Agenda

Review Quiz 7
Lecture Multiple Integrals
Examples of Multiple Integrals on Connect
Project: Express the volume of the Building
3D Printer

Review Quiz 7
The distance from a point \((x, y)\) to the point \((5, -3, 1)\) is
\[
d(x, y) = \sqrt{(x - 5)^2 + (y + 3)^2 + (z - 1)^2}.
\]
To minimize this it is useful to note that we can minimize \(g(x, y) = d(x, y)^2 = D(x, y)\)

instead.

\[
\begin{align*}
\delta_x &= 2(x - 5) - 4x(3 - x^2 - y^2) = 0 \\
\delta_y &= 2(y + 3) - 4y(3 - x^2 - y^2) = 0 \\
\delta_{xx} &= -10 + 12x^2 + 4y^2 \\
\delta_{xy} &= -10 + 12x^2 + 4x^2 \\
\delta_{yy} &= 8y
\end{align*}
\]

\(\delta_x = \delta_y = 0\) numerically yields \((1.7073, -1.0244)\).

\(D(1.7073, -1.0244) = 15.6747\) and \(x, y = (1.7073, -1.0244) = 15.1761\).

Therefore this point is a minimum.

The closest point on the paraboloid to the point \((5, -3, 1)\) is \((1.7073, -1.0244, 0.0357)\)
A box is to be constructed out of 96 square feet of material. Find the dimensions $x$, $y$, and $z$ that maximize the volume of the box.

$x \cdot y \cdot z$ maximizes the volume.

$V = x \cdot y \cdot z$

$xy + yz + zx = 48$

$\nabla V = \langle yz, xz, xy \rangle$

$\nabla g = \langle yz, xz, xy \rangle$

$\nabla V = \lambda \nabla g$

$\lambda (xy + xz + yz) = \lambda (xy + yz) = \lambda (xz + yz) = \lambda (yz + zx) = \lambda (xy + xz + yz)$

$xy + xz + yz = xy + yz$

$xy = yz$

$x = y$  

$x = y > z$
For a business that produces three products, suppose that when producing \( x, y, \) and \( z \) thousand units of the products, the profit of the company (in thousands of dollars) can be modeled by \( P(x, y, z) = 2x - 6y + 7z \). Manufacturing constraints force

\( x^2 + 4y^2 + z^2 \leq 800 \)

Find the maximum profit for the company. Round your answers to three decimal places.

The maximum profit is ________.

\[ \nabla^2 = \langle 2, 3, 7 \rangle \quad \nabla g = \langle 2x, 8y, 4z \rangle \]

\[ 2 = 2x \lambda \quad x = \frac{\lambda}{2} \]

\[ 9 = 8y \lambda \quad y = \frac{9\lambda}{8} \]

\[ 7 = 4z \lambda \quad z = \frac{7\lambda}{4} \]

\[ \left( \frac{\lambda}{2} \right)^2 + 4 \left( \frac{9\lambda}{8} \right)^2 + 2 \left( \frac{7\lambda}{4} \right)^2 = 800 \]

\[ x = \frac{\lambda}{2} = 8.10307, \quad y = \frac{9\lambda}{8} = 9.11595, \quad \text{and} \quad z = \frac{7\lambda}{4} = 14.18037. \]

Setting \( \nabla f = \lambda \nabla g + \mu \nabla h \):

Minimize \( f(x, y, z) = x^2 + y^2 + z^2 \), subject to the constraints \( x - 2y + 4z = 0 \) and \( y + z = 0 \).

\[ \nabla f = \langle 2x, 2y, 2z \rangle \quad \nabla g = \langle 1, 2, 4 \rangle \quad \nabla h = \langle 0, 1, 1 \rangle \]

\[ 2x = \lambda + 0 \]

\[ 2y = 2\lambda + 4\mu \]

\[ 2z = 4\lambda + \mu \]
Lecture: Multiple Integrals
DEFINITE INTEGRAL--

\[ A = \int_{a}^{b} f(x) \, dx \]

INDEFINITE--

\[ F(x) = \int f(x) \, dx \quad \text{-- Anti-derivative} \]

\[ F(x,y,z) = \int \int f(x,y) \, dx \, dy \]

Not so easy?

\[ F_{xy} = f(x,y) \]
\[
\int \int (3x + y) \, dy \, dx \\
\int 3xy + \frac{y^2}{2} \, dx
\]

\[F(x, y) = 3y \frac{x^2}{2} + \frac{y^2}{2}x + C\]

Let \(x\) be a constant

Let \(y\) be a constant

\[
V = \int \int \int \frac{\partial f(x, y)}{\partial x} \, dA
\]

\[
\int_a^b \int_c^d f(x, y) \, dx \, dy
\]
Examples
If \( R = \{(x, y)| 0 \leq x \leq 2 \text{ and } 1 \leq y \leq 4\} \), evaluate

\[
\int_R \int (6x^2 + 7xy^3) \, dA.
\]

We have

\[
\int_R \int (6x^2 + 7xy^3) \, dA = \int_1^4 \int_0^2 (6x^2 + 7xy^3) \, dx \, dy
\]

\[
= \int_1^4 \left[ \int_0^2 (6x^2 + 7xy^3) \, dx \right] \, dy
\]

\[
= \int_1^4 \left( \frac{6x^3}{3} + 7 \frac{x^2y^3}{2} \right) \bigg|_{x=0}^{x=2} \, dy
\]

\[
= \int_1^4 \left( 16 + 14y^3 \right) \, dy
\]

\[
= \left[ 16y + \frac{14y^4}{4} \right]_1^4 = \left( 64 + \frac{224}{4} \right) - 16 - \frac{14}{4} = 48
\]

Let \( R \) be the region bounded by the graphs of \( y = \sqrt{x}, \ x = 0 \) and \( y = 7 \). Evaluate

\[
\int_R \int (8xy^2 + 2y \cos x) \, dA.
\]

Round your answer to one decimal place.

The value is \boxed{ }
\[
\int_{x=0}^{49} \int_{y=0}^{\sqrt{x}} 2y + 2y \cos x \, dy \, dx = \int_{x=0}^{49} \left[ \frac{8x^3}{3} + 2y^2 \cos x \right]_{y=0}^{y=\sqrt{x}} \, dx
\]

\[
\int_{x=0}^{49} \left( \frac{8x^3}{3} + (x^2) \cos x - \frac{8x(\sqrt{x})^3}{3} - (\sqrt{x})^2 \cos x \right) \, dx
\]

\[
\int_{y=0}^{7} \left( \frac{8y^3}{3} + 2y \sin x \right) \, dy
\]

\[
\left( 4y^6 + 2y \sin y \right) \bigg|_{y=0}^{y=7} - \left( 0 \right) \bigg|_{y=0}^{y=7}
\]

\[
\int_{y=0}^{7} \left( 4y^6 + 2y \sin y \right) \, dy
\]

\[
\frac{4y^7}{7} - \cos(9) \bigg|_{y=0}^{y=7}
\]

\[
4 \cdot 7 - \cos(9)
\]
Compute the Riemann sum for the given function and region, a partition with \( n \) equal-sized rectangles and the given evaluation rule.

\[ f(x, y) = 11x^2 - 11y, \ 1 \leq x \leq 5, \ 0 \leq y \leq 1, \ n = 4, \ \text{evaluate at midpoint}. \]

Evaluate the iterated integral by first changing the order of integration.

\[
\int_0^4 \int_0^{\sqrt{20 + y^3}} \frac{1}{x} \, dy \, dx + \int_0^{\sqrt{4}} \int_0^{\sqrt{4}} \frac{1}{20 + y^3} \, dx \, dy + \int_0^{\sqrt{4}} \int_0^{\sqrt{4}} \frac{1}{20 + y^3} \, dx \, dy
\]
Project
Volume of Building

3D Printer update

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Do they work?