Match the vector–valued function

\[ f(t) = \left< t \cos t, t \sin t, t \right> \]

with the corresponding computer–generated graph.

\[
\begin{align*}
x^2 &= t^2 \cos^2 t \\
y^2 &= t^2 \sin^2 t \\
z^2 &= t^2 + y^2 
\end{align*}
\]

Find the derivative of \( r(t) = \left< \cos 3t, \tan t, 9 \sin t \right> \).

\[ r'(t) = \left< -3\sin(3t), \sec^2(t), 9\cos(t) \right> \]
**Arclength**

\[ r(t) = \langle t^2 + 1, 2t, t^2 - 1 \rangle, \quad 0 \leq t \leq 2 \]

\[ s = \int_{a}^{b} \sqrt{\left( r'(t) \right)^2} \, dt \]

\[ r'(t) = \langle 2t, 2, 2t \rangle \]

\[ s = \int_{0}^{2} \sqrt{4t^2 + 4 + 4t^2} \, dt \approx 7.25 \]

\[ r(0) = \langle 1, 0, -1 \rangle, \quad r(2) = \langle 5, 4, 3 \rangle \]

**Example**

The arclength of the curve \( r(t) = \langle \cos t, \sin t, t \rangle \) from \( t = 0 \) to \( 2\pi \)

\[ s = \int_{0}^{2\pi} \sqrt{1 + 1} \, dt \]

\[ s = \int_{0}^{2\pi} \sqrt{2} \, dt \]

\[ s = \sqrt{2} \cdot 2\pi \]
Lecture

$s(t)$ position now $\vec{r}(t)$

$s'(t)$ velocity now $\vec{v}(t) = \vec{r}'(t)$

$s''(t)$ acceleration now $\vec{r}''(t)$

Parameterization:

circle $x(t) = \cos(t)$ $y(t) = \sin(t)$  

$t$ is in radians

$x(s) = \cos(s)$ $y(s) = \sin(s)$

$s = \text{arc length}$

$s = r \cdot t$

$s = t$
Radians: 
\[ y = \sin(t) \Rightarrow y' = \cos(t) \]

Degrees: 
\[ y = \sin(at) \Rightarrow y' = a \cos(t) \]

\[ a = \frac{\pi}{180} \]
Find an arc length parameterization of the circle of radius 8 centered at the origin.

A. \( C : x = 8 \sin \left( \frac{s}{8} \right), y = 8 \cos \left( \frac{s}{8} \right), 0 \leq s \leq 16\pi \)

B. \( C : x = 8 \cos \left( \frac{s}{8} \right), y = 8 \sin \left( \frac{s}{8} \right), 0 \leq s \leq 16\pi \)  

C. \( C : x = 8 \cos \left( \frac{s}{8} \right), y = 8 \sin \left( \frac{s}{8} \right), 0 \leq s \leq 16\pi \)

D. \( C : x = 8 \sin \left( \frac{s}{8} \right), y = 8 \sin \left( \frac{s}{8} \right), 0 \leq s \leq 16\pi \)

\[ t = \frac{s}{8} \]
\[ \mathbf{r}(t) = \langle t, 1-t, t+2 \rangle \quad 0 \leq t \leq 1 \]

\[ \mathbf{r}' = \langle 1, -1, 1 \rangle \]

\[ s = \int_{0}^{\frac{\sqrt{3}}{3}} \sqrt{1+t^2+1} \, dt \]

\[ s = \frac{\sqrt{3}}{3} t \]

\[ s = \frac{\sqrt{3}}{2} \]

graphical representation with points and lines indicating the path.
\[ \int_0^2 \sqrt{4t^2 + 2} \, dt = S(t) \]
\[ \frac{1}{2} \sqrt{4t^2 + 2} + \frac{1}{2} \arcsinh (\sqrt{2} t) \]

\[ T = S(t) \]

\[ r = \langle x, y, z \rangle \]

\[ r' = \langle x, y', z \rangle \]

\[ x = t^2 \quad y = \frac{1}{2} \quad t = \frac{1}{2} y^2 \]

Radius of Curvature
Find the curvature of the straight line \( r(t) = \langle 8t + 4, 3t + 2, 5t + 7 \rangle \).

First, think about what we're asking. Straight lines are, well, straight, so their curvature should be zero at every point. Let's see. We have

\[ r'(t) = \langle 8, 3, 5 \rangle, \]

so that

\[ ||r'(t)|| = \sqrt{8^2 + 3^2 + 5^2} = \sqrt{98}. \]

The unit tangent vector is then

\[ T(t) = \frac{r'(t)}{||r'(t)||} = \frac{\langle 8, 3, 5 \rangle}{\sqrt{98}}, \]

which is a constant vector. This gives us \( T'(t) = 0 \), for all \( t \). We now have

\[ \kappa = \frac{||T'(t)||}{||r'(t)||} \frac{\langle 0 \rangle}{\sqrt{98}} = 0, \]

as expected.

Lit Search:

Vectors are lots of places.

Look in your STEM? field for a current journals

https://www.youtube.com/watch?v=RXJKdh1KZ0w

Objective: Easy to find (251 is awesome)

Hard to read (cause that life)

I can use on letters of recommendation
For Applications

Arch length of the spiral
\[ S = \int_0^{2\pi} \sqrt{(x')^2 + (y')^2 + (z')^2} \, dt \]

\[ x = 75 \cos t \cos t \rightarrow x' \]
\[ y = 75 \sin t \cos t \rightarrow y' \]
\[ z = 75 \cos t \rightarrow z' \]

Do this for next time.