Compute the exact work done by the force field \( \mathbf{F}(x, y, z) = (3y, 6z, 4x) \) acting on an object as it moves along the helix defined parametrically by \( x = 3 \cos t \), \( y = \sin t \), and \( z = 2t \), from the point \((3, 0, 0)\) to the point \((-3, 0, 0)\).

\[
\mathbf{F} = \int_0^\pi \mathbf{F}(\mathbf{r}(t)) \cdot d\mathbf{r}
\]

\[\mathbf{r}(t) = \langle 3\cos t, \sin t, 2t \rangle\]
\[d\mathbf{r} = \langle -3\sin t, \cos t, 2 \rangle dt\]
\[\mathbf{F} = \langle 9\sin t, 3\cos t, 3y \rangle \cdot \langle 3\cos t, \sin t, 2 \rangle
\]
\[
= \int_0^\pi 27 \sin^2 t + 10 \cos^2 t + 24 \sin t \, dt
\]

\[= 8\pi.
\]

Evaluate the line integral.
\[
\int_C 7x \, ds, \text{ where } C \text{ is the line segment from (1, 1) to (7, 6)}
\]

\( \mathbf{r}(t) = \langle 1 + 6t, 2 + 4t \rangle \)

\(0 \leq t \leq 1
\]

\[
\int_0^1 7(1 + 6t) \sqrt{(6)^2 + (4)^2} \, dt
\]

\[= \frac{7}{2} \left[ (7 + 42t) - 0 \right]_0^1
\]

\[= 28\pi.
\]

Evaluate the line integral.
\[
\int_C 7x \, ds, \text{ where } C \text{ is the line segment from (1, 2) to (7, 6)}
\]

\[
x = 1 + 6t, \quad y = 2 + 4t, \quad 0 \leq t \leq 1
\]
\[
dt = \frac{dt}{\sqrt{(6)^2 + (4)^2}}
\]
\[
= \frac{dt}{2 \sqrt{13}}
\]
\[
\int_C 7x \, ds = \int_0^1 (1 + 6t)(2 \sqrt{13}) \, dt
\]

\[= 28\sqrt{13}.
\]
Evaluate the line integral.
\[ \int_C 8z \; ds \], where \( C \) is the line segment from (4, 0, 1) to (5, -8, 8)

\[ x = 4 + t, \; y = -8t, \; z = 1 + 7t \]
\[ 0 \leq t \leq 1 \]

\[
ds = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} \; dt = \sqrt{1^2 + (-8)^2 + 7^2} \; dt = \sqrt{114} \; dt
\]

\[
\int_C 8z \; ds = \int_0^1 8(1 + 7t)\sqrt{114} \; dt = 36\sqrt{114}
\]

Compute the work done by the force field \( F \) along the curve \( C \).
\( F(x, y) = \langle 2x, 2y \rangle \), \( C \) is the line segment from (2, 1) to (6, 4)

\[
X = 2 + 4t
\]
\[
Y = 1 + 3t
\]

\[
\int_0^1 \langle 2(2+4t), 2(1+3t) \rangle \cdot \langle 4, 3 \rangle \; dt
\]
\[
\int_0^1 \langle 8(2+4t) + 6(1+3t) \rangle \; dt
\]
### 2D

\[ \nabla \mathbf{P} \]
\[ \left\langle \begin{array}{c} p_x \\ p_y \end{array} \right\rangle \]
\[ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \]
\[ p_{xy} = p_{yx} \]

### 3D

\[ \nabla \mathbf{P} \]
\[ \left\langle \begin{array}{c} m \\ n \\ p \end{array} \right\rangle \]
\[ \left\langle \begin{array}{c} 2p_x \\ 2p_y \\ 2p_z \end{array} \right\rangle \]
\[ \left\langle \begin{array}{c} M_y - N_x \\ M_z - P_x \\ N_z - P_y \end{array} \right\rangle \]
\[ \left\langle \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right\rangle \]
\[ \left\langle \begin{array}{c} 1 \\ -0 \\ 0 \\ 0 \\ 0 \end{array} \right\rangle \]
Using the graph, determine if the work done is positive, negative, or zero.

\[
\langle 0, 1 \rangle \cdot \langle 0, -1 \rangle = -1
\]

Find the mass of a spring in the shape of the helix defined parametrically by

\[x = 8 \cos t, \quad y = 8t, \quad z = 8 \sin t, \quad \text{for } 0 \leq t \leq 10\pi, \quad \text{with density } \rho(x, y, z) = 4y.\]

- A. \(1797 \pi^2 \sqrt{5}\)
- B. \(1290 \pi^2 \sqrt{3}\)
- C. \(1280 \pi^2 \sqrt{2}\)
- D. \(1797 \pi^2 \sqrt{2}\)

\[
M = \int_{\pi}^{2\pi} \rho(x(t), y(t), z(t)) \, dt
\]

\[
= \int_{\pi}^{2\pi} 4y(t) \, dt
\]

\[
= \left. \frac{4}{9} \left( \frac{y(t)^3}{3} + 2 \right) \right|_{\pi}^{2\pi}
\]

\[
= \frac{4}{9} \left( \frac{8^3}{3} + 2 - \frac{8^3}{3} - 2 \right)
\]

\[
= \frac{4}{9} \cdot \frac{512}{3}
\]

\[
= \frac{2048}{27}
\]

\[
= \frac{256 \pi^2}{3}
\]

\[
\int_0^{10\pi} \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2} \, dt
\]

\[
= \int_0^{10\pi} \sqrt{8^2 + 8^2 + 8^2} \, dt
\]

\[
= \int_0^{10\pi} \frac{4}{\sqrt{3}} \, dt
\]

\[
= \frac{4}{\sqrt{3}} \left( 10\pi - \pi \right)
\]

\[
= \frac{256 \pi^2}{3}
\]
Show that for \( F(x, y) = (2x + 2y, x^2 + y^2) \), the line integral \( \int_C F \cdot dr \) is independent of path. Then, evaluate the line integral for any curve \( C \) with initial point at \((-3, 4)\) and terminal point at \((3,7)\).

**Your Answer:**

\[
\begin{align*}
M_y &= 2y \\
N_x &= 2x \\
\end{align*}
\]

Conservative \( \iff \) Path Independent

**Fundamental Theorem of Line Integrals**

\[
\int_C F \cdot dr = f(3, 7) - f(-3, 4)
\]

\[
\begin{align*}
f &= x^2 y + 5x \\
f_x &= x^2 + 6y + 3 \\
f_y &= x^2 y + 3y + 3 \\
\end{align*}
\]

\[
\begin{align*}
f(3, 7) - f(-3, 4) &= (3^2(7) + 12 + 6) - (-3^2(-4) + 12 + 6) \\
&= 34
\end{align*}
\]

\( \mathbf{C} \cdot \mathbf{v} = -81 \)

Determine whether or not the line integral \( \int_C (e^{3x} + 2x \sin y) \, dx + (6x^2 \cos y) \, dy \) is independent of path.

\( \bigcirc \) A. The line integral is independent of path.

\( \bigcirc \) B. The line integral is not independent of path.

\[
\begin{align*}
M_y &= N_x \\
2x \cos y &= 12x \cos y
\end{align*}
\]

Not Conservative.
Evaluate \( \int_C F \cdot dr \), where

\[
F(x, y, z) = \frac{(1.74x^2, 4.09y^2, 2.04z)}{\sqrt{1.74x^2 + 4.09y^2 + 2.04z^2}}
\]

and \( C \) runs from \((0, 5, 2)\) to \((6, 4, 0)\).

A potential function exists, so the integral is independent of the path.

A potential function is

\[
f(x, y, z) = \sqrt{1.74x^2 + 4.09y^2 + 2.04z^2}.
\]

Thus

\[
\int_C F \cdot dr = \left[ \sqrt{1.74x^2 + 4.09y^2 + 2.04z^2} \right]_{(0, 5, 2)}^{(6, 4, 0)} \approx 0.869.
\]

Evaluate \( \int_C F \cdot dr \), where

\[
F(x, y) = (2.77x^2y + 7, 2.77xy^2)
\]

and \( C \) is the bottom half-circle from \((1, 0)\) to \((-1, 0)\).

\[
M_y = 2.77x^2 \quad \text{and} \quad N_x = 2.77y^2,
\]

so the vector field is not conservative

and the line integral is not independent of the path.

For \(0 \leq t \leq \pi\),

\[
\begin{align*}
x &= \cos t, \\
y &= -\sin t.
\end{align*}
\]

\[
\int_C F \cdot dr
\]
\[
- \int_C (2.77x^2y + 7) \, dx + (2.77x y^2) \, dy \\
- \int_0^\pi \left[ 2.77(\cos^2 t)(-\sin t)(-\sin t) + 7(-\sin t) + 2.77(\cos t)(\sin^2 t)(-\cos t) \right] \, dt \\
- \int_0^\pi -\sin t \, dt \\
= -14
\]

Evaluate the line integral \( \oint_C (6y - e^{\sin x}) \, dx + [16x - \sin (y^3 + 5y)] \, dy \), where \( C \) is the circle of radius 4 centered at the point \((2, -8)\), as shown below.

\[
\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} \int_R \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right) N \\
\int_R (16 - 6) \, dy \\
10 \int_R e^y \, dy = 10 \cdot 16\pi \\
= 160\pi
\]
Evaluate the indicated line integral, using Green's Theorem.
\[ \oint_C (x^2 - 8y) \, dx + y^2 \, dy \], where \( C \) is the circle \( x^2 + y^2 = 36 \) oriented counterclockwise.

\[ \frac{\partial M}{\partial x} = 0 \quad \frac{\partial N}{\partial y} = -8 \]

\[ = 8 \iint_R dv = 8 \cdot 36\pi = 836.8 \]
Evaluate the line integral \( \int_C x^2 \, dx - x^3 \, dy \) where \( C \) is the square contour from (0,0) to (0,1) to (1,1) to (1,0) using Green's Theorem.

Your Answer:

\[
\begin{align*}
\frac{2N}{\partial x} &= -3x^2 & \frac{2M}{\partial y} &= 0 \\
\iint_{0}^{1} -3x^2 \, dy \, dx &= \int_{0}^{1} \int_{0}^{1} -3x^2 \, dy \, dx \\
&= -x^3 \bigg|_{0}^{1} = -1 \quad \text{(Change to 1)}
\end{align*}
\]

Use Green's theorem to evaluate the line integral

\[
\int_C (\tan x - 3y^3) \, dx + (3x^3 - \sin y) \, dy,
\]

where \( C \) is the circle \( x^2 + y^2 = 2 \).

\[
\begin{align*}
\frac{2N}{\partial x} &= 9x^2 & \frac{2M}{\partial y} &= -9y^2 \\
9 \iint_{R} (x^2 + y^2) \, dx \, dy &= 9 \int_{0}^{2\pi} \int_{0}^{\sqrt{2}} r^2 \, r \, dr \, d\theta \\
&= 9 \cdot 2\pi \cdot \frac{r^4}{4} \bigg|_{0}^{\sqrt{2}} \\
&= 18\pi
\end{align*}
\]