7. Finding solutions in an interval for a basic equation involving sine or cosine

Find all solutions of the equation in the interval $[0,2\pi)$.

$$2\sin\theta - 1 = 0$$

Write your answer in radians in terms of π.
If there is more than one solution, separate them with commas.

$Y_1 = 2\sin(x) - 1$

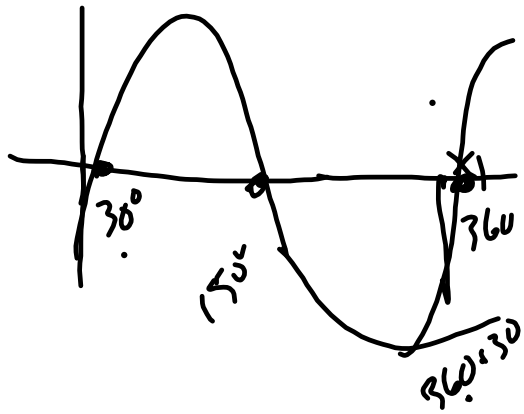
$Y_2 = 0$

Mode: degrees

Window:
Xmin: 0
Xmax: 360

Calc 5: Intersect

$30 \cdot \frac{\pi}{180} = \frac{\pi}{6}$ $\frac{5\pi}{6}$



9. Solving a basic trigonometric equation involving sine or cosine

Find all solutions to the equation.

$$\cos\theta + 1 = 0$$

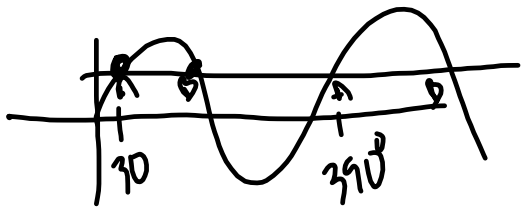
Write your answer in radians in terms of π, and use the "**or**" button as necessary.

Example: $\theta = \frac{2\pi}{5} + k\pi$, or $\theta = \frac{8\pi}{7} + 2k\pi$,

$\theta = \frac{6\pi}{6} + 2\pi \cdot K$

$30° + K \cdot 360°$

$150° + K \cdot 360°$

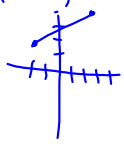


Show that for $\mathbf{F}(x, y) = \langle 3x^2y + 5,\ x^3 + 16y^3 - 2 \rangle$, the line integral $\int_C \mathbf{F}(x, y) \cdot d\mathbf{r}$ is independent of path. Then, evaluate the line integral for any curve C with initial point at $(-2, 2)$ and terminal point at $(3, 4)$.

$\int_0 \langle \ldots \rangle \cdot \langle 5, 2 \rangle \, dt$

$\vec{r}(t) = \langle -2+5t, 2+2t \rangle$

Your Answer:



$\nabla f = F$

$\frac{\partial f}{\partial x} = 3x^2y + 5$

$f = x^3y + 5x + C(y)$

$\frac{\partial f}{\partial y} = x^3 + 16y^3 - 2$

$f = x^3y + 4y^4 - 2y + C_2(x)$

$f = x^3y + 5x + 4y^4 - 2y + C$

$(-2, 2) \to (3, 4)$

$f(3,4) - f(-2,2)$

$(27 \cdot 4 + 15 + 4^5 - 8) - (-16 + -10 + 64 - 4)$

We now have that

$$f(x, y) = x^3y + 5x + 4y^4 - 2y + c$$

is a potential function for $\mathbf{F}(x, y)$, for any constant c. Now that we have found a potential function, we have that for any path $(-2, 2)$ to $(3, 4)$,

$$\int_C \mathbf{F}(x, y) \cdot d\mathbf{r} = f(x, y)\Big|_{(-2,2)}^{(3,4)}$$

$$= [(3)^3(4) + 5(3) + 4(4)^4 - 2(4) + c] - [(-2)^3(2) + 5(-2) + 4(2)^4 - 2(2) + c]$$

$$= 1105.$$

Determine whether $\mathbf{F}(x, y) = \langle 7x^6y - 5,\ x^7 \rangle$ is conservative.
If it is, find a potential function f.

- A. F is not conservative.
- B. F is conservative and $f(x, y) = x^7y - 5y + c$ is a potential function.
- C. F is conservative and $f(x, y) = x^7y + c$ is a potential function.
- D. F is conservative and $f(x, y) = x^7y - 5x + c$ is a potential function.

$\langle f_x, f_y \rangle$

$\langle M, N \rangle$

$f_{xy} = M_y = N_x$

$= 7x^6 = 7x^6$

$f = x^7y - 5x$

$f_x = 7x^6y - 5$

$f_y = x^7$

Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where

$\mathbf{F}(x, y) = \langle 2.91x^2y + 1, 2.91xy^2 \rangle$

and C is the bottom half-circle from $(1, 0)$ to $(-1, 0)$.

$F = \langle -2.91\cos^2 t \sin t + 1, 2.91 \cos t \sin^2 t \rangle$

$r = \langle \cos t, -\sin t \rangle$

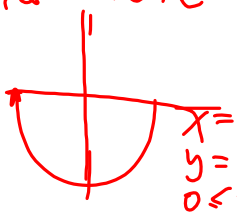
$dr = \langle -\sin t, -\cos t \rangle dt$

$\int_0^{\pi} \cancel{+2.91\cos^2 t \sin^2 t} - \sin t + \cancel{-2.91\cos^2 t \sin^2 t}\, dt$

$-\int_0^{\pi} \sin t\, dt$

$\cos t \Big|_0^{\pi}$

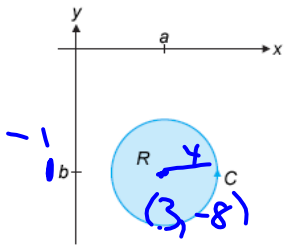
$-1 - 1 = -2$



$x = \cos t$
$y = -\sin t$
$0 \le t \le \pi$

M N

Evaluate the line integral $\oint_C (3y - e^{\sin x})\, dx + [14x - \sin(y^3 + 4y)]\, dy$, where C is th circle of radius 4 centered at the point $(3, -8)$, as shown below.



$a = 3$ and $b = -8$

Your Answer: π

$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

$(14 - 3)$

$11 \iint_R dA$

$11 \cdot \pi(4)^2$

$(x-3)^2 + (y+8)^2 = 16$

$y = \sqrt{16 - (x-3)^2} - 8$

$y = -\sqrt{16 - (x-3)^2} - 8$

$-1 \le x \le 7$

Evaluate the line integral $\oint_c \overset{M}{x^2}\,dx - \overset{N}{x^3}\,dy$ where C is the square contour from (0,0) to (0,5) to (5,5) to (5,0) to (0,0) using Green's Theorem.

Your Answer:

Not left hand

$$= \int_0^5 \int_0^5 \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$$\iint_R (-3x^2 - 0)\,dx\,dy$$

$$\int_0^5 dy \int_0^5 3x^2\,dx$$

$$(5)\left(x^3\Big|_0^5\right) =$$

$$= 625$$

Select the graph of the parametric surface.

$x = 88 \sin u \cos v$, $y = 88 \sin u \sin v$, $z = 88 \cos u$

$$x^2 = 88 \sin^2 u \cos^2 v$$

$$y^2 = 88 \sin^2 u \sin^2 v$$

$$x^2 + y^2 = 88 \sin^2 u$$

$$z^2 = 88 \cos^2 v$$

$$x^2 + y^2 + z^2 = 88$$

Find the surface area of the given surface.

The portion of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 7$.

$$r = \langle \underset{x}{r\cos\theta}, \underset{y}{r\sin\theta}, \underset{z}{r} \rangle$$

$$0 \le \theta \le 2\pi \qquad 0 \le r \le 7$$

$$\iint \| r_r \times r_\theta \| \, dA$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & \sin\theta & 1 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix}$$

$$\| \langle -r\cos\theta, r\sin\theta, r \rangle \|$$

$$\sqrt{r^2\cos^2\theta + r^2\sin^2\theta + r^2}$$

$$\int_{\theta=0}^{2\pi} \int_{r=0}^{7} \sqrt{2r^2} \, r \, dr \, d\theta$$

$$2\pi \cdot \sqrt{2} \, \frac{49}{2} = 49\sqrt{2}\pi$$

Use the Divergence Theorem to compute $\iint_{\partial Q} \mathbf{F} \cdot \mathbf{n} \, dS$, if Q is the cube $-1 \le x \le 1$, $-1 \le y \le 1$, $-1 \le z \le 1$ and $\mathbf{F} = \langle 9y^2, 4z - \cos x, z^3 - x \rangle$.

Your Answer:

$\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}$

$$F = \langle M, N, P \rangle$$

$$\iint_{\partial Q} F \cdot n \, dS = \iiint_Q \operatorname{div} F \, dQ$$

$$\operatorname{div} F = \nabla \cdot F = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$$

$$1 + 1 = 3z^2$$

$$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

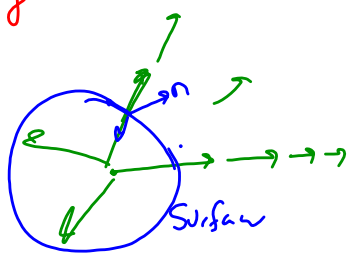
$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} 3z^2 \, dz \, dy \, dx$$

$$\langle M, N, P \rangle$$

$$4 \int_{-1}^{1} 3z^2 \, dz = 4z^3 \Big|_{-1}^{1}$$

$$\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$$

$$= 8$$



Question

Use Stokes' Theorem to compute $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$. $= \int$

S is the portion of the tetrahedron bounded by $x + y + 2z = 2$ and the coordinate planes with $z > 0$, $\mathbf{n}$ upward, $\mathbf{F} = \langle z^4 - y^2, y - x^3, z^3 \rangle$ $= \langle 1, 0, 0 \rangle = 0$ $C_1 : 0$

$\langle -t^2, t-(2-t)^4, 0 \rangle \langle -1, 1, 0 \rangle$

$t^2 + t - (2-t)^3$

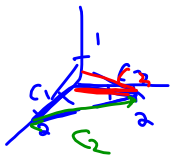
Stokes and surface independence:

Stokes:

$$\oint_C \vec{\mathbf{F}} \cdot d\vec{r} = \iint_{S_1} (\nabla \times \vec{\mathbf{F}}) \cdot \hat{\mathbf{n}}\, dS = \iint_{S_2} (\nabla \times \vec{\mathbf{F}}) \cdot \hat{\mathbf{n}}\, dS$$

why same?

$$\iint_{S_1} (\nabla \times \vec{\mathbf{F}}) \cdot \hat{\mathbf{n}}\, dS - \iint_{S_2} = \iint_{S = S_1 - S_2} (\nabla \times \vec{\mathbf{F}}) \cdot \hat{\mathbf{n}}\, dS$$



C_2 $0 \le t < 2$

$x = 2 - t$

$y = t$

$z = 0$

$\int_0^2 t^2 + t - (2-t)^4 dt$

C_1

$0 \le x \le 2$ $x = t$

$y = 0$

$\frac{t^3}{3} + \frac{t^2}{2} \Big|_0^2 + \frac{(2-t)^4}{4} \Big|_0^2$

$\frac{8}{3} + 2 + -4 = 2/3$ $= 2/3$

C_3:

$x = 0$ $d_1 = \langle 0, 1, 1 \rangle$

$z = 0$

$y = t$ $F \cdot d_1 =$

C_1	C_2	C_3
0	$2/3$	-2
	$-4/3$	

$-\int_0^2 t\, dt = -\frac{t^2}{2} \Big|_0^2$

$= -2$