The distance from a point \((x, y, 4 - x^2 - y^2)\) to the point \((5, -3, 1)\) is
\[
d(x, y) = \sqrt{(x - 5)^2 + (y + 3)^2 + (4 - x^2 - y^2)^2}.
\]
To minimize this it is useful to note that we can minimize \(g(x, y) = d(x, y)^2 = D(x, y)\) instead.

\[
\begin{align*}
\sigma_x &= 2(x - 5) - 4x(4 - x^2 - y^2) = 0 \\
\sigma_y &= 2(y + 3) - 4y(4 - x^2 - y^2) = 0 \\
\sigma_{xx} &= -10 + 12x^2 + 4y^2 \\
\sigma_{xy} &= -10 + 12x^2 + 4x^2 \\
\sigma_{yy} &= 8y
\end{align*}
\]
\(
\sigma_x = \sigma_y = 0 \text{ numerically yields } (1.7073, -1.0244).
\)
\(
D(1.7073, -1.0244) = 15.6747 \text{ and } \sigma_{xx}(1.7073, -1.0244) = 29.1761.
\)

Therefore this point is a minimum.

The closest point on the paraboloid to the point \((5, -3, 1)\) is \((1.7073, -1.0244, 0.0365)\).
A box is to be constructed out of 96 square feet of material. Find the dimensions $x$, $y$, and $z$ that maximize the volume of the box.

$$V = x \cdot y \cdot z$$

$$x^2 + 2xy + y^2 = 48$$

$$\nabla V = \langle y^2, x^2, xy \rangle$$

$$\nabla g = \langle y + x + 2z, x + y, z \rangle$$

$$\nabla V = \lambda \nabla g$$

$$x^2 + y^2 = \lambda (y + 2)$$

$$x^2 + y^2 = \lambda (x + 2)$$

$$z^2 = \lambda (x + y)$$

$$\lambda (x^2 + y^2 + z^2) = \lambda (x^2 + y^2 + z^2)$$

$$x = y = z$$

$$x = y = z$$

$$x = y = z$$
For a business that produces three products, suppose that when producing $x$, $y$, and $z$ thousand units of the products, the profit of the company (in thousands of dollars) can be modeled by $P(x, y, z) = 2x - 3y + 5z$. Manufacturing constraints force $x^2 + 4y^2 + z^2 \leq 800$. Find the maximum profit for the company. Round your answers to two decimal places. The maximum profit is 

\[ \nabla f = \langle 2, 9, 7 \rangle \quad \nabla g = \langle 2x, 8y, 4z \rangle \]

\[
\begin{align*}
2 &= 2x\lambda & x &= \lambda \\
9 &= 8y\lambda & y &= \frac{9}{8}\lambda \\
7 &= 4z\lambda & z &= \frac{7}{4}\lambda \\

\left(\frac{1}{\lambda}\right)^2 + 4\left(\frac{9}{8\lambda}\right)^2 + 2\left(\frac{7}{4\lambda}\right)^2 &= 800
\end{align*}
\]

Setting $\nabla f = \lambda \nabla g + \mu \nabla h$,

Minimize $f(x, y, z) = x^2 + y^2 + z^2$, subject to the constraints $x - 2y + 4z = 9$ and $y + z = 0$.
DEFINITE INTEGRAL--

\[ \text{Area} = \int_a^b \]

INDEFINITE--

\[ f(x) = \int f(x) \, dx \quad \text{Antiderivative} \]

\[ V = \iint_A f(x, y) \, dA \]

\[ \iint_{A} \int_{x_{0}}^{x_{1}} \int_{y_{0}}^{y_{1}} f(x, y) \, dx \, dy \]
If \( R = \{(x, y) | 0 \leq x \leq 2 \text{ and } 1 \leq y \leq 4\} \), evaluate

\[
\int_R \int (6x^2 + 7xy^3) \, dA.
\]

We have

\[
\int_R \int (6x^2 + 7xy^3) \, dA = \int_1^4 \int_0^2 (6x^2 + 7xy^3) \, dx \, dy
\]

\[
= \int_1^4 \left[ \int_0^2 (6x^2 + 7xy^3) \, dx \right] \, dy
\]

\[
= \int_1^4 \left( \frac{6x^3}{3} + 7\frac{x^2y^3}{2} \right)|_{x=0}^{x=2} \, dy
\]

\[
= \int_1^4 (16 + 14y^3) \, dy
\]

\[
= \left[ 16y + 14y^4 \right]_1^4
\]

Let \( R \) be the region bounded by the graphs of \( y = \sqrt{x} \), \( x = 0 \) and \( y = 7 \). Evaluate

\[
\int_R \int (8xy^2 + x \cos x) \, dA.
\]

Round your answer to one decimal place.

The value is _________.