$.

$\langle \square, \square, \square \rangle$

r'(t) = < -3sin(3t), sec^2(t), 9cos(t) >

$$\mathbf{r}(t) = \langle t^2 + 1, 2t, t^2 - 1 \rangle, 0 \le t \le 2$$

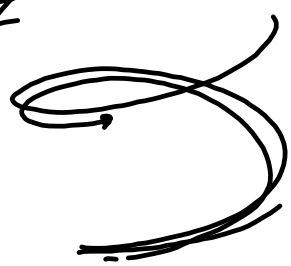


$$\mathbf{r}'(t) = \langle 2t, 2, 2t \rangle \longleftarrow \sqrt{x'^2 + y'^2 + z'^2}$$

$$s = \int_0^2 \sqrt{4t^2 + 4 + 4t^2}\, dt$$

$$\approx 7.25$$

Ex Arclength.



$r = \langle \cos t, \sin t, t \rangle$

From $t = 0$ to 2π

$r' = \langle -\sin t, \cos t, 1 \rangle$

$$\int_0^{2\pi} \sqrt{1 + 1}\, dt$$

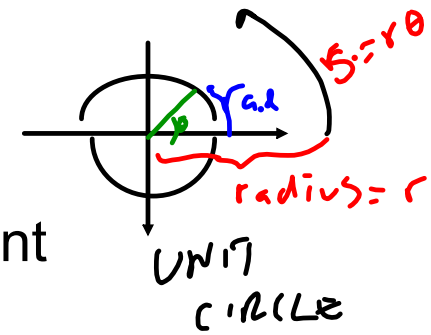
$$\sqrt{2} \cdot 2\pi$$

s(t) position now $\overrightarrow{r(t)}$

s'(t) velocity now $\overrightarrow{v(t)} = \overrightarrow{r'(t)}$

s''(t) accerlation now $\overrightarrow{r''(t)}$

Parameterization:



circle x(t) = cost y(t) = sint

t is in radians

s = arc length

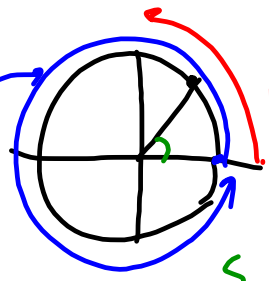
$s = r \cdot t$

$\frac{s}{r} = t$

x(s) = cos (s) y (s) = sin (s)

Find an arc length parameterization of the circle of radius 8 centered at the origin.

- A. $C : x = 8 \sin\left(\frac{s}{8}\right),\ y = 8 \cos\left(\frac{s}{8}\right),\ 0 \le s \le 16\pi$
- B. $C : x = 8 \cos\left(\frac{s}{8}\right),\ y = 8 \cos\left(\frac{s}{8}\right),\ 0 \le s \le 16\pi$
- C. $C : x = 8 \cos\left(\frac{s}{8}\right),\ y = 8 \sin\left(\frac{s}{8}\right),\ 0 \le s \le 16\pi$
- D. $C : x = 8 \sin\left(\frac{s}{8}\right),\ y = 8 \sin\left(\frac{s}{8}\right),\ 0 \le s \le 16\pi$

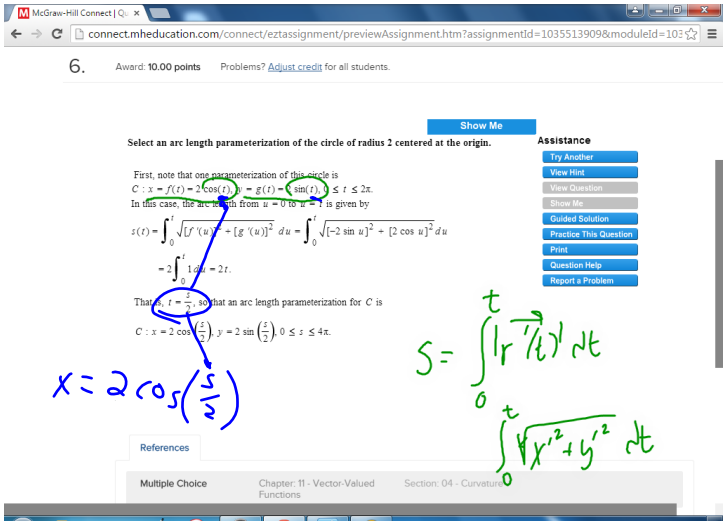


$0 \le s \le 4\pi$

$\frac{\pi}{2} \cdot 8$

radian $= \frac{s}{8}$

$t = \frac{s}{8}$



$x = 2\cos\left(\frac{s}{2}\right)$

$s = \int_0^t |\vec{r}\,'(t)|\, dt$

$\int_0^t \sqrt{x'^2 + y'^2}\, dt$

$x = 2\cos t \quad y = 2\sin t$

$x' = -2\sin t \quad y' = 2\cos t$

$\sqrt{4\sin^2 t + 4\cos^2 t}$

$\sqrt{4} = 2$

$s = \int_0^t 2\, dt = s = 2t$

$\frac{s}{2} = t$

$r = \langle t, 1-t, t+7 \rangle \rightarrow$ Replace $t = \frac{s}{\sqrt{3}}$

$0 \le t \le 1$

$r' = \langle 1, -1, 1 \rangle$

$\vec{r}(s) = \langle \frac{s}{\sqrt{3}}, 1 - \frac{s}{\sqrt{3}}, \frac{s}{\sqrt{3}} + 7 \rangle$

$s = \int_0^t \sqrt{1+1+1}\, dt$

$0 \le s \le \sqrt{3}$

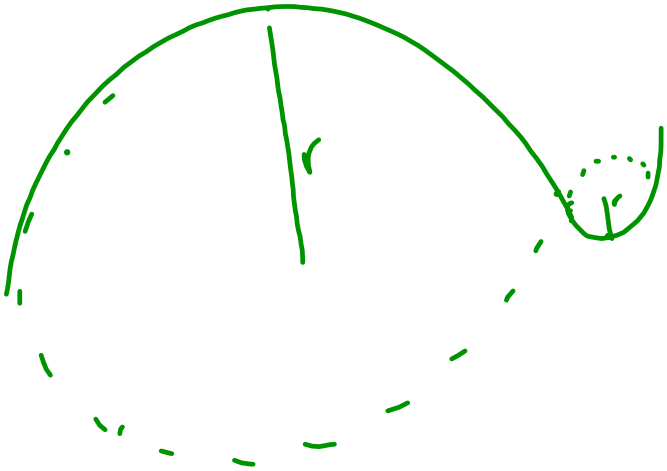
$s = \sqrt{3}\, t$

$\frac{s}{\sqrt{3}} = t$

$$\int_0^T \sqrt{4t^2 + 2}\, dt$$

$$\underbrace{\frac{1}{2} T \sqrt{4T^2 + 2} + \frac{1}{2} \operatorname{arcsinh}(\sqrt{2}\, T)}_{T = f(s)} = s$$

Radius of Curvature



Show Me

Find the curvature of the straight line $\mathbf{r}(t) = \langle 8t + 4, 3t + 2, 5t + 7 \rangle$.

First, think about what we're asking. Straight lines are, well, straight, so their curvature should be zero at every point. Let's see. We have

$$\mathbf{r}'(t) = \langle 8, 3, 5 \rangle,$$

so that

$$\|\mathbf{r}'(t)\| = \sqrt{8^2 + 3^2 + 5^2} = \sqrt{98}.$$

$T'(t) = 0$

The unit tangent vector is then

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = \frac{\langle 8, 3, 5 \rangle}{\sqrt{98}},$$

which is a constant vector. This gives us $\mathbf{T}'(t) = \mathbf{0}$, for all t. We now have

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{0}\|}{\sqrt{98}} = 0,$$

as expected.