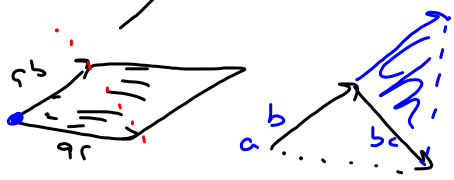
Connect Quiz

Question

Find the area of the triangle with vertices a = (2, 3, 5), b = (1, 4, 2), and c = (0, 0, 0). Round your final answer to three decimal places.

The area of the triangle is [ ] .

$\frac{1}{2}\|\vec{ab}\times\vec{ac}\|$



Find the area of the triangle with vertices a = (2, 3, 5), b = (1, 4, 2), and c = (0, 0, 0). Round your final answer to three decimal places.

$\vec{ca}$ $\vec{cb}$

6−20 4−5 8−3

First notice that

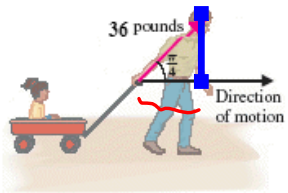
$$\vec{a}\times\vec{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 5 \\ 1 & 4 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 5 \\ 4 & 2 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}\mathbf{k} = \langle -14, 1, 5\rangle.$$

The area of the parallelogram with two adjacent sides formed by $\langle 2, 3, 5\rangle$ and $\langle 1, 4, 2\rangle$ is equal to $\|\mathbf{a}\times\mathbf{b}\|$.

Therefore the area of the triangle is given by

$$\frac{1}{2}\|\mathbf{a}\times\mathbf{b}\| = \frac{1}{2}\|\langle -14, 1, 5\rangle\| = \frac{1}{2}\sqrt{222} \approx 7.450.$$

You exert a constant force of 36 pounds in the direction of the handle of the wagon pictured in the figure below. If the handle makes an angle of $\frac{\pi}{4}$ with the horizontal and you pull the wagon along a flat surface for 3 miles (15840 feet), find the work done. Round your final answer to the nearest whole number.



W=FD

D=15840ft

F= $36\cdot\frac{1}{\sqrt{2}}$

The work done is [ ] **foot−pounds.**

If we apply a constant force F for a distance d, the work done is given by $W = Fd$. Unfortunately, the force exerted in the direction of motion is not given. Since the magnitude of the force is 36, the force vector must be

$$\begin{aligned}\mathbf{F} &= 36\left\langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4}\right\rangle \\ &= 36\left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right\rangle \\ &= \langle 18\sqrt{2}, 18\sqrt{2}\rangle.\end{aligned}$$

The force exerted in the direction of motion is simply the component of the force along the vector $\mathbf{i}$ (that is, the horizontal component of $\mathbf{F}$) or $18\sqrt{2}$.

The work done is then

$$W = Fd = 18\sqrt{2}(15840) \approx 403221 \text{ foot−pounds.}$$

Question

Find the volume of the parallelepiped with three adjacent edges formed by the vectors $\mathbf{a} = \langle 8, 1, 9 \rangle$, $\mathbf{b} = \langle 4, 3, 1 \rangle$ and $\mathbf{c} = \langle 7, 9, 0 \rangle$.

The volume of the parallelepiped is $\boxed{}$.

Show Me

Find the volume of the parallelepiped with three adjacent edges formed by the vectors $\mathbf{a} = \langle 8, 1, 9 \rangle$, $\mathbf{b} = \langle 4, 3, 1 \rangle$ and $\mathbf{c} = \langle 7, 9, 0 \rangle$.

First, note that Volume $= |\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})|$. We have that

$$\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = \begin{vmatrix} 7 & 9 & 0 \\ 8 & 1 & 9 \\ 4 & 3 & 1 \end{vmatrix} = 7\begin{vmatrix} 1 & 9 \\ 3 & 1 \end{vmatrix} - 9\begin{vmatrix} 8 & 9 \\ 4 & 1 \end{vmatrix} + 0\begin{vmatrix} 8 & 1 \\ 4 & 3 \end{vmatrix}$$

$$= 7(-26) - 9(-28) = 70.$$

So, the volume of the parallelepiped is Volume $= |\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})| = |70| = 70$.

Line

$\vec{v} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

vector

slope

$\langle a, b, c \rangle$

Point

$\langle x_1, y_1, z_1 \rangle + t\vec{v}$

$y = a + bx$

$x = x_1 + at$ $t = 0$ (x_1, y_1, z_1)

$y = y_1 + bt$ slope: $\langle a, b, c \rangle$

$z = z_1 + ct$

$\langle -1, 2, 2 \rangle$

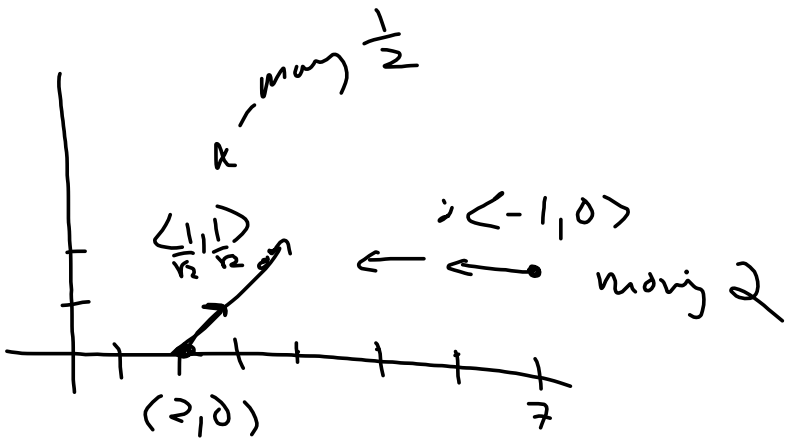
Determine if the lines

$$\ell_1: x - 7 = -t, \quad y - 3 = 2t, \quad \text{and} \quad z - 9 = 2t$$

$$\ell_2: x - 8 = s, \quad y - 2 = -s, \quad \text{and} \quad z - 2 = 2s$$

are parallel or intersect. $\langle 1, -1, 2 \rangle$

- A. The lines are not parallel and do intersect.
- B. The lines are ~~parallel~~ and do not intersect.
- C. The lines are ~~parallel~~ and intersect.
- D. The lines are not parallel, yet do not intersect.



$x = 2 + t\left(\frac{1}{\sqrt{2}}\right)$ $\quad x = 7 - 2s$

$y = 0 + t\left(\frac{1}{\sqrt{2}}\right)$ $\quad y = 2$

$x = 2 + \frac{t}{\sqrt{2}} = 7 - 2t$

$y = \frac{t}{\sqrt{2}} = 2$

ex) $a = (1, 2, 3)$ $b = (5, 7, -8)$

$\vec{ab} = \langle 5-1, 7-2, -8-3 \rangle$

$\vec{ab} = \langle 4, 5, -11 \rangle$

$x = 1 + 4t$ $\quad x - 1 = 4t$ $\quad \frac{x-1}{4} = t$

$y = 2 + 5t$ $\quad y - 2 = 5t$ $\quad \frac{y-2}{5} = t$

$z = 3 + -11t$ $\quad z - 3 = -11t$ $\quad \frac{z-3}{-11} = t$

$x = 1 + 4t$ $\quad x = 1 - 2s$

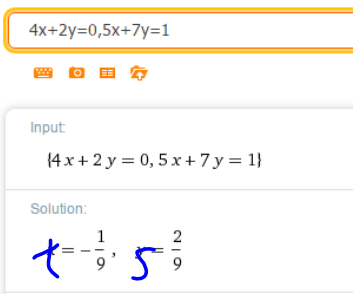
$y = 2 + 5t$ $\quad y = 3 - 7s$

$\boxed{z = 3 + 6t \quad z = 6 + 2s}$

$4t + 2s = 0$

$5t + 7s = 1$

$< \begin{array}{l} 1 + 4t = 1 - 2s \\ 2 + 5t = 3 - 7s \end{array}$



$z = 3 + 6\left(-\frac{1}{9}\right) = 2\frac{1}{3}$

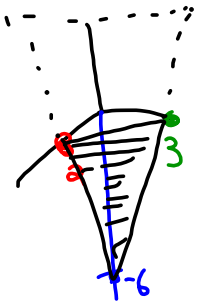
$z = 6 + 2\left(\frac{2}{9}\right) = 6\frac{4}{9}$

Planes

$$Ax + By + Cz = D$$

Intercept for Plane

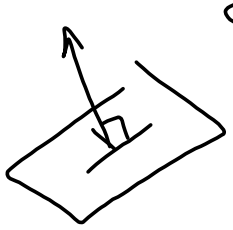
Ex $3x + 2y - 2z = 6$



$x = 2$

$y = 3$

Normal



$\langle 3, 2, -1 \rangle$

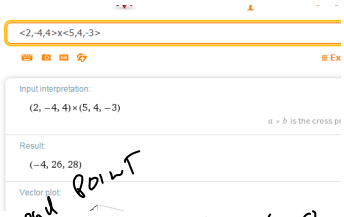
Normal To Plane

$\vec{n} = \vec{A} \times \vec{B}$

Ex

Head $(1, 3, 2)$
Doorab $(3, -1, 6)$ $\rangle$ $\langle 2, -4, 4 \rangle$
Ghost $(6, 3, 3)$ $\rangle$ $\langle 5, +4, -3 \rangle$

$\vec{n} = \langle 2, -4, 4 \rangle \times \langle 5, +4, -3 \rangle$



sub point

$-4x + 26y + 28z = D$

sub $\rightarrow$ $-4(1) + 26(3) + 28(2) = D$

$-4x + 26y + 28z = 130$

$x + y + \cancel{z} = 7$ $\quad n_1 = \langle 1,1,1 \rangle$

$2x + 5y + \cancel{z} = 5$ $\quad n_2 = \langle 2,5,1 \rangle$

Point of intersection

$n_1 \times n_2 = \vec{n}$

Floor $(z=0)$

$x + z = 7$ or $y = 0$

$2x + z = 5$ $\quad x = -2$

$z = 9$

$(-2, 0, 9)$

find

Correct Response:

The thrust of an airplane's engine produces a speed of 555 mph in still air. The plane is aimed in the direction of $\langle 2, 2, 1 \rangle$ and the wind velocity is $\langle 12, -23, 0 \rangle$ mph. Find the velocity vector of the plane with respect to the ground and find the speed (rounded to two decimal places).

The net velocity with wind speed is $\langle$ [ ] [ ] [ ] $\rangle$

The speed is [ ] mph.

$\frac{1110}{3} + 12$ $\quad \frac{1110}{3} - 23$ $\quad \frac{555}{3}$

$\sqrt{a^2 + b^2 + c^2}$

$\frac{555 \cdot \langle 2,2,1 \rangle}{3}$

$\langle \frac{1110}{3}, \frac{1110}{3}, \frac{555}{3} \rangle + \langle 12, -23, 0 \rangle$

Correct Response:

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The net velocity with wind speed is $\langle$ 382 , 347 , 185 $\rangle$.

The speed is 548.23 mph.

$\left| \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle \right|$

$\sqrt{\frac{4}{9} + \frac{4}{9} + \frac{1}{9}} = 1$

$\langle 1,1,1 \rangle$ $\quad \langle 2,2,2 \rangle$

$\sqrt{3}$ $\quad \sqrt{12} = 2\sqrt{3}$