Agenda

Lecture: Limits, Functions, Continuity, Differentiability

Review: Quizzes for Test 1

Multivariable Functions

$A(l, w)$

z = f(x,y) in 3D

Like $y = f(x)$

w = f(x,y,z) in 4D

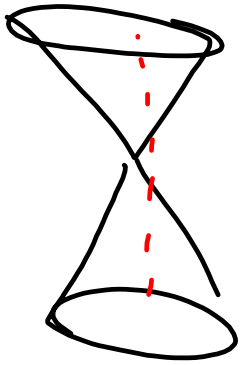
Data $w(x,y,z)$

$(1,2,3) \to 2$

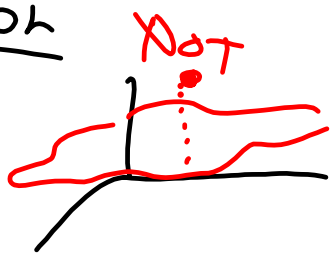
$(1,2,3) \to 4$ NOT

Eqn

$z^2 = x^2 + y^2$

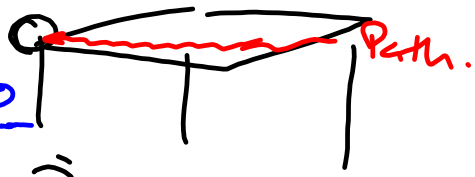


Graph



Limits

$$\lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y} = 2 \;??$$

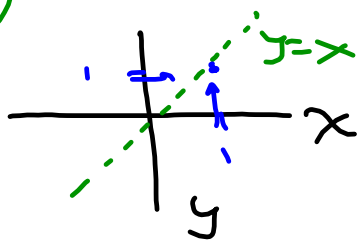


$\lim = 0$

$\lim = 1$

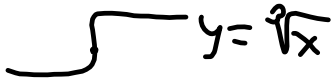
$\lim_{x\to 1}$ $y = x$

$y = x^2$



Derivatives

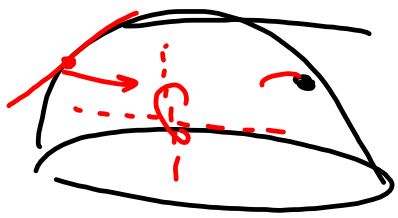
To exist
1. Continuous
2. No Corners
3. No Cusps.



Partial Derivatives

$$\frac{\partial f}{\partial x} \qquad f_x$$

Slope of Tangent Line



$$\frac{\partial z}{\partial x} \qquad \frac{\partial z}{\partial y}$$

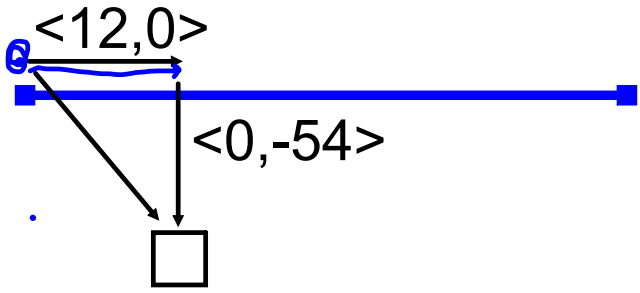
Review Problems

1. Award: 10.00 points Problems? Adjust credit for all students.

1 out of 1 attempt

A paperboy is riding at 12 ft/s on a bicycle and tosses a paper over his left shoulder at 54 ft/s. If the porch is 54 ft off the road, how far up the street should the paperboy release the paper to hit the porch?

He should release the paper 12 feet up the road.



2. Award: 10.00 points Problems? Adjust credit for all students.

Identify the geometric shape described by the given equation.

$(x-2)^2 + y^2 + (z+2)^2 = 64$

0

- A. A sphere of radius 64 and center (2, 0, −2).
- B. A sphere of radius 64 and center (−2, 0, 2).
- C. A sphere of radius 8 and center (−2, 0, 2).
- D. A sphere of radius 8 and center (2, 0, −2).

3. Award: 10.00 points Problems? Adjust credit for all students.

Compute the dot product a • b for a = 7i − 4k and b = −4i − 4k.

a • b = [ ]

<7,0,-4>dot<-4,0,-4>

-28 + 0 + 16 = -12

4. Award: 10.00 points Problems? Adjust credit for all students.

1 out of 1 attempt

Find the volume of the parallelepiped with three adjacent edges formed by the vectors $a = \langle 3, 7, 8 \rangle$, $b = \langle 4, 6, 5 \rangle$ and $c = \langle 4, 8, 0 \rangle$.

The volume of the parallelepiped is 84 .

$$\| \vec{c} \text{ dot } (\vec{a} \times \vec{b}) \|$$

$$\det \begin{vmatrix} 4 & 8 & 0 \\ 3 & 7 & 8 \\ 4 & 6 & 5 \end{vmatrix}$$

$$4 (35 - 48) - 8 (15 - 32) + 0$$

$$- 52 + 136 = 84$$

5. Award: 10.00 points Problems? Adjust credit for all students.

1 out of 1 atte

Please round your answer to three decimals and remember answer is in radians.

Use the cross product to determine the angle between $a = 4i + 2k$ and $b = 2j + 7k$.

Your Answer:

$$\begin{vmatrix} i & j & k \\ 4 & 0 & 2 \\ 0 & 2 & 7 \end{vmatrix} = \langle -4, -28, 8 \rangle$$

$$\cos t = \frac{(\vec{a} \text{ dot } \vec{b})}{|\vec{a}|\,|\vec{b}|}$$

$$|\vec{a}| = \|\vec{a}\|$$

$$\cos t = 14 / \sqrt{(20)(53)}$$

$$t = \arccos (14/\sqrt{1060})$$

$$t = 64.53\ldots \text{degres}$$

$$1.126 \text{ rad}$$

6. Award: 10.00 points Problems? Adjust credit for all students.

1 out of 1 attempt

Find the parametric equations for the line passing through $(-2, 1, 3)$ and normal to the plane $3x - 5y + 5z = 10$.

$x =$, $y =$, $z =$

normal = < 3, -5, 5>

x = -2 + 3t y = 1 - 5t z = 3 + 5t

7. Award: 10.00 points Problems? Adjust credit for all students.

1 out

Find the distance between the point $(4, 0, 8)$ and the plane $7x - y + 2z = 28$.

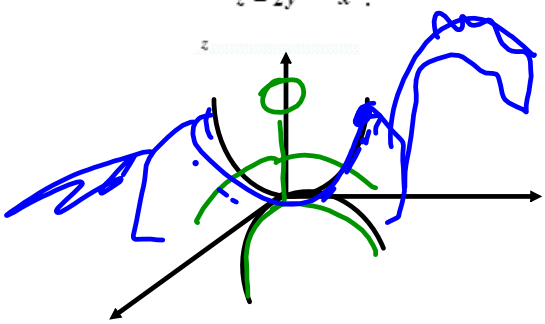
$$\frac{|Ax + By + Cz - D|}{\sqrt{A^2 + B^2 + C^2}}$$

$$\frac{28+16-28}{\sqrt{54}}$$

8. Award: 10.00 points Problems? Adjust credit for all students.

Sketch the graph of the quadric surface defined by the equation

$$z = 2y^2 - x^2.$$



hyperbolic paraboloid

9. Award: 10.00 points Problems? Adjust credit for all students.

Find parametric equations for the surface $z = 10 - x^2 - y^2$. $= 10 - (x^2+y^2) = 10 - s^2$

A. $z = \frac{1}{10} - s^2$, $x = \frac{1}{4}\cos t$, and $y = \frac{1}{4}\sin t$

B. $z = 10 - s^2$, $x = s\cos t$, and $y = s\sin t$

C. $z = 10 - s^2$, $x = \frac{1}{4}\cos t$, and $y = \frac{1}{4}\sin t$

D. $z = \frac{1}{10} - s^2$, $x = s\cos t$, and $y = s\sin t$

$x^2 = s^2\cos^2 t$ $y^2 = s^2\sin^2 t$

$x^2 + y^2 = s^2$

elim. makes no sense

$z = 10 - (s\cos t)^2 - (s\sin t)^2$

$10 - s^2(1)$

10

Find $\lim_{t \to 0} \langle e^{2t} + 6,\ t^2 + 2t - 7,\ \frac{5}{t} \rangle$. $= \langle 7, -7, \frac{5}{0} \rangle$

- A. $\langle 12, -5, 5 \rangle$
- B. $\langle 6, -7, 5 \rangle$
- C. $\langle 6, 7, 0 \rangle$
- D. The limit does not exist.

11. Award: 10.00 points Problems? Adjust credit for all students.

Find the derivative of $\mathbf{r}(t) = \langle \cos 9t,\ \tan t,\ 2 \sin t \rangle$.

$\langle \square, \square, \square \rangle$

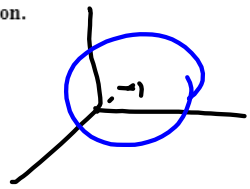
$\langle -9\sin 9t,\ \sec^2 t,\ 2\cos t \rangle$

12. Award: 10.00 points Problems? Adjust credit for all students.

1 out of 1

Choose the sketch of the curve traced out by the given vector–valued function.

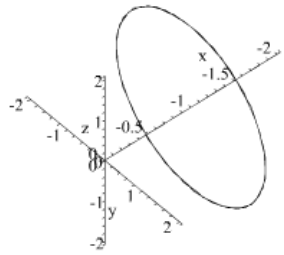
$r(t) = \langle -1, 2\cos t, 2\sin t \rangle$



$4\cos^2 t + 4\sin^2 t$

$x = -1$ $y^2 + z^2 = 4$

B.



13. Award: 10.00 points Problems? Adjust credit for all students.

1 out c

Select the correct answer for t such that $r(t)$ and $r'(t)$ are perpendicular.

$r(t) = \langle 3\cos t, 9\sin t \rangle$

$r'(t) = \langle -3\sin t, 9\cos t \rangle$

A. $r(t) \perp r'(t)$ when $t = \frac{n\pi}{2}$ for any even integer n.

B. $r(t) \perp r'(t)$ when $t = \frac{n\pi}{4}$ for any integer n.

C. $r(t) \perp r'(t)$ when $t = \frac{n\pi}{2}$ for any integer n.

D. $r(t) \perp r'(t)$ when $t = n\pi$ for any integer n.

perp means r dot r' = 0

$-9\cos t \sin t + 81\cos t \sin t = 0$

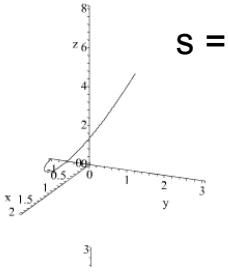
$\cos t \sin t = 0$

14. Award: 10.00 points Problems? Adjust credit for all students.

Choose the sketch of the curve and estimate its arc length.

$\mathbf{r}(t) = \langle t, t^2 - 1, t^3 \rangle, 0 \le t \le 2$

A.



$$s = \int_0^2 \sqrt{x'^2 + y'^2 + z'^2}\, dt$$

$$\int_0^2 \sqrt{1^2 + (2t)^2 + (3t^2)^2}\, dt$$

$$\int_0^2 \sqrt{1 + 4t^2 + 9t^4}\, dt$$

fnint(√(1+4X²+9X⁴),X,0,2)

Y₁ = √(1+4X²+9X⁴)

Calc 7: ∫f(x)dx

Lower: 0 Upper: 2 →

9.57..

15. Award: 10.00 points Problems? Adjust credit for all students.

Find the position function given.

$\mathbf{a}(t) = \langle 64e^{-4t}, 12t, -\cos t \rangle$

$\mathbf{v}(0) = \langle 4,0,4 \rangle$

$\mathbf{r}(0) = \langle 9, 0, -7 \rangle$

A. $\langle 16e^{-4t} + 5, 2t^3, \cos t + 4t + 8 \rangle$

B. $\langle 4e^{-4t} + 20t + 5, 2t^3, -\sin t + 4t + 8 \rangle$

C. $\langle 4e^{-4t} + 5, 2t^3, \cos t + 4t + 8 \rangle$

D. $\langle 4e^{-4t} + 20t + 5, 2t^3, \cos t + 4t - 8 \rangle$

$v(t) = \langle -16e^{-4t}, 6t^2, -\sin t \rangle + C$

$v(0) = \langle -16, 0, 0 \rangle + C = \langle 4, 0, 4 \rangle$

$C = \langle 20, 0, 4 \rangle$

$v(t) = \langle -16e^{-4t} + 20, 6t^2, -\sin t + 4 \rangle$

$r(t) = \langle 4e^{-4t} + 20t, 2t^3, \cos t + 4t \rangle + C$

$r(0) = \langle 9, 0, -7 \rangle$

$\langle 4, 0, 1 \rangle + \langle c_1, c_2, c_3 \rangle = \langle 5, 0, -8 \rangle$

16. Award: 10.00 points Problems? Adjust credit for all students.

1 out of 1 attempt

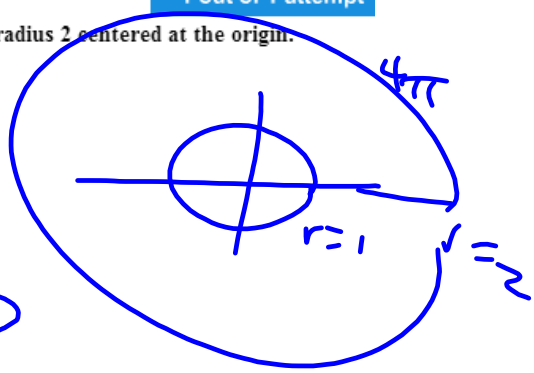
Select an arc length parameterization of the circle of radius 2 centered at the origin.

A. $C: x = 4\sin\left(\frac{s}{2}\right),\ y = 2\cos\left(\frac{s}{4}\right),\ 0 \le s \le 2\pi$

B. $C: x = 4\cos\left(\frac{s}{4}\right),\ y = 2\cos\left(\frac{s}{2}\right),\ 0 \le s \le 2\pi$

C. $C: x = 4\sin\left(\frac{s}{4}\right),\ y = 2\sin\left(\frac{s}{2}\right),\ 0 \le s \le 4\pi$

D. $C: x = 2\cos\left(\frac{s}{2}\right),\ y = 2\sin\left(\frac{s}{2}\right),\ 0 \le s \le 4\pi$



17. Award: 10.00 points Problems? Adjust credit for all students.

$(u\cos v)^2 + (u\sin v)^2 \rightarrow u^2(\cos^2 v + \sin^2 v)$

$(x)^2 + (y)^2 \rightarrow x^2 + y^2$

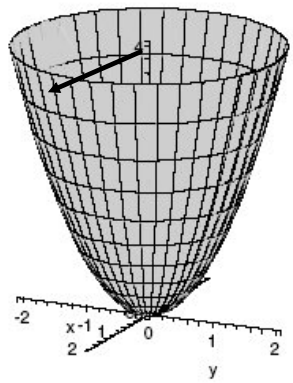
Select the graph of the parametric surface.

$x = u\cos v,\ y = u\sin v,\ z = u^2$

$u^2 = z$

$z = x^2 + y^2$

A.

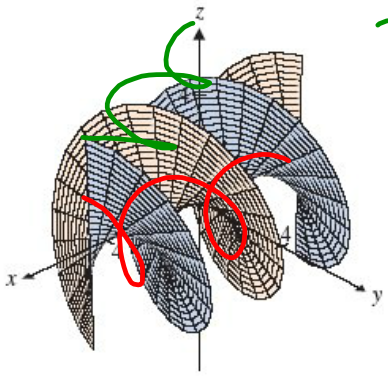


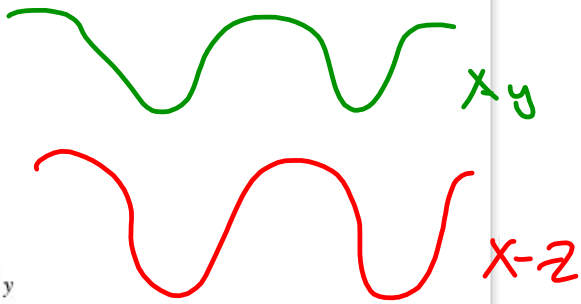
paraboloid

18. Award: 10.00 points Problems? Adjust credit for all students.

Show Me

Select the parametric equation for the surface.





$x = v$, $y = u \cos v$, $z = u \sin v$ is the parametric equation for the surface, since the position along the $x-$ axis determines the direction of a line in the yz cross section.

19. Award: 10.00 points Problems? Adjust credit for all students.

1 out of 1

Select parametric equations for the plane through the point $(5, 3, 10)$ and containing the vectors $\langle 4, -4, 2 \rangle$ and $\langle 9, 9, 8 \rangle$.

A. $\mathbf{r} = \langle 0, 0, 10 \rangle + u\langle 4, 6, -2 \rangle + v\langle -1, -7, -8 \rangle$

B. $\mathbf{r} = \langle -1, -7, -8 \rangle + u\langle 5, 3, 10 \rangle + v\langle 4, 6, -2 \rangle$

C. $\mathbf{r} = \langle 4, 6, -2 \rangle + u\langle 5, 3, 10 \rangle + v\langle -1, -7, -8 \rangle$

D. $\mathbf{r} = \langle 5, 3, 10 \rangle + u\langle -1, -7, -8 \rangle + v\langle 4, 6, -2 \rangle$

$\langle 4,-4,2\rangle - \langle 5,3,10\rangle$

$\langle -1,-7,-8\rangle$

$\langle 9,9,8\rangle - \langle 5,3,10\rangle$

$\langle 4,6,-2\rangle$