Midterm Review for Final

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Mid. #2

Please round your answer to three decimals and remember answer is in radians.

Use the cross product to determine the angle between \( \mathbf{a} = 9\mathbf{i} + 4\mathbf{k} \) and \( \mathbf{b} = 4\mathbf{j} + 3\mathbf{k} \).

\[
\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| \cdot |\mathbf{b}|}
\]

\[
\theta = \cos^{-1} \left( \frac{<9, 0, 4> \cdot <0, 4, 3>}{\sqrt{81 + 16} \cdot \sqrt{16 + 9}} \right)
\]

\[
= \cos^{-1} \left( \frac{12}{5\sqrt{97}} \right)
\]

\[
\cos^{-1} \left( \frac{12}{5\sqrt{97}} \right) \approx 0.324634719
\]
Choose the sketch of the curve and estimate its arc length

\[ r(t) = \langle t^2 + 1, 2t, t^2 - 1 \rangle, 0 \leq t \leq 2 \]

\[ r'(t) = \langle 2t, 2, 2t \rangle \]

\[ |r'(t)| = \sqrt{4t^2 + 4 + 4t^2} \]

\[
\int_{0}^{2} |r'(t)| \, dt = \int_{0}^{2} \sqrt{8t^2 + 4} \, dt
\]

Find the unit tangent vector to the curve

\[ r(t) = \langle 5 \cos t, 3 \sin t \rangle \]

at \( t = -\pi \).

- A. \( T(-\pi) = \langle 0, 0 \rangle \)
- B. \( T(-\pi) = \langle 5, 0 \rangle \)
- C. \( T(-\pi) = \langle 0, -1 \rangle \)
- D. \( T(-\pi) = \langle 5, 5 \rangle \)

\[ r'(-\pi) = \langle -5\sin(-\pi), 3\cos(-\pi) \rangle \]

\[ \left| r'(-\pi) \right| = \sqrt{(-5)^2 + 3^2} = \sqrt{25 + 9} = \sqrt{34} \]

\[ \frac{r'(-\pi)}{\left| r'(-\pi) \right|} = \frac{\langle -5\sin(-\pi), 3\cos(-\pi) \rangle}{\sqrt{34}} \]

\[ = \langle 0, -\frac{3}{\sqrt{34}} \rangle \]
Compute the linear approximation of the function \( f(w, x, y, z) = 8w^2xy - e^{2wy}z \) at the point \((-2, 5, 2, 0)\).

\[
\frac{df}{dw} = 16wx - e^{2wy} \cdot 2z
\]

\[
L(-2, 5, 2, 0) = \left( \frac{df}{dw} \right)(w + 2) + \left( \frac{df}{dx} \right)(x - 5) + \left( \frac{df}{dy} \right)(y - 2) + \left( \frac{df}{dz} \right)z
\]
#11

\[ l = y - 4x^2 \]
\[ y = 4x^2 + l \]

#17

Minimize \( f(x, y, z) = x^2 + y^2 + z^2 \), subject to the constraints \( x + 2y + 3z = 35 \) and \( y + z = 0 \).

\[
\begin{align*}
\nabla f &= \begin{pmatrix} \partial_x f \\ \partial_y f \\ \partial_z f \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix} \\
\nH(x, y, z) &= x + 2y + 3z + 15 = 0 \\

\text{Let } x &= \lambda \\

y &= -2 \\
z &= -3 \lambda \\

x + 2y + 3z &= -15 \\
\frac{\partial f}{\partial x} &= 2x = \lambda \\
\frac{\partial f}{\partial y} &= 2y = -2 - 3\lambda \\
\frac{\partial f}{\partial z} &= 2z = -3 \lambda \\
\frac{\lambda}{2} + 2\lambda - 3\frac{\lambda}{4} &= -15 \\
\frac{2\lambda}{4} + 2\lambda - 3\frac{\lambda}{4} &= -15 \\
\lambda &= -15 \frac{4}{3} \\
2\lambda &= -15 \\
\lambda &= -15 \frac{4}{3} \\
\lambda &= -15 \\
\frac{\lambda}{2} &= -15 \\
\lambda &= -15 \\
x &= \frac{\lambda}{2} = -10 \\
\lambda &= -15 \\
\frac{\partial f}{\partial y} &= \frac{\partial f}{\partial z} = 15 \\
\frac{\partial f}{\partial z} &= -15 \\
-3x^2 + y^2 + z^2 &= 30^2 + 15^2 + 15^2 =
\end{align*}
\]
#20

Compute the volume of the solid bounded by the given surfaces.

\[ x + 15y - 10z = 150 \]

and the three coordinate planes.

\[ \int \int_{\text{Top - Bottom}} \text{d}A \]

\[ \int_{x=0}^{150} \left( \int_{y=0}^{150-15y} \frac{150-x}{10} \frac{15y}{10} \text{d}x \right) \text{d}y \]

\[ \int_{y=0}^{10} \left( 15z - \frac{x^2}{20} - 1.5xy \right) \text{d}z \]

\[ \int_{y=0}^{15} (150-15y) - \frac{(150-15y)^2}{20} - 1.5(150-15y)y \text{d}y \]