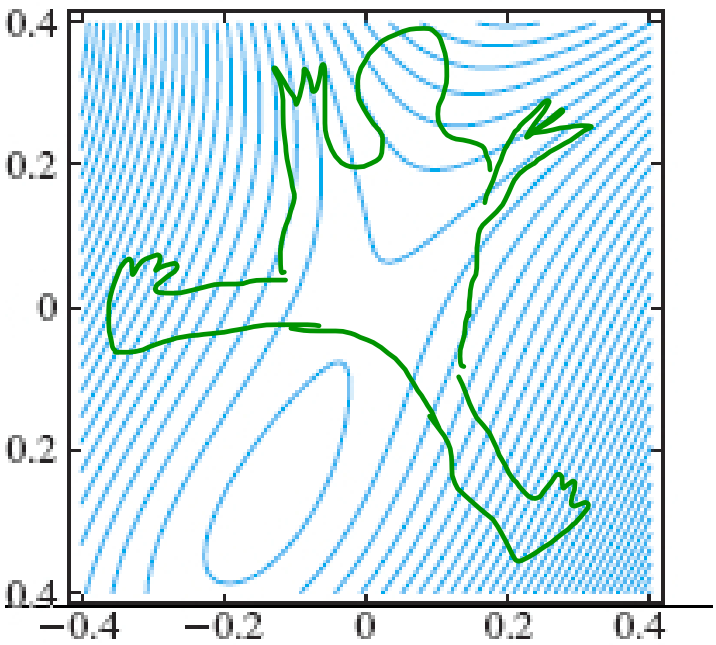



The distance from a point $(x, y, 4 - x^2 - y^2)$ to the point $(5, -3, 1)$ is

$d(x, y) = \sqrt{(x-5)^2 + (y+3)^2 + (3 - x^2 - y^2)^2}$.

To minimize this it is useful to note that we can minimize $g(x, y) = d(x, y)^2 = D(x, y)$ instead.

$$g_x = 2(x-5) - 4x(3 - x^2 - y^2) = 0$$
$$g_y = 2(y+3) - 4y(3 - x^2 - y^2) = 0$$
$$g_{xx} = -10 + 12x^2 + 4y^2$$
$$g_{yy} = -10 + 12y^2 + 4x^2$$
$$g_{xy} = 8xy$$

$D = g_{xx}g_{yy} - g_{xy}^2$

$g_x = g_y = 0$ numerically yields $(1.7073, -1.0244)$.

$D(1.7073, -1.0244) \approx 15.6747$ and $g_{xx}(1.7073, -1.0244) \approx 29.1761$.

Therefore this point is a minimum.

The closest point on the paraboloid to the point $(5, -3, 1)$ is $(1.7073, -1.0244, 0.0357)$.

$$\frac{2(x-5)}{4x} = \frac{2(y+3)}{4y}$$

$$\frac{x-5}{x} = \frac{y+3}{y}$$

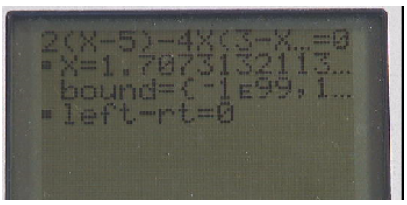
$$1 - \frac{5}{x} = 1 + \frac{3}{y}$$

$$-\frac{5}{x} = \frac{3}{y}$$

$$-5y = 3x$$

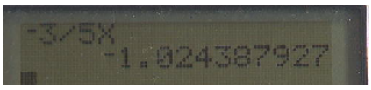
$$y = -\tfrac{3}{5}x$$

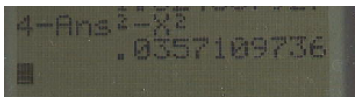
$$2(x-5) - 4x\left(3 - x^2 - \tfrac{9}{25}x^2\right) = 0$$




$x = 1.707..$

$y = -1.02...$



**A box is to be constructed out of 96 square feet of material. Find the dimensions $x$, $y$, and $z$ that maximize the volume of the box.**

$x =$ [ ], $y =$ [ ], and $z =$ [ ] maximize the volume.


$V = xyz$ $\qquad x = y = 4$

$2xz + 2yz + 2xy = 96$

$xz + yz + xy = 48$

$z = \dfrac{48 - xy}{x+y} = \dfrac{48-16}{8} = 4$

$V = xy\left(\dfrac{48-xy}{x+y}\right)$

$V = \dfrac{48xy - x^2y^2}{x+y}$

$V_x = \dfrac{(x+y)(48y - 2xy^2) - (48xy - x^2y^2)}{\cancel{(x+y)^2}} = 0$

$V_y = \dfrac{(x+y)(48x - 2x^2y) - (48xy - x^2y^2)}{} = 0$

$48y - 2xy^2 = 48x - 2x^2y$

$24y - xy^2 = 24x - x^2y$

$y(24 - xy) = x(24 - xy)$

$x = y$

$2x(48x - 2x^3) - (48x^2 - x^4) = 0$

$x = 4$


**A box is to be constructed out of 96 square feet of material. Find the dimensions $x$, $y$, and $z$ that maximize the volume of the box.**

$x =$ [4], $y =$ [4], and $z =$ [4] maximize the volume.

$xy + yz + xz = 48 \qquad V = xyz$

$\nabla V = \langle yz, xz, xy \rangle$

$\nabla g = \langle y+z, x+z, x+y \rangle$

$\nabla V = \lambda \nabla g$

$x \quad yz = \lambda(y+z) \quad x$

$y \quad xz = \lambda(x+z) \quad y$

$z \quad xy = \lambda(x+y) \quad z$

$\underbrace{\lambda(xy+xz) = \lambda(xy+yz)}_{} = \lambda(xz+yz)$

$xz = yz \qquad y = z$

$x = y \qquad x = y = z$

$3x^2 = 48$

$x^2 = 16$

$x = 4$

For a business that produces three products, suppose that when producing $x$, $y$, and $z$ thousand units of the products, the profit of the company (in thousands of dollars) can be modeled by $P(x, y, z) = 2x + 9y + 7z$. Manufacturing constraints force $x^2 + 4y^2 + 2z^2 \le 800$. Find the maximum profit for the company. Round your answers to five decimal places.

The maximum profit is $\boxed{\phantom{00000}}$.

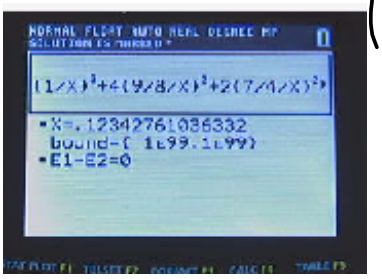
$\nabla P = \langle 2, 9, 7 \rangle \quad \nabla g = \langle 2x, 8y, 4z \rangle$

$2 = 2\lambda x \qquad x = 1/\lambda$

$9 = 8\lambda y \qquad y = 9/8\lambda$

$7 = 4\lambda z \qquad z = 7/4\lambda$

$(1/\lambda)^2 + 4(9/8\lambda)^2 + 2(7/4\lambda)^2 = 800$




$x = \frac{1}{\lambda} \approx 8.10307$, $y = \frac{1.125}{\lambda} \approx 9.11595$, and $z = \frac{1.75}{\lambda} \approx 14.18037$,

Setting $\nabla f = \lambda \nabla g + \mu \nabla h$ ;

Minimize $f(x, y, z) = x^2 + y^2 + z^2$, subject to the constraints $x + 2y + 4z = 9$ and $y + z = 0$.

DEFINITE INTEGRAL--

$$\text{Area} = \int_a^b$$

INDEFINITE--

$$f(x) = \int f(x)\,dx \longleftarrow \text{Anti-derivative}$$

$$V = \iint_A f(x,y)\,dA$$

Volume

$$\int_{y=} \boxed{\int_{x=} f(x,y)\,dx}\,dy$$

If $R = \{(x, y) \mid 0 \le x \le 2 \text{ and } 1 \le y \le 4\}$, evaluate

$$\int_R \int (6x^2 + 7xy^3)\, dA.$$

We have

$$\int_R \int (6x^2 + 7xy^3)\, dA = \int_1^4 \int_0^2 (6x^2 + 7xy^3)\, dx\, dy$$

$$= \int_1^4 \left[ \int_0^2 (6x^2 + 7xy^3)\, dx \right] dy$$

$$= \int_1^4 \left( 6\frac{x^3}{3} + 7\frac{x^2}{2} y^3 \right) \Big|_{x=0}^{x=2} dy$$

$$= \int_1^4 (16 + 14y^3)\, dy$$

$$= 16y + \frac{14y^4}{4} \Big|_1^4$$

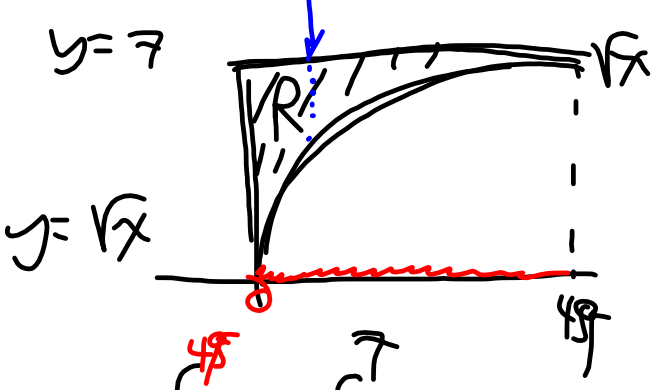
Let $R$ be the region bounded by the graphs of $y = \sqrt{x}$, $x = 0$ and $y = 7$. Evaluate

$$\int_R \int (8xy^2 + 2y \cos x)\, dA.$$

Round your answer to one decimal place.

The value is $\boxed{\phantom{000}}$.

$$\iint 8xy^2 + 2y\cos x \, dA$$




$$\int_{x=0}^{49} \int_{y=\sqrt{x}}^{7} f(x,y)\, dy$$