Agenda

Review Quiz 7

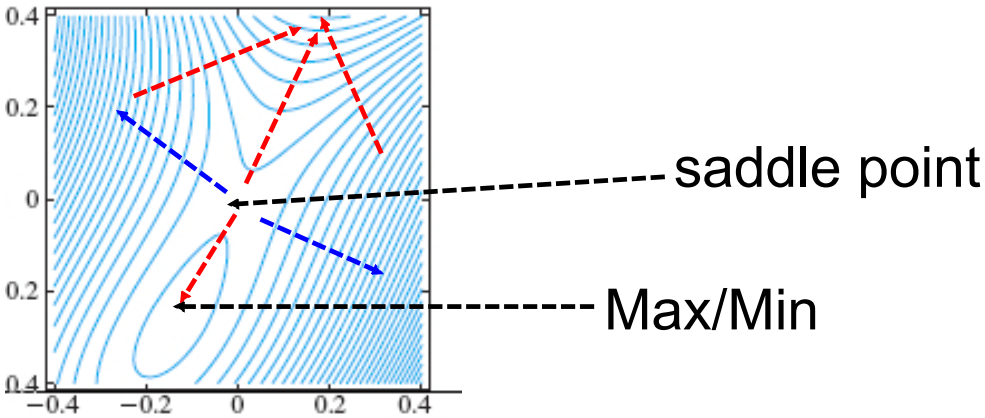
Lecture Multiple Integrals

Examples of Multiple Integrals on Connect

Project: Express the volume of the Building

3D Printer

Review Quiz 7



The distance from a point $(x, y, 4 - x^2 - y^2)$ to the point $(5, -3, 1)$ is

$d(x, y) = \sqrt{(x-5)^2 + (y+3)^2 + (3 - x^2 - y^2)^2}$.

To minimize this it is useful to note that we can minimize $g(x, y) = d(x, y)^2 = D(x, y)$ instead.

$$g_x = 2(x-5) - 4x(3 - x^2 - y^2) = 0$$
$$g_y = 2(y+3) - 4y(3 - x^2 - y^2) = 0$$
$$g_{xx} = -10 + 12x^2 + 4y^2$$
$$g_{yy} = -10 + 12y^2 + 4x^2$$
$$g_{xy} = 8xy$$

$D = g_{xx} g_{yy} - g_{xy}^2$

$g_x = g_y = 0$ numerically yields $(1.7073, -1.0244)$.

$D(1.7073, -1.0244) \approx 15.6747$ and $g_{xx}(1.7073, -1.0244) \approx 29.1761$.

Therefore this point is a minimum.

The closest point on the paraboloid to the point $(5, -3, 1)$ is $(1.7073, -1.0244, 0.0357)$.

$$\frac{2(x-5)}{4x} = \frac{2(y+3)}{4y}$$

$$\frac{x-5}{x} = \frac{y+3}{y}$$

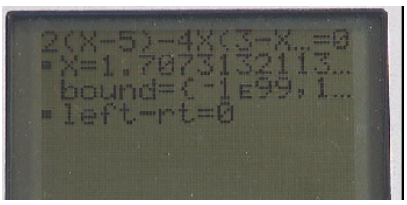
$$1 - \frac{5}{x} = 1 + \frac{3}{y}$$

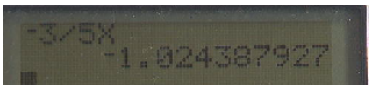
$$-\frac{5}{x} = \frac{3}{y}$$

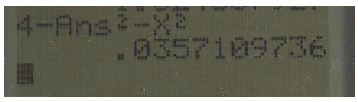
$$-5y = 3x$$

$$y = -\tfrac{3}{5}x$$

$$2(x-5) - 4x\left(3 - x^2 - \tfrac{9}{25}x^2\right) = 0$$







$x = 1.707\ldots$

$y = -1.02\ldots$

A box is to be constructed out of 96 square feet of material. Find the dimensions x, y, and z that maximize the volume of the box.

$x =$ [ ], $y =$ [ ], and $z =$ [ ] maximize the volume.

$V = xyz$ $\quad x = y = 4$

$2xz + 2yz + 2xy = 96$

$xz + yz + xy = 48$

$z = \frac{48 - xy}{x+y} = \frac{48-16}{8} = 4$

$V = xy\left(\frac{48 - xy}{x+y}\right)$

$V = \frac{48xy - x^2y^2}{x+y}$

$V_x = \frac{(x+y)(48y - 2xy^2) - (48xy - x^2y^2)}{\cancel{(x+y)^2}} = 0$

$V_y = \frac{(x+y)(48x - 2x^2y) - (48xy - x^2y^2)}{} = 0$

$48y - 2xy^2 = 48x - 2x^2y$

$24y - xy^2 = 24x - x^2y$

$y(24 - xy) = x(24 - xy)$

$x = y$

$2x(48x - 2x^3) - (48x^2 - x^4) = 0$

$x = 4$

A box is to be constructed out of 96 square feet of material. Find the dimensions x, y, and z that maximize the volume of the box.

$x =$ [4], $y =$ [4], and $z =$ [4] maximize the volume.

$xy + yz + xz = 48 \qquad V = xyz$

$\nabla V = \langle yz, xz, xy \rangle$

$\nabla g = \langle y+z, x+z, x+y \rangle$

$\nabla V = \lambda \nabla g$

$x \quad yz = \lambda(y+z) \quad x$

$y \quad xz = \lambda(x+z) \quad y$

$z \quad xy = \lambda(x+y) \quad z$

$\lambda(xy + xz) = \lambda(xy + yz) = \lambda(xz + yz)$

$\cancel{\lambda xy} + \lambda xz = \cancel{\lambda xy} + \lambda yz \qquad xz = yz \qquad y = z$

$x = y$

$x = y = z$

$3x^2 = 48$

$x^2 = 16$

$x = 4$

For a business that produces three products, suppose that when producing x, y, and z thousand units of the products, the profit of the company (in thousands of dollars) can be modeled by $P(x, y, z) = 2x + 9y + 7z$. Manufacturing constraints force $x^2 + 4y^2 + 2z^2 \le 800$. Find the maximum profit for the company. Round your answers to five decimal places.

The maximum profit is $\boxed{}$.

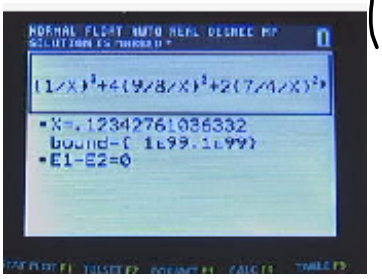
$\nabla P = \langle 2, 9, 7 \rangle \quad \nabla g = \langle 2x, 8y, 4z \rangle$

$2 = 2\lambda x \qquad x = 1/\lambda$

$9 = 8\lambda y \qquad y = 9/8\lambda$

$7 = 4\lambda z \qquad z = 7/4\lambda$

$(1/\lambda)^2 + 4(9/8\lambda)^2 + 2(7/4\lambda)^2 = 800$



$x = \frac{1}{\lambda} \approx 8.10307$, $y = \frac{1.125}{\lambda} \approx 9.11595$, and $z = \frac{1.75}{\lambda} \approx 14.18037$,

Setting $\nabla f = \lambda \nabla g + \mu \nabla h$;

Minimize $f(x, y, z) = x^2 + y^2 + z^2$, subject to the constraints $x + 2y + 4z = 9$ and $y + z = 0$.

5

$\nabla f = \langle 2x, 2y, 2z \rangle$

$\nabla g = \langle 1, 2, 4 \rangle$

$\nabla h = \langle 0, 1, 1 \rangle$

4

① $2x = \lambda + 0$

② $2y = 2\lambda + 4\mu$

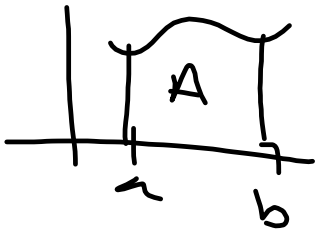
③ $2z = 4\lambda + \mu$

- jasonyvon
- John
- Kiefer Ahn
- Kristen Gavenda
- Saleem Saniya
- Seth Doolittle
- Yauheni

Lecture: Multiple Integrals

DEFINITE INTEGRAL--

$$A = \int_a^b f(x)\,dx$$



INDEFINITE--

$$F(x) = \int f(x)\,dx$$ ← Anti-derivative

$$F(x,y)_x = \int \boxed{\int f(x,y)\,dx}\,dy$$

Not so Easy?

$$F_{xy} = f(x,y)$$

Ex

$$\int \boxed{\int 3x+y \; dy} \, dx$$

Let x be like a constant

$$\int 3xy + \frac{y^2}{2} \; dx$$

Let y be a constant

$$F(x,y) = \frac{3yx^2}{2} + \frac{y^2}{2}x + c$$

$$V = \int_a^b \int_c^d f(x,y) \, \frac{dA}{dx\,dy}$$

R

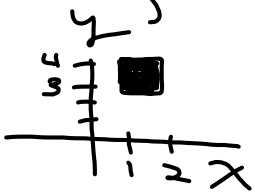
$$\int_{y=c}^{d} \boxed{\int_{x=a}^{b} f(x,y) \; dx} \, dy$$

Ex

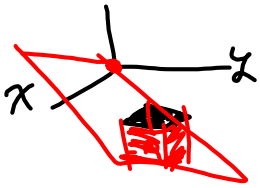
$$\int_1^2 \int_3^4 (x - 2y)\, dy\, dx$$

$z = x - 2y$

Region



$$\int_{x=1}^{2} xy - y^2 \Big|_{y=3}^{4} dx$$



$$\int_{x=1}^{2} (4x-16) - (3x-9)\, dx$$

$$\int_1^2 x - 7\, dx = \frac{x^2}{2} - 7x \Big|_1^2$$

$$= (2-14) - \left(\tfrac{1}{2} - 7\right)$$

$$= -5\tfrac{1}{2}$$

Examples

If $R = \{(x, y) \mid 0 \le x \le 2 \text{ and } 1 \le y \le 4\}$, evaluate

$$\int_R \int (6x^2 + 7xy^3)\, dA.$$

We have

$$\int_R \int (6x^2 + 7xy^3)\, dA = \int_1^4 \int_0^2 (6x^2 + 7xy^3)\, dx\, dy$$

$$= \int_1^4 \left[\int_0^2 (6x^2 + 7xy^3)\, dx \right] dy$$

$$= \int_1^4 \left(6\frac{x^3}{3} + 7\frac{x^2}{2} y^3 \right) \Big|_{x=0}^{x=2} dy$$

$$= \int_1^4 (16 + 14y^3)\, dy$$

$$= 16y + \frac{14y^4}{4} \Big|_1^4 = 16(4) + \frac{14(4)^4}{4} - 16 - \frac{14}{4} = 48$$

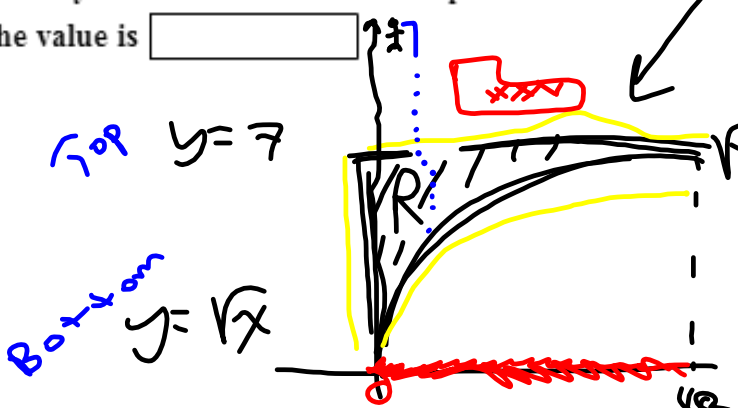
$= \#$

Let R be the region bounded by the graphs of $y = \sqrt{x}$, $x = 0$ and $y = 7$. Evaluate

$$\int_R \int (8xy^2 + 2y \cos x)\, dA.$$

Round your answer to one decimal place.

The value is



$$\int_0^{49} \int_{\sqrt{x}}^{7} 8xy^2 + 2y\cos x \, dA$$

top $y = 7$

Bottom $y = \sqrt{x}$

$$\int_0^{49} (7 - \sqrt{x})\, dx$$

$$\int_{x=0}^{49} \int_{y=\sqrt{x}}^{7} f(x,y)\, dy$$

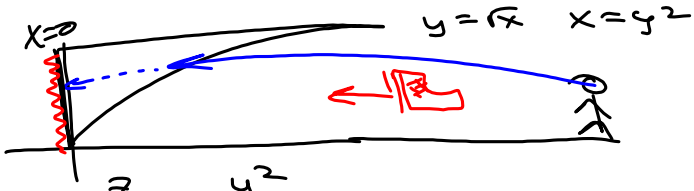
$$\int_0^{49} \int_{\sqrt{x}}^{7} dy\, dx$$

$$\int_0^{49} y \Big|_{\sqrt{x}}^{7} dx$$

$$\int_0^{49} (7 - \sqrt{x})\, dx$$

$$\int_{x=0}^{49}\int_{y=\sqrt{x}}^{7} 8xy^2+2y\cos x \, dA = \int_0^{49}\left[\frac{8xy^3}{3}+\frac{2y^2}{2}\cos x\right]_{y=\sqrt{x}}^{7} dx$$

$$\int_0^{49} \underbrace{\frac{8x}{3}(7)^3+(7)^2\cos x}_{y=7} - \underbrace{\frac{8x}{3}(\sqrt{x})^3-(\sqrt{x})^2\cos x}_{y=\sqrt{x}} \, dx$$



$$\int_{y=0}^{7}\int_{x=0}^{y^2} 8xy^2+2y\cos x \, dx \, dy$$

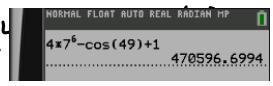
$$\int_0^7 \frac{8x^2}{2}y^2+2y\sin x \Big|_0^{y^2} dy$$

$$\int_0^7 (4y^6+2y\sin y^2)-(0)\, dy$$

$$\boxed{\int_0^7 4y^6+2y\sin(y^2)\, dy}$$

$$\frac{4y^7}{7}-\cos(y^2)\Big|_0^7$$

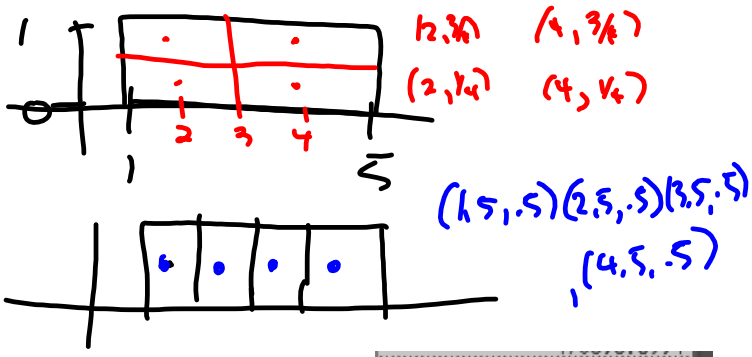
$$4\cdot 7^6-\cos(4$$

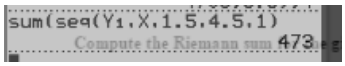


Compute the Riemann sum for the given function and region, a partition with n equal-sized rectangles and the given evaluation rule.

$f(x, y) = 11x^2 + 11y$, $1 \le x \le 5$, $0 \le y \le 1$, $n = 4$, evaluate at midpoint.

$f(x,y)$ Region





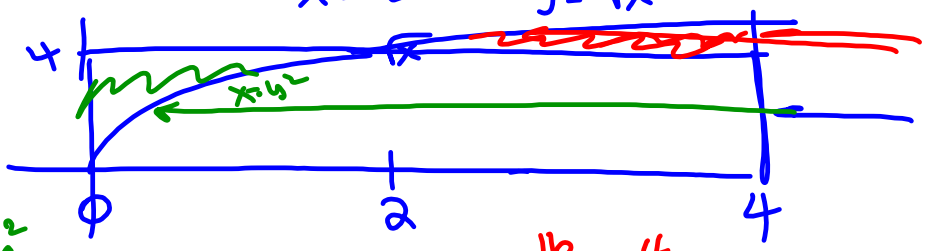
Evaluate the iterated integral by first changing the order of integration.

$$\int_0^4 \int_{\sqrt{x}}^4 \frac{1}{20 + y^3}\, dy\, dx$$

$x = 4$ $y = 4$

$x = 0$ $y = \sqrt{x}$

$R =$



$$\int_0^4 \int_0^{y^2} \frac{1}{20+y^3}\, dx\, dy + \int_4^{16} \int_{y^2}^4 \frac{1}{20+y^3}\, dx\, dy$$

Project

Volume of Building $\int\int_{\text{Footprint}} \text{Roof}$

3D Printer update .obj .stl do they work?

bittnerh@mccc.edu