Agenda

Overview

Review Quiz 6

Lecture Optimization

-Boundary Value Problems

Lecture Error

Project: 3D printer?

Introduce Lagrange Multipiers

Calc 3 Overview-Multidimensional

Vectors(magnitude/direction=slope), Parametric lines, surfaces

Partial Derivatives

-Tangent lines/planes, "Linearization",Chain Rule, Implicit, Max/Min, Boundaries, Constraints

Integrals (later)

Quiz 6 Review

Question

Please fully reduce the final answer.

Compute the linear approximation of the function $f(w, x, y, z) = 3w^2xy - e^{6wyz}$ at the point $(-9, 4, 3, 0)$.

$L(-9, 4, 3, 0) =$ $\boxed{}$

$-648 = \frac{\partial f}{\partial w} = 6wxy - e^{6wyz} \cdot \cancel{6yz} = 6(-9)(4)(3)$

$729 = \frac{\partial f}{\partial x} = 3w^2y = 3(81)(3)$

$972 = \frac{\partial f}{\partial y} = 3w^2x - e^{6wyz} \cdot \cancel{6wz} = 3(81)(4)$

$+162 = \frac{\partial f}{\partial z} = -e^{6wyz} \cdot 6wy = -6(-9)(3)$

$L = -648(w+9) + 729(x-4) + 972(y-3) + 162z$

Use implicit differentiation on the equation

$$6e^{xyz} - 4xz^2 + x\cos y = 3$$

to find $\dfrac{\partial z}{\partial x}$ **and** $\dfrac{\partial z}{\partial y}$.

$$F(x,y,z) = 6e^{xyz} - 4xz^2 + x\cos y - 3 = 0$$

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$0 = \frac{\partial F}{\partial x}\cancel{\frac{\partial x}{\partial x}} + \frac{\partial F}{\partial y}\cancel{\frac{\partial y}{\partial x}} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial x}$$

$$0 = F_x + F_z\frac{\partial z}{\partial x}$$

Find equations of the tangent plane and normal line to the surface at the given point.

$z^2 = x^2 + y^2$ **at (3, 5, −5)**

$$f(x, y, z) = x^2 + y^2 - z^2$$

$$\nabla f = \langle 2x, 2y, -2z \rangle$$

$$\nabla f(3, 5, -5) = \langle 6, 10, 10 \rangle$$

$$6(x - 3) + 10(y - 5) + 10(z + 5) = 0$$

The normal line has parametric equations

$x = 3 + 6t$

$y = 5 + 10t$

$z = -5 + 10t$

$z(x,y)$

$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

Sec. Ex. 31 - 12.4 Section Exercise 31

Your response:

Find the total differential of $f(x, y) = y e^{6x} + \sin 2x$.

A. $dz = (6y e^{6x} + 2\cos 2x)dx + (e^{x})dy$

B. $dz = (y e^{6x} + 2\cos 2x)dx + (e^{6x})dy$

C. $dz = (6y e^{6x} + 2\cos x)dx + (e^{6x})dy$

D. $dz = (6y e^{6x} + 2\cos 2x)dx + (e^{6x})dy$

$\frac{\partial z}{\partial x}$ $\frac{\partial z}{\partial y}$

Connect? #9

Sec. Ex. 3 - Vector Calculus

f_x $36x^2 - 36y = 0$

f_y $-36x + 36y^2 = 0$

Your response:

Locate all critical points and classify them.

$f(x, y) = 12x^3 - 36xy + 12y^3$

A. (0, 0) is a saddle point and (1, 1) is a local minimum. ✓

B. (0, 0) is a local maximum and (−1, −1) is a saddle point.

C. (0, 0) is a saddle point and (−1, −1) is a local minimum.

D. (0, 0) is a local minimum and (1, 1) is a local maximum. ✓

$D = f_{xx}f_{yy} - f_{xy}^2$

$f_{yy} = 72y$ $f_{xx} = 72x$ > 0 concave up

$f_{xy} = -36$ $D = (72x)(72y) - 36^2$

$D(1,1) = 72^2 - 36^2 > 0$

Sec. Ex. 34 - Extrema

Your response:

Round your answers to three decimal places, if needed.

Find the absolute extrema of the function

$f(x, y) = 4.4x^2 + 2.8y^2 - 3.2x - 3.1y$

on the region bounded by $y = x$, $y = 3$, and $x = 0$.

The absolute maximum is 45.9

The absolute minimum is -1.44

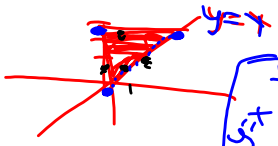
$f_x = 8.8x - 3.2 = 0$

$x = 3.2/8.8 = .36$

$f_y = 5.6y - 3.1 = 0$

$y = \frac{3.1}{5.6} = .55$

$f(.36, .55)$



$y=x$

$f(x) = 4.4x^2 + 2.8x^2 - 3.2x - 3.1x$

$7.2x^2 - 6.3x$

$f'(x) = 14.4x - 6.3 = 0$

$x = \frac{6.3}{14.4} = .4375$

$f(.4375, .4375)$

$x=0$ $f = 2.8y^2 - 3.1y$

$f' = 5.6y - 3.1 = 0$

$y = .55$

$f(0, .55)$

$y=3$

$f(x, y) = 4.4x^2 + 2.8y^2 - 3.2x - 3.1y$

$f(x) = 4.4x^2 - 3.2x + 15.9$

$f' = 8.8x - 3.2 = 0$

$f(0, .55) = -.88$

$f(.36, 3) = 15.318..$

$f(.36, .55) = -1.44$

$f(.4375, .4375) =$

$f(0, 0) = 0$

$f(0, 3) = 15.9$

$f(3, 3) = 45.9$

Optimization

Boundary Problems

① Find $\nabla f = 0$

$\langle f_x, f_y \rangle = \langle 0, 0 \rangle$

② Each Boundary

$x = 0$ or $y = 3$

or

$y = x$

Use regular $f'(x) = 0$

③ Find the END POINTS (or corners)

④ Plug em all in

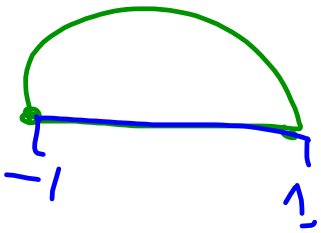
Find big & SMALL

What if boundary is a circle of radius 3, centered at origin?

② Max/Min on Boundary??

$x = 3\cos t$ $\quad 0 \le t \le 2\pi$
$y = 3\sin t$

If Maybe



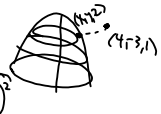
$x = \cos t$ $\quad 0 \le t \le \pi$
$y = \sin t$

$x = x$ $\quad -1 \le x \le 1$
$y = 0$

Construct the function $d(x, y)$ giving the distance from a point (x, y, z) on the paraboloid $z = 4 - x^2 - y^2$ to the point $(4, -3, 1)$. Then determine the point that minimizes $d(x, y)$. Round all your intermediate calculations to four decimal places and round your final answer to one decimal place.

The closest point on the paraboloid to the point $(4, -3, 1)$ is approximately ([ ], [ ], [ ]).

$d(x,y,z) = \sqrt{(x-4)^2+(y+3)^2+(z-1)^2}$



$d(x,y) = \sqrt{(x-4)^2+(y+3)^2+(4-x^2-y^2-1)^2}$

$d = (x-4)^2+(y+3)^2+(3-x^2-y^2)^2$

$d_x = 2(x-4)^1 + 2(3-x^2-y^2)(-2x) = 0$

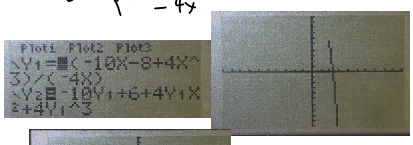
$2x-8-12x+4x^3+4xy^2=0$

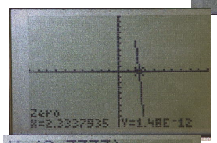
$d_y = 2(y+3) + 2(3-x^2-y^2)(-2y) = 0$

$2y+6-12y+4yx^2+4y^3=0$

$y = \dfrac{-10x-8+4x^3}{-4x}$

$y = \sqrt{\dfrac{-10x-8+4x^3}{-4x}}$

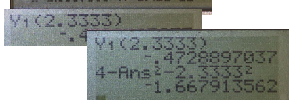




x=2.333

y= -.47288

z=-



$$g_x = 2(x-2) - 4x(3 - x^2 - y^2) = 0$$
$$g_y = 2(y+3) - 4y(3 - x^2 - y^2) = 0$$

$$2(x-2) = 4x(\ \ ) \rightarrow \frac{2(x-2)}{4x} = (\ \ )$$

$$2(y+3) = 4y(\ \ ) \quad \frac{2(y+3)}{4y} = (\ \ )$$

$$\frac{2(x-2)}{4x} = \frac{2(y+3)}{4y}$$

$$\cancel{\frac{1}{2}} - \frac{1}{2x} = \cancel{\frac{1}{2}} + \frac{3}{4y}$$

$$-\frac{1}{2x} = \frac{3}{4y}$$

$$y = \frac{-6x}{4} = -\frac{3x}{2}$$

$g_x = g_y = 0$ numerically yields $(1.033, -1.5495)$.

$D(1.033, -1.5495) \approx 3.2581$ and $g_{xx}(1.033, -1.5495) \approx 12.4089$.

Therefore this point is a minimum.

The closest point on the paraboloid to the point $(2, -3, 1)$ is $(1.033, -1.5495, 0.532)$.

Round your answers to three decimal places, if needed.

Find the absolute extrema of the function

$f(x, y) = 2.1x^2 + 1.5y^2 - 1.2x - 4.9y$

on the region bounded by $y = x$, $y = 3$, **and** $x = 0$.

The absolute maximum is $\boxed{}$.

The absolute minimum is $\boxed{}$.

$f_x = 4.2x - 1.2 = 0$

$4.2x = 1.2$

$x = 1.2/4.2$

$x = .285$

$x = 0$

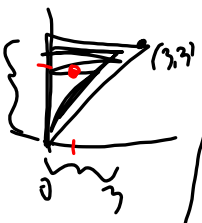
$f = 1.5y^2 - 4.9y$

$0 \le y \le 3$

$f_y = 3y - 4.9 = 0$

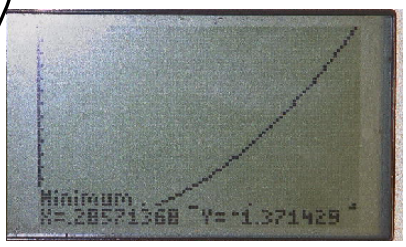
$3y = 4.9$

$y = \frac{4.9}{3} = 1.63$



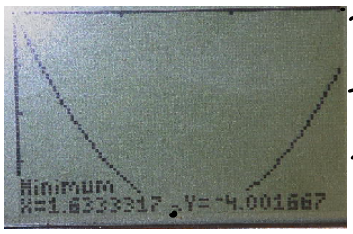
$y = 3$

$2.1x^2 + 1.5(9) - 1.2x - 4.9(3)$



$(.285, 1.63)$

$x = (.285?, 3)$



$f(0, 1.633) = -4.01$ $f(.847, .847) = -2.82$

$f(0, 0) = 0$ $f(.285, 1.63) = -4.17$

$f(0, 3) = -1.2$

$f(3, 3) = 14.1$ $f(.285, 3) = -1.371$

Round your answers to three decimal places, if needed.

Find the absolute extrema of the function

$f(x, y) = 2.1x^2 + 1.5y^2 - 1.2x - 4.9y$

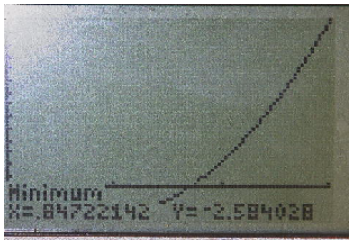
on the region bounded by $y = x$, $y = 3$, and $x = 0$.

The absolute maximum is $\boxed{}$.

The absolute minimum is $\boxed{}$.

$Y = 3.6x^2 - 6.1x$

$(.847, .847)$



The distance from a point $(x, y, z = 3 - x^2 - y^2)$ to the point $(2, -4, 1)$ is

$d(x, y) = \sqrt{(x-2)^2 + (y+4)^2 + (3 - x^2 - y^2)^2}$.

To minimize this it is useful to note that we can minimize $g(x, y) = d(x, y)^2 = D(x, y)$ instead.

$$\begin{aligned} g_x &= 2(x-2) - 4x(3 - x^2 - y^2) \\ g_y &= 2(y+4) - 4y(3 - x^2 - y^2) \\ g_{xx} &= -10 + 12x^2 + 4y^2 \\ g_{yy} &= -10 + 12y^2 + 4x^2 \\ g_{xy} &= 8xy \end{aligned}$$

$g_x = g_y = 0$ numerically yields $(0.8565, -1.7129)$.

$D(0.8565, -1.7129) \approx 6.9841$ and $g_{xx}(0.8565, -1.7129) \approx 10.5392$.

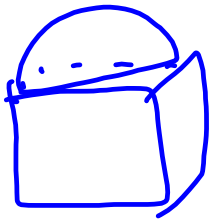
Therefore this point is a minimum.

The closest point on the paraboloid to the point $(2, -4, 1)$ is $\left(0.8565, -1.7129, 0.3324\right)$.

Project:

Get Autocad

(windows or Mac)

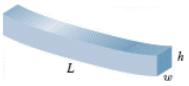


ERROR dS

Suppose that the sag in a beam of length L, width w and height h is given by

$S(L, w, h) = 0.0006\frac{L^4}{wh^3}$, with all lengths measured in inches. We illustrate the beam in the figure below.

$$S(L,w,h) = \frac{\partial S}{\partial L}\underbrace{dL}_{2} + \frac{\partial S}{\partial w}\underbrace{dw}_{.3} + \frac{\partial S}{\partial h}\underbrace{dh}_{.8}$$



A typical beam.

A beam is supposed to measure $L = 32$, $w = 5$ and $h = 9$ with a corresponding sag of 0.1726 inches. Due to weathering and other factors, the manufacturer only guarantees measurements with error tolerances $L = 32 \pm 2$, $w = 5 \pm 0.3$ and $h = 9 \pm 0.8$. Use a linear approximation to estimate the possible range of sags in the beam. Round your intermediate calculations and your final answer to five decimal places.

Based on the linear approximation, the sag is 0.1726 ± [ ] .

dL dw dh

$$dS = \frac{\partial S}{\partial L}dL + \frac{\partial S}{\partial w}dw + \frac{\partial S}{\partial h}dh$$

$$\frac{4L^3}{wh^3}dL + \frac{-L^4}{w^2h^3}dw + \frac{3L^4}{wh^4}dh$$

$$.0006\frac{L^3}{wh^3}\left(4dL - \frac{L}{w}dw - \frac{3L}{h}dh\right)$$

$$.0006\frac{32^3}{5\cdot 9^3}\left((4)(2) + \frac{32}{5}(.3) + \frac{3\cdot 32}{9}(.8)\right)$$

We first compute

$$\frac{\partial S}{\partial L} = 0.0016\frac{L^3}{w\,h^3},\ \frac{\partial S}{\partial w} = -0.0004\frac{L^4}{w^2 h^3} \text{ and } \frac{\partial S}{\partial h} = -0.0012\frac{L^4}{w\,h^4}.$$

At the point (37, 3, 7), we then have

$$\frac{\partial S}{\partial L}(37, 3, 7) = 0.07876,\ \frac{\partial S}{\partial w}(37, 3, 7) = -0.24285 \text{ and } \frac{\partial S}{\partial h}(37, 3, 7) = -0.31223.$$

The linear approximation of the sag is then given by

$$S \approx 0.7285 + 0.07876(L - 37) - 0.24285(w - 3) - 0.31223(h - 7).$$

From the stated tolerances, $L - 37$ must be between -1 and 1, $w - 3$ must be between -0.3 and 0.3 and $h - 7$ must be between -0.7 and 0.7. Notice that the maximum sag then occurs with $L - 37 = 1$, $w - 3 = -0.3$ and $h - 7 = -0.7$. The linear approximation predicts that

$$S - 0.7285 \approx 0.07876 + 0.07286 + 0.21856 = 0.37018.$$

Similarly, the minimum sag occurs with $L - 37 = -1$, $w - 3 = 0.3$ and $h - 7 = 0.7$. The linear approximation predicts that

$$S - 0.7285 \approx -0.07876 - 0.07286 - 0.21856 = -0.37018.$$

Based on the linear approximation, the sag is 0.7285 ± 0.37018, or between 1.09868 and

Intro to Lagrange Multipliers/ Constrain Problems

Show Me

Use Lagrange multipliers to find the maximum and minimum of the function $f(x, y)$ subject to the constraint $g(x, y) = c$.

$f(x, y) = x^2y^2$ subject to $x^2 + 4y^2 = 56$

$$\nabla f = \lambda \nabla g$$

$$\begin{aligned} g(x, y) &= x^2 + 4y^2 - 56 = 0 \\ \nabla f &= \langle 2xy^2, 2x^2y \rangle \\ \nabla g &= \langle 2x, 8y \rangle \\ \boxed{\nabla f = \lambda \nabla g} \\ 2xy^2 &= 2x\lambda \\ 2x^2y &= 8y\lambda \end{aligned}$$

$y^2 = \frac{x^2}{4}$

$\frac{x^2}{4} = \lambda$

$y^2 = \frac{x^2}{4}$

$y = \pm \frac{1}{2} x$

Eliminating λ we get $y = \pm\frac{1}{2}x$.

CONSTRAINT g

$$\boxed{z = x^2 + y^2 - 3}$$

$(2, -4, 1)$

$0 = x^2 + y^2 - z - 3$

$$\boxed{(x-2)^2 + (y+4)^2 + (z-1)^2 = D^2 = f}$$

$\nabla f = \langle 2(x-2), 2(y+4), 2(z-1) \rangle$

$\nabla g = \langle 2x, 2y, -1 \rangle$

$\nabla f = \lambda \nabla g$

$2x - 4 = 2\lambda x$

$2y + 8 = 2\lambda y$

$2z - 2 = -\lambda$

$z = x^2 + y^2 - 3$