7. Finding solutions in an interval for a basic equation involving sine or cosine

Find all solutions of the equation in the interval \([0,2\pi)\).

\[2\sin \theta - 1 = 0\]

Write your answer in radians in terms of \(\pi\).
If there is more than one solution, separate them with commas.

\[y_1 = 2 \sin (x) - 1, \quad y_2 = 0\]

\[\text{Window: } x_{\min} = 0, \quad x_{\max} = 2\pi, \quad y_{\min} = -2, \quad y_{\max} = 2\]

9. Solving a basic trigonometric equation involving sine or cosine

Find all solutions to the equation.

\[\cos \theta + 1 = 0\]

Write your answer in radians in terms of \(\pi\), and use the "or" button as necessary.

Example: \(\theta = \frac{2\pi}{3} + k\pi\) or \(\theta = \frac{4\pi}{3} + 2k\pi\).

\[\theta = \frac{2\pi}{3} + 2\pi \cdot k, \quad 90^\circ + k \cdot 360^\circ, \quad 150^\circ + k \cdot 360^\circ\]
Show that for \( F(x, y) = \langle 3x^2y^5, x^3 + 16x^3y^3 - 2 \rangle \), the line integral 
\[ \int_C F(x, y) \cdot dr \] is independent of path \( C \) and, therefore, the line integral for any curve \( C \) with initial point \( (-5, 2) \) and terminal point \( (3, 4) \) is \[ \int_C F(x, y) \cdot dr = \text{constant} \]

where
\[ f_x = F_x = x^3 + 16x^3y^3 - 2 \\
\]
\[ f_y = F_y = 3x^2y^5 + C(y) \]

\[ f_x = x^3 + 16x^3y^3 - 2x \\
\]
\[ f_y = x^3 + 5x^3y + 4y^2 - 2y + C \]

\[ \langle 2, 2 \rangle \rightarrow \langle 3, 4 \rangle \]

\[ f(3, 4) = f(2, 2) \]

\[ (3^3 + 15 + 4^3 - 8) - \left(-\frac{16}{2} + 10 + 4 - 4 \right) = 101 \]

Determine whether \( F(x, y) = \langle 7x^6y - 5, x^7 \rangle \) is conservative.
If it is, find a potential function \( f \).

- A. \( F \) is not conservative.
- B. \( F \) is conservative and \( f(x, y) = x^7y - 5y + c \) is a potential function.
- C. \( F \) is conservative and \( f(x, y) = x^7y + c \) is a potential function.
- D. \( F \) is conservative and \( f(x, y) = x^7y - 5x + c \) is a potential function.
Evaluate \( \int_C F \cdot dr \), where

\[ F(x, y) = (2.91x^2y + 1, 2.91y^2) \]

and \( C \) is the bottom half-circle from \((1, 0)\) to \((-1, 0)\).

\[
\begin{align*}
  f &= \langle 2.91 \cos^2 t \sin t + 1, 2.91 \cos^2 t \rangle \\
  r &= \langle \cos t, -\sin t \rangle \\
  dr &= \langle -\sin t, -\cos t \rangle \, dt \\
  \int_0^{\pi} 2.91 \cos^2 t \sin t - \sin t + 2.91 \cos^2 t \, dt \\
  \int_0^{\pi} \sin t \sin t - \sin t \, dt = 0 \\
  -1 - 1 = -2
\end{align*}
\]

Evaluate the line integral \( \int_C (3y - e^{x+y}) \, dx + (14x - \sin (y^3 - 4y)) \, dy \), where \( C \) is the circle of radius 4 centered at the point \((3, -8)\), as shown below.

\[
\begin{align*}
  \int_R \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right) \, dA \\
  \int_R (14 - 3) \\
  11 \int_D dA \\
  11 \cdot \pi (4^2)
\end{align*}
\]

Your Answer: \( \pi \)
Evaluate the line integral \( \int_C x^2 \, dx - x^3 \, dy \) where \( C \) is the square contour from \((0,0)\) to \((0.5, 0)\) to \((5.0, 0)\) to \((0.5, 0)\) using Green's Theorem.

Your Answer:

\[
\begin{align*}
\mathcal{R} & \quad \text{not left hand} \\
\int_0^5 \int_0^5 \left( \frac{2N}{2x} - \frac{2M}{2y} \right) \, dx \, dy \\
& \quad \int_0^5 \int_0^5 (3x^2 - 0) \, dx \, dy \\
& \quad \int_0^5 \int_0^5 3x^2 \, dx \\
& \quad (5)(x^3 \bigg|_0^5) \\
& \quad = 625
\end{align*}
\]

Select the graph of the parametric surface.

\( x = 88 \sin \, u \cos \, v \), \( y = 88 \sin \, u \sin \, v \), \( z = 88 \cos \, u \)

\[
\begin{align*}
X^2 &= 88 \sin^2 \, u \cos^2 \, v \\
Y^2 &= 88 \sin^2 \, u \sin^2 \, v \\
X^2 + Y^2 &= 88 \sin^2 \, u \\
Z^2 &= 88 \cos^2 \, v \\
X^2 + Y^2 + Z^2 &= 88
\end{align*}
\]
Find the surface area of the given surface.

The portion of the cone \( z = \sqrt{x^2 + y^2} \) below the plane \( z = 7 \).

\[
\mathbf{r} = \langle r \cos \theta, r \sin \theta, z \rangle = \langle r \cos \theta, r \sin \theta, \sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta} \rangle
\]

\[
\mathbf{r}_\theta = \langle -r \sin \theta, r \cos \theta, 0 \rangle
\]

\[
\mathbf{r}_r \times \mathbf{r}_\theta = \langle \sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta}, 0, r \rangle
\]

\[
\iint_S \mathbf{r}_r \times \mathbf{r}_\theta \cdot d\mathbf{A} = \iint_0^{2\pi} \int_0^7 r \sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta} \, dr \, d\theta
\]

\[
= 2\pi \cdot 7 \cdot \frac{49}{2} = 49\sqrt{2}\pi
\]

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Use the Divergence Theorem to compute \( \int \int_S F \cdot n \, dS \), if \( S \) is the cube \( -1 \leq x \leq 1, -1 \leq y \leq 1, -1 \leq z \leq 1 \) and \( F = \langle 2x, y, z \rangle \).

Your Answer: \( \int \int_S F \cdot n \, dS = \iiint_Q \nabla \cdot F \, dV \)

\[ \nabla \cdot F = \frac{\partial}{\partial x} F_x + \frac{\partial}{\partial y} F_y + \frac{\partial}{\partial z} F_z = 2 + 1 + 1 = 4 \]

\[ \iiint_Q 4 \, dV = \left[ \frac{1}{2} x^2 + y + z \right]_{-1}^{1} = 4 \]

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[Diagram of a vector field]
Use Stokes' Theorem to compute\[\int_S \left( \nabla \times \mathbf{F} \right) \cdot \mathbf{n} \, d\sigma.\]

\(S\) is the portion of the tetrahedron bounded by \(x + y + z = 2\) and the coordinate planes with \(z > 0\), \(u\)-apex, \(F = \langle -z, y, -x \rangle\).

\[\begin{align*}
&\mathbf{F} = \left\langle 1, 1, 0 \right\rangle \\
&\mathbf{n} = \left\langle 1, 1, 0 \right\rangle \\
&t^2 + t^2 - (2-t)^2
\end{align*}\]

Stokes and curl/independence:

\[
\begin{align*}
&\int_S \left( \nabla \times \mathbf{F} \right) \cdot \mathbf{n} \, d\sigma \quad \text{why basis?}
&\mathbf{F} = \mathbf{F} \quad \mathbf{n} = \mathbf{n}
\end{align*}
\]

\[
\begin{align*}
&C_1: x = 2-t, \quad y = t, \quad z = 0
&C_2: 0 \leq x \leq 2, \quad x = t
\end{align*}
\]

\[
\int_0^2 t^2 + t^2 - (2-t)^2 \, dt
\]

\[
\int_0^2 \left( \frac{t^4}{2} + \frac{(2-t)^4}{4} \right) \, dt
\]

\[
\frac{8}{3} + \frac{4}{3} = \frac{12}{3}
\]

\[
\frac{1}{3} x^3 + \frac{2}{3} y^3 - \frac{1}{3} z = \frac{1}{3}
\]

\[
\begin{align*}
&C_1: x = 0, \quad y = 0, \quad \mathbf{F}(0,0,0) = 0
&C_2: y = 0, \quad \mathbf{F}(0,0,0) = 0
&C_3: z = 0, \quad \mathbf{F}(0,0,0) = 0
\end{align*}
\]

\[-\int_0^2 t \cdot t^2 = \frac{1}{2} \left( \frac{2}{3} \right)^3 \]

\[-2\]