Agenda

Lecture: Limits, Functions, Continuity, Differentiability

Review: Quizzes for Test 1

Multivariable Functions

\[ z = f(x,y) \text{ in 3D} \]
\[ w = f(x,y,z) \text{ in 4D} \]
Data
\( w(x, y, z) \)
\( (1, 2, 3) \rightarrow 2 \)
\( (1, 2, 3) \rightarrow 4 \)

Graph
\( \text{Not} \)

Equation
\( z^2 = x^2 + y^2 \)

Limits
\( \lim_{(x, y) \to (1, 1)} \frac{x^2 - y^2}{x - y} = ? \)
\( \lim_{x \to 1} y = x \)
\( \lim_{y \to x^2} y = x^2 \)
Derivatives

Partial Derivatives

\[ \frac{df}{dx} \]

To exist
- Continuous
- No corners
- No cusps

\[ y = f(x) \]

Slope of Tangent Line

\[ \frac{dz}{dx} \quad \frac{dz}{dy} \]
A paperboy is riding at 12 ft/s on a bicycle and tosses a paper over his left shoulder at 54 ft/s. If the porch is 54 ft off the road, how far up the street should the paperboy release the paper to hit the porch? He should release the paper \( <12,0> \) feet up the road.
2. Award: 10.00 points  Problems? Adjust credit for all students.

Identify the geometric shape described by the given equation.

\[(x - 2)^2 + y^2 + (z + 2)^2 - 64 = 0\]

- A. A sphere of radius 8 and center \((2, 0, -2)\).
- B. A sphere of radius 0 and center \((-2, 0, 2)\).
- C. A sphere of radius 8 and center \((-2, 0, 2)\).
- D. A sphere of radius 8 and center \((2, 0, -2)\).

3. Award: 10.00 points  Problems? Adjust credit for all students.

Compute the dot product \(a \cdot b\) for \(a = 7i - 4k\) and \(b = -4i + 4k\).

\[a \cdot b = \]

\[<7, 0, -4> \cdot <-4, 0, -4>\]

\[-28 + 0 + 16 = -12\]
4. Award: 10.00 points  Problems? Adjust credit for all students.

Find the volume of the parallelepiped with three adjacent edges formed by the vectors 
\( \mathbf{a} = \langle 3, 7, 8 \rangle \), \( \mathbf{b} = \langle 4, 6, 5 \rangle \) and \( \mathbf{c} = \langle 4, 8, 0 \rangle \).
The volume of the parallelepiped is 
\[ || \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) || \]

\[
\begin{vmatrix}
4 & 8 & 0 \\
3 & 7 & 8 \\
4 & 6 & 5
\end{vmatrix}
\]

\[ 4(35 - 48) - 8(15 - 32) + 0 \]
\[ -52 + 136 = 84 \]

5. Award: 10.00 points  Problems? Adjust credit for all students.

Please round your answer to three decimals and remember answer is in radians.

Use the cross product to determine the angle between \( \mathbf{a} = 4\mathbf{i} + 2\mathbf{k} \) and \( \mathbf{b} = 3\mathbf{j} + 7\mathbf{k} \).

Your Answer: 

\[
\begin{vmatrix}
i & j & k \\
4 & 0 & 2 \\
0 & 2 & 7
\end{vmatrix} = <-4, -28, 8>
\]

\[ \cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}|| ||\mathbf{b}||} \]

\[ \cos t = \frac{14}{\sqrt{20} \cdot 53} \]

\[ t = \arccos \left( \frac{14}{\sqrt{1060}} \right) \]

\[ t = 64.53...\text{degrees} \]

\[ 1.126 \text{ rad} \]
6. Award: 10.00 points. Problems? Adjust credit for all students.

Find the parametric equations for the line passing through \((-2, 1, 3)\) and normal to the plane \(3x - 5y + 5z = 10\).

\[ x = \boxed{\quad} \quad y = \boxed{\quad} \quad z = \boxed{\quad} \]

normal = \(<3, -5, 5>\)

\[ x = -2 + 3t \quad y = 1 - 5t \quad z = 3 + 5t \]

7. Award: 10.00 points. Problems? Adjust credit for all students.

Find the distance between the point \((4, 0, 8)\) and the plane \(7x - y + 2z = 28\).

\[
\frac{|Ax + By + Cz - D|}{\sqrt{A^2 + B^2 + C^2}} = \frac{28 + 16 - 28}{\sqrt{54}}
\]
8. Sketch the graph of the quadric surface defined by the equation
\[ z = 2y^2 - x^2. \]

9. Find parametric equations for the surface \( z = 10 - x^2 - y^2 \).

- A. \( z = \frac{1}{10} - s^2 \), \( x = \frac{1}{4} \cos t \), and \( y = \frac{1}{4} \sin t \)
- B. \( z = 10 - s^2 \), \( x = s \cos t \), and \( y = s \sin t \)
- C. \( z = 10 - s^2 \), \( x = \frac{1}{4} \cos t \), and \( y = \frac{1}{4} \sin t \)
- D. \( z = \frac{1}{10} - s^2 \), \( x = s \cos t \), and \( y = s \sin t \)

elim. makes no sense
\[ z = 10 - (s \cos t)^2 - (s \sin t)^2 \]
\[ 10 - s^2 \ (1) \]
Find \( \lim_{t \to 0} \left\{ e^{2t} + 6, t^2 + 2t - 7, \frac{5}{t} \right\} = \langle 1, -7, 5 \rangle \).

- A. \( \langle 12, -5, 5 \rangle \)
- B. \( \langle 6, -7, 5 \rangle \)
- C. \( \langle 6, 7, 0 \rangle \)
- D. The limit does not exist.

11. Award: 10.00 points  Problems? Adjust credit for all students.

Find the derivative of \( r(t) = \langle \cos 9t, \tan t, 2 \sin t \rangle \).

\[ \langle -9 \sin 9t, \sec^2 t, 2 \cos t \rangle \]
12. Award: 10.00 points  Problems? Adjust credit for all students.

Choose the sketch of the curve traced out by the given vector-valued function.

\[ r(t) = (-1, 2\cos t, 2\sin t) \]

\[ 4\cos^2 t + 4\sin^2 t = 4 \]

\[ x = -1 \]

\[ y^2 + z^2 = 4 \]

Select the correct answer for \( t \) such that \( r(t) \) and \( r'(t) \) are perpendicular.

\[ r'(t) = \langle -3\sin t, 9\cos t \rangle \]

- A. \( r(t) \perp r'(t) \) when \( t = \frac{n\pi}{2} \) for any even integer \( n \).
- B. \( r(t) \perp r'(t) \) when \( t = \frac{n\pi}{4} \) for any integer \( n \).
- C. \( r(t) \perp r'(t) \) when \( t = \frac{n\pi}{4} \) for any integer \( n \).
- D. \( r(t) \perp r'(t) \) when \( t = n\pi \) for any integer \( n \).

\[ -9\cos t\sin t + 81\cos t\sin t = 0 \]

\[ \cos t\sin t = 0 \]
Choose the sketch of the curve and estimate its arc length.

\[ s = \int_{0}^{2} \sqrt{1 + (2t)^2 + (3t^2)^2} \, dt \]

\[ r(t) = \begin{cases} \sqrt{t} + y \cdot z^2 \, dt & \text{if } 0 \leq t \leq 2 \\ \sqrt{1 + 4t^3 + 9t^4} \, dt & \text{if } t \geq 2 \end{cases} \]

\[ \int_{\text{int} \left( \left( 1 + 4x^2 + 9y^2 \right) \right)} \left( \frac{y - 1}{\sqrt{1 + 4x^2 + 9y^2}} \right) \, dx \]

\[ C = \int_{0}^{2} \frac{1}{x^{3/2}} \, dx \]

\[ v(t) = \langle -16e^{-4t}, 6t^2, -\sin t \rangle + C \]

\[ v(0) = \langle -16, 0, 0 \rangle + C = \langle 4, 0, 4 \rangle \]

\[ C = \langle 20, 0, 4 \rangle \]

\[ v(t) = \langle -16e^{-4t} + 20t, 2t^3, -\sin t + 4 \rangle \]

\[ r(t) = \langle 4e^{-4t} + 20t, 2t^3, \cos t \rangle + C \]

\[ r(0) = \langle 9, 0, -7 \rangle \]

\[ \langle 4, 0, 1 \rangle + \langle 1, 1, 1 \rangle = \langle 5, 1, 2 \rangle \]
16. Award: 10.00 points  Problems? Adjust credit for all students.

Select an arc length parameterization of the circle of radius 1, centered at the origin.

- A. \( C : x = \sin \left( \frac{s}{2} \right), y = 2 \cos \left( \frac{s}{4} \right), 0 \leq s \leq 2\pi \)
- B. \( C : x = \cos \left( \frac{s}{4} \right), y = 2 \cos \left( \frac{s}{2} \right), 0 \leq s \leq 2\pi \)
- C. \( C : x = \sin \left( \frac{s}{4} \right), y = 2 \sin \left( \frac{s}{2} \right), 0 \leq s \leq 4\pi \)
- D. \( C : x = 2 \cos \left( \frac{s}{2} \right), y = 2 \sin \left( \frac{s}{2} \right), 0 \leq s \leq 4\pi \)

17. Award: 10.00 points  Problems? Adjust credit for all students.

Select the graph of the parametric surface.

\[
x = u \cos v, \quad y = u \sin v, \quad z = u^2
\]

\[
u^2 = \frac{x^2 + y^2}{2}
\]

- A. paraboloid

\[
z = x^2 + y^2
\]
18. Award: 10.00 points  Problems? Adjust credit for all students.

Select the parametric equation for the surface.

\[ x = v, \quad y = u \cos v, \quad z = u \sin v \] is the parametric equation for the surface, since the position along the x-axis determines the direction of a line in the yz cross section.

19. Award: 10.00 points  Problems? Adjust credit for all students.

Select parametric equations for the plane through the point \((5, 3, 10)\) and containing the vectors \(\langle 4, -1, 2 \rangle\) and \(\langle 9, 9, 3 \rangle\).

- A. \( r = \langle 0, 0, 10 \rangle + u \langle 6, -2 \rangle + v \langle -1, -7, -8 \rangle \)
- B. \( r = \langle -1, -7, 6 \rangle + u \langle 5, 3, 10 \rangle + v \langle 4, 6, -2 \rangle \)
- C. \( r = \langle 4, 6, -2 \rangle + u \langle 5, 3, 10 \rangle + v \langle -1, -7, -8 \rangle \)
- D. \( r = \langle 5, 3, 10 \rangle + u \langle -1, -7, -8 \rangle + v \langle 4, 6, -2 \rangle \)