Connect Quiz

Find the area of the triangle with vertices \(a = (2, 3, 5), b = (1, 4, 2),\) and \(c = (0, 0, 0).\) Round your final answer to three decimal places.

The area of the triangle is \(F = \frac{1}{2} \| \mathbf{a} \times \mathbf{b} \|.\)

Find the area of the triangle with vertices \(a = (2, 3, 5), b = (1, 4, 2),\) and \(c = (0, 0, 0).\) Round your final answer to three decimal places.

First notice that

\[
\mathbf{a} \times \mathbf{b} = \begin{vmatrix}
1 & 2 & 3 \\
2 & 4 & 5 \\
3 & 5 & 6
\end{vmatrix} = 1 \begin{vmatrix}
4 & 5 \\
5 & 6
\end{vmatrix} - 2 \begin{vmatrix}
2 & 5 \\
3 & 6
\end{vmatrix} + 3 \begin{vmatrix}
2 & 4 \\
3 & 5
\end{vmatrix} = (14, 1, 5).
\]

The area of the parallelogram with two adjacent sides formed by \((2, 3, 5)\) and \((1, 4, 2)\) is equal to \(\| \mathbf{a} \times \mathbf{b} \|.
\]

Therefore the area of the triangle is given by

\[
\frac{1}{2} \| \mathbf{a} \times \mathbf{b} \| = \frac{1}{2} \| (14, 1, 5) \| = \frac{1}{2} \sqrt{217} = 7.450.
\]

You exert a constant force of 36 pounds in the direction of the handle of the wagon pictured in the figure below. If the handle makes an angle of \(\frac{\pi}{4}\) with the horizontal and you pull the wagon along a flat surface for 3 miles (15840 feet), find the work done. Round your final answer to the nearest whole number.

\[
W = FD = 36 \cdot \frac{1}{18} = \frac{36}{18} = 2
\]

The work done is \(2\) foot-pounds.

If we apply a constant force \(F\) for a distance \(d,\) the work done is given by \(W = Fd.\)

Unfortunately, the force exerted in the direction of motion is not given. Since the magnitude of the force is 36, the force vector must be

\[
\mathbf{F} = 36 \left( \cos \frac{\pi}{4}, \sin \frac{\pi}{4} \right) = 36 \left( \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)
\]

\[
= \left( 18 \sqrt{2}, 18 \sqrt{2} \right)
\]

The force exerted in the direction of motion is simply the component of the force along the vector \(\mathbf{F}\) (that is, the horizontal component of \(\mathbf{F}\)) or \(18 \sqrt{2}.\)

The work done is then

\[
W = Fd = 18 \sqrt{2}(15840) = 403221\text{ foot-pounds}.
\]
Find the volume of the parallelepiped with three adjacent edges formed by the vectors 
\( a = \langle 8, 1, 9 \rangle \), \( b = \langle 4, 3, 1 \rangle \) and \( c = \langle 7, 9, 0 \rangle \).
The volume of the parallelepiped is \( \quad \).

Find the volume of the parallelepiped with three adjacent edges formed by the vectors 
\( a = \langle 8, 1, 9 \rangle \), \( b = \langle 4, 3, 1 \rangle \) and \( c = \langle 7, 9, 0 \rangle \).

First, note that \( \text{Volume} = |c \times (a \times b)| \). We have that 
\[
\begin{vmatrix}
7 & 9 & 0 \\
8 & 1 & 9 \\
4 & 3 & 1
\end{vmatrix} - 7 \begin{vmatrix}
1 & 9 \\
3 & 1
\end{vmatrix} - 9 \begin{vmatrix}
1 & 9 \\
4 & 1
\end{vmatrix} = \begin{vmatrix}
8 & 1 \\
4 & 3
\end{vmatrix}
\]
\[
= 7(-26) - 9(-28) = 70.
\]
So, the volume of the parallelepiped is \( \text{Volume} = |c \times (a \times b)| = 70 \).
Determine if the lines

$\ell_1: x - 7 = -t, \quad y - 3 = 2t, \quad \text{and} \quad z - 9 = 2t$

$\ell_2: x - 8 = s, \quad y - 2 = -s, \quad \text{and} \quad z - 2 = 2s$

are parallel or intersect.

- A. The lines are not parallel and do intersect.
- B. The lines are parallel and do not intersect.
- C. The lines are parallel and intersect.
- D. The lines are not parallel, yet do not intersect.
\( x = 1 + 4t \quad \frac{x-1}{4} = t \)

\( y = 2 + 5t \quad \frac{y-2}{5} = t \)

\( z = 3 + 11t \quad \frac{z-3}{11} = t \)

\[ 4x + 2y = 0, 5x + 7y = 1 \]

\[ \begin{align*}
4x + 2y &= 0, \\
5x + 7y &= 1
\end{align*} \]

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Planes

\[ A x + B y + C z = D \]

Intersection for plane

\[ 3x + 2y - 2z = 6 \]

\[ x = 2 \]

\[ y = 3 \]

Normal vector \( \langle 3, 2, -1 \rangle \)

Normal to plane

\[ \hat{n} = \vec{a} \times \vec{b} \]

Example:

Normal: \( \langle 1, 3, 2 \rangle \)
Point: \( \langle 3, -1, 6 \rangle \)

\[ \hat{n} = \langle 3, -4, 4 \rangle \times \langle 5, 7, 4 \rangle \]

To find the equation of the plane:

\[ -4x + 26y + 28z = D \]

\[ -4(1) + 26(3) + 28(2) = 130 \]
The thrust of an airplane's engine produces a speed of 555 mph in still air. The plane is aimed in the direction of \( \langle 2, 1, 1 \rangle \) and the wind velocity is \( \langle 12, -20, 3 \rangle \) mph. Find the velocity vector of the plane with respect to the ground and find the speed (rounded to two decimal places).

The net velocity with wind speed is

\[
\text{The speed is } \frac{555}{3} \text{ mph.}
\]

\[
\frac{555}{3} \cdot \langle 2, 1, 1 \rangle
\]

\[
\langle \frac{110}{3}, \frac{110}{3}, \frac{555}{3} \rangle + \langle -40, 20, 3 \rangle
\]

\[
\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle
\]

\[
\sqrt{\frac{4}{9} + \frac{4}{9} + \frac{1}{9}} = 1
\]

\[
\langle 1, 1, 1 \rangle \quad \langle 2, 2, 2 \rangle
\]

\[
\sqrt{3} \quad \sqrt{13} = 3.60
\]