Match the vector–valued function

$$f(t) = \langle t \cos t,\ t \sin t,\ t \rangle$$

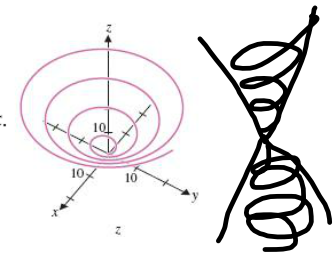
with the corresponding computer–generated graph.

$x^2 = t^2 \cos^2 t$

$y^2 = t^2 \sin^2 t$

$x^2 + y^2 = t^2 = z^2$

C.



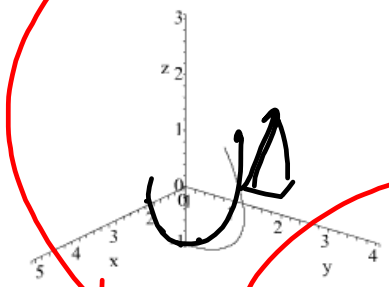
Find the derivative of $\mathbf{r}(t) = \langle \cos 3t,\ \tan t,\ 9 \sin t \rangle$.

$\langle \square, \square, \square \rangle$

$r'(t) = \langle -3\sin(3t),\ \sec^2(t),\ 9\cos(t) \rangle$

arclength

$\mathbf{r}(t) = \langle t^2+1, 2t, t^2-1 \rangle, 0 \le t \le 2$



$$S = \int_a^b \|\vec{r}\,'\|\,dt$$

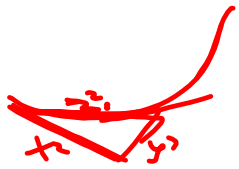
$$\mathbf{r}'(t) = \langle 2t, 2, 2t \rangle$$

$$s = \int_0^2 \sqrt{4t^2+4+4t^2}\,dt$$

$$\approx 7.25$$

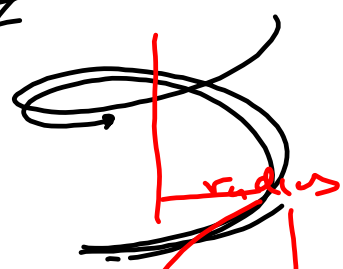
$$\sqrt{x'^2+y'^2+z'^2}$$

$$(2t)^2+2^2+(2t^2)$$



$$\sqrt{x^2+y^2} = |r'|$$

Ex Arclength.



$$r = \langle r\cos t, r\sin t, t \rangle$$

From $t=0$ to 2π

$$\|r'\| = \|\langle -\sin t, \cos t, 1 \rangle\|$$

$$\int_0^{2\pi} \sqrt{1+1}\,dt$$

$$\sqrt{2}\cdot 2\pi$$

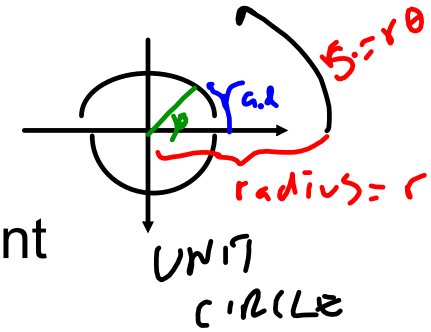
Lecture

s(t) position now $\overrightarrow{r(t)}$

s'(t) velocity now $\overrightarrow{v(t)} = \overrightarrow{r'(t)}$

s''(t) accerlation now $\overrightarrow{r''(t)}$

Parameterization:



circle x(t) = cost y(t) = sint

t is in radians

t (time)

s = arclength

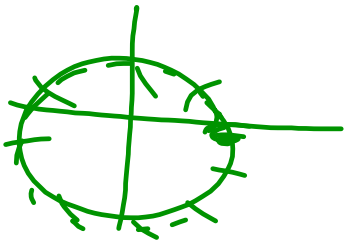
$s = r \cdot t$

$\frac{s}{r} = t$

1 x(s) = ~~r~~cos (s) y (s) = ~~r~~sin (s)

$2\pi r$

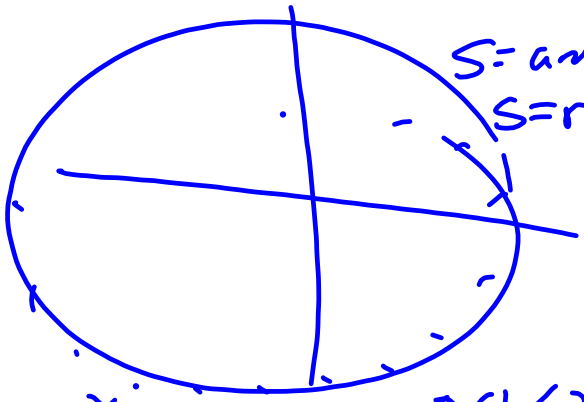
s = arclength



$x = \cos t$

$y = \sin t$

$0 \le t \le 2\pi$



$s = ar$

$s = rt$

$x = 7\cos t$ $\quad 0 \le t \le 2\pi$

$y = 7\sin t$

$x = 7\cos\left(\frac{s}{7}\right)$ $\quad 0 \le s \le 14\pi$

$y = 7\sin\left(\frac{s}{7}\right)$

In Radians

$y = \sin(t) \Rightarrow y' = \cos(t)$

In degrees

$y = \sin(at) \Rightarrow y' = a\cos(t)$

$a = \frac{\pi}{180}$ $\quad$ t in degrees $\quad$ $\frac{\pi}{180}$

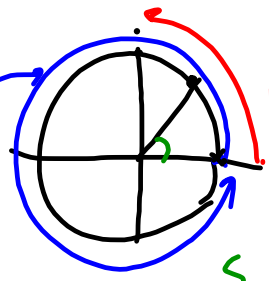
Find an arc length parameterization of the circle of radius 8 centered at the origin.

A. $C : x = 8\sin\left(\frac{s}{8}\right),\ y = 8\cos\left(\frac{s}{8}\right),\ 0 \le s \le 16\pi$

B. $C : x = 8\cos\left(\frac{s}{8}\right),\ y = 8\cos\left(\frac{s}{8}\right),\ 0 \le s \le 16\pi$

C. $C : x = 8\cos\left(\frac{s}{8}\right),\ y = 8\sin\left(\frac{s}{8}\right),\ 0 \le s \le 16\pi$

D. $C : x = 8\sin\left(\frac{s}{8}\right),\ y = 8\sin\left(\frac{s}{8}\right),\ 0 \le s \le 16\pi$

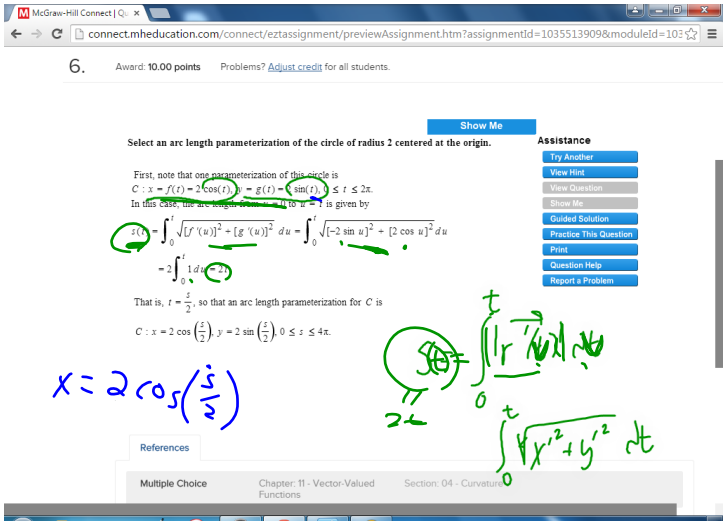


$0 \le s \le 4\pi$

$\frac{\pi}{2} \cdot 8$

radian $= \frac{s}{8}$

$t = \frac{s}{8}$



$x = 2\cos\left(\frac{s}{2}\right)$

$s(t) = \int_0^t |\vec{r}\,'(u)|\, du$

$= 2t$

$\int_0^t \sqrt{x'^2 + y'^2}\, dt$

$x = 2\cos t \quad y = 2\sin t$

$x' = -2\sin t \quad y' = 2\cos t$

$\sqrt{4\sin^2 t + 4\cos^2 t}$

$\sqrt{4} = 2$

$s = \int_0^t 2\, dt = s = 2t$

$\frac{s}{2} = t$

$(0,1,7) \to (1,0,8)$

$$\vec{r}(t) = \langle t, 1-t, t+7 \rangle$$

$$0 \le t \le 1$$

Replace $t = \frac{s}{\sqrt{3}}$

$$\vec{r}(s) = \left\langle \frac{s}{\sqrt{3}}, 1-\frac{s}{\sqrt{3}}, \frac{s}{\sqrt{3}}+7 \right\rangle$$

$\vec{r}(0) = \langle 0,1,7 \rangle$

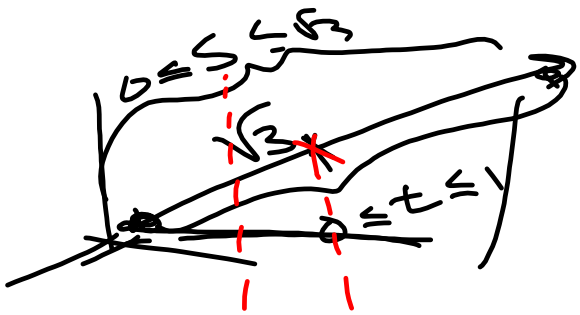
$0 \le s \le \sqrt{3}$

$$r' = \langle 1,-1,1 \rangle$$

$$s = \int_0^t \underbrace{\sqrt{1+1+1}}_{\|r'\|}\, dt$$

$$s = \sqrt{3}\, t$$

$$\frac{s}{\sqrt{3}} = t$$



$t = 1/2$

$s = \frac{\sqrt{3}}{2}$

$$\int_0^T \sqrt{4t^2+2}\,dt = s(T)$$

$$\frac{1}{2}T\sqrt{4T^2+2} + \frac{1}{2}\operatorname{arcsinh}(\sqrt{2}\,T) = s$$

$\|r'\|$

$T = f(s)$

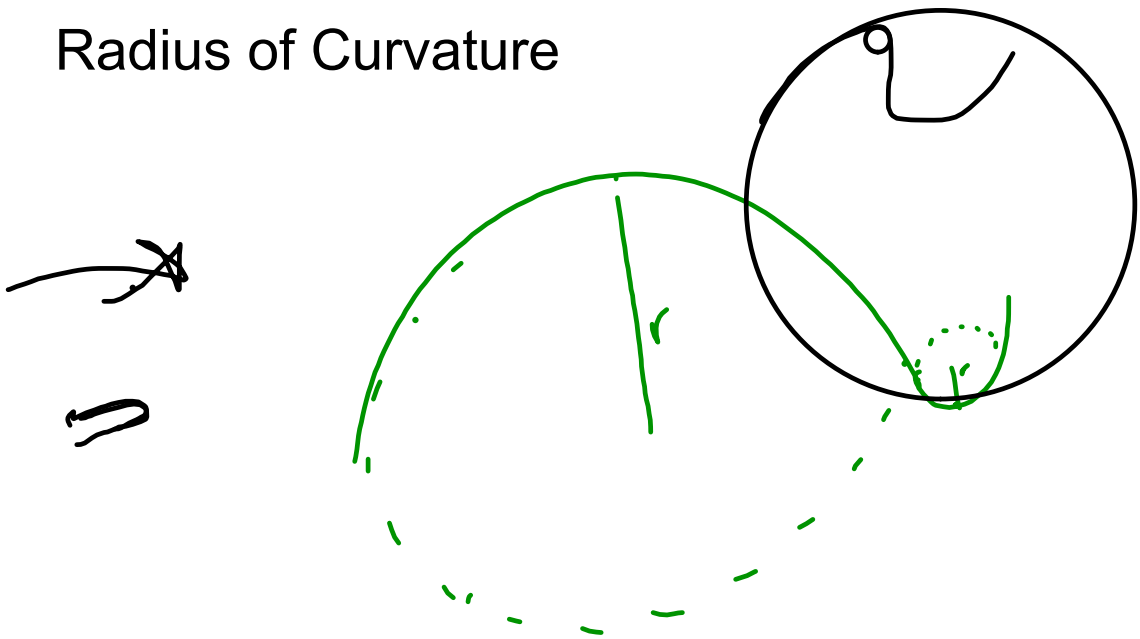
$r' = \langle 2t, \sqrt{2} \rangle$

$r = \langle t^2, \sqrt{2}t \rangle$

$x = t^2$ $\quad x = \frac{y^2}{2}$ $\quad x = \frac{1}{2}y^2$

$y = \sqrt{2}t$ $\quad t = y/\sqrt{2}$

Radius of Curvature



Show Me

Find the curvature of the straight line $\mathbf{r}(t) = \langle 8t + 4, 3t + 2, 5t + 7 \rangle$.

First, think about what we're asking. Straight lines are, well, straight, so their curvature should be zero at every point. Let's see. We have

$$\mathbf{r}'(t) = \langle 8, 3, 5 \rangle,$$

so that

$$\|\mathbf{r}'(t)\| = \sqrt{8^2 + 3^2 + 5^2} = \sqrt{98}.$$

The unit tangent vector is then

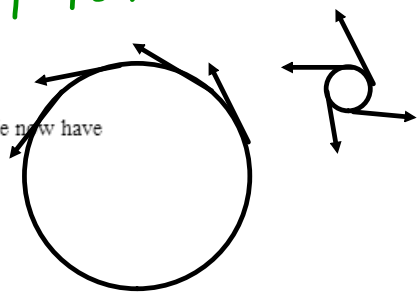
$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = \frac{\langle 8, 3, 5 \rangle}{\sqrt{98}},$$

which is a constant vector. This gives us $\mathbf{T}'(t) = \mathbf{0}$, for all t. We now have

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{0}\|}{\sqrt{98}} = 0,$$

as expected.

$T'(t) = 0$



Lit Search:

Vectors are lots of places.

Look in your STEMB? field for a current journals

https://www.youtube.com/watch?v=RXJKdh1KZ0w

Objective: Easy to find (251 is awesome)

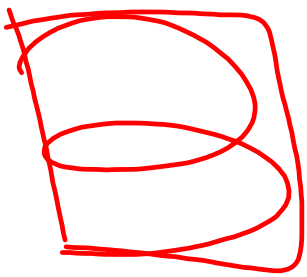
Hard to read (cause that life)

I can use on letters of recommnedation

$$\frac{\delta f}{\delta x}$$

$$\iiint f(x,y,z)$$

For Applications



Arclength of the Spiral

$$S = \int_0^{\pi/2} \sqrt{(x')^2 + (y')^2 + (z')^2}\, dt$$

$x = 75 \cos t \cos t \rightarrow x'$

$y = 75 \sin t \cos t \rightarrow y'$

$z = 75 \cos t \rightarrow z'$

Do this for next time