Find the tangential and normal components of acceleration for an object with position vector \( \mathbf{r}(t) = \langle 3 \sin t, 3 \cos t, 7t \rangle \). Use \( \kappa = \frac{3}{58} \).

We have \( \mathbf{r}'(t) = \langle 3 \cos t, -3 \sin t, 7 \rangle \), so that
\[
\frac{ds}{dt} = ||\mathbf{r}'(t)|| = \sqrt{58}
\]
and so, \( \frac{d^2s}{dt^2} = 0 \), for all \( t \). We have that the acceleration is
\[
\mathbf{a}(t) = \frac{d^2s}{dt^2} \mathbf{T}(t) + \kappa \left( \frac{ds}{dt} \right)^2 \mathbf{N}(t)
\]
\[
= (0)\mathbf{T}(t) + \frac{3}{58} \left( \sqrt{58} \right)^2 N(t) = 3N(t).
\]
So, here we have \( a_T = 0 \) and \( a_N = 3 \).
Find the unit tangent and principal unit normal vectors at the given points.

\( r(t) = \langle t, t^2 \rangle \) at \( t = 0, t = 1 \)

\[ T(0) = \frac{1}{\sqrt{2}} \langle 1, 0 \rangle, \quad T(1) = \frac{1}{\sqrt{2}} \langle 1, 2 \rangle \]

\[ N(0) = \langle 0, 1 \rangle \quad \text{and} \quad N(1) = \langle 2, -1 \rangle \]

\[ T'(t) = \frac{T'(1)}{\|T'(1)\|}, \quad N'(0) = \frac{N'(1)}{\|N'(1)\|} \]

\[ T'(1) = \langle 4, 2 \rangle \quad \text{and} \quad N'(1) = \langle -1, 2 \rangle \]

\[ T(0) = \frac{1}{\sqrt{17}} \langle 1, 2 \rangle = \frac{1}{\sqrt{17}} \langle 0, 1 \rangle \]

\[ \|T'(1)\| = \sqrt{2} \]

\[ T'(1) = \frac{1}{\sqrt{2}} \langle 2, 1 \rangle \quad \text{unit vector} \]
Find the unit tangent and principal unit normal vectors at the given points.

\[ r(t) = \langle t, t^2 \rangle \text{ at } t = 0, t = 1 \]

\[ T(0) = \left\langle 1, 0 \right\rangle \text{ and } T(1) = \left\langle \frac{1}{\sqrt{10}}, \frac{2}{\sqrt{10}} \right\rangle \]

\[ N(0) = \left\langle 1, 2t \right\rangle \text{ and } N(1) = \left\langle 1 + 4t^2, 2t(1 + 4t^2) \right\rangle \]
\( r'(t) = \langle 1, 2t \rangle \) and \( \| r'(t) \| = \sqrt{1 + 4t^2} \), so we have \( T(t) = \left\langle \frac{1}{\sqrt{1 + 4t^2}}, \frac{2t}{\sqrt{1 + 4t^2}} \right\rangle \).

This yields \( T(0) = \langle 1, 0 \rangle \) and \( T(1) = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle \).

Also, since \( T'(t) = \left\langle \frac{-4t}{(1 + 4t^2)^{3/2}}, \frac{2}{(1 + 4t^2)^{3/2}} \right\rangle \) and \( \| T'(t) \| = \frac{2}{1 + 4t^2} \), we have \( N(t) = \left\langle \frac{-2t}{\sqrt{1 + 4t^2}}, \frac{1}{\sqrt{1 + 4t^2}} \right\rangle \).

This yields \( N(0) = \langle 0, 1 \rangle \) and \( N(1) = \langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \rangle \).
Find the tangential and normal components of acceleration for the given position function at the given points.

\[ r(t) = \langle 8t, 16t - 16t^2 \rangle \text{ at } t = 0, t = 1 \]

\[ v(t) = \langle 8, 16 - 32t \rangle \text{ and } a(t) = \langle 0, -32 \rangle. \]

\[ ||v(t)|| = \frac{ds}{dt} = 8\sqrt{5 - 16t + 16t^2}, \]

so the tangential component is

\[ a_T = \frac{\frac{d^2s}{dt^2}}{\sqrt{5 - 16t + 16t^2}} = \frac{-64(2t - 1)}{\sqrt{5 - 16t + 16t^2}}. \]

The normal component is

\[ a_N = \sqrt{||a(t)||^2 - a_T^2} = \frac{32}{\sqrt{5 - 16t + 16t^2}}. \]

At \( t = 0, \) \( a_T = \frac{-64}{\tau c} \) and \( a_N = \frac{32}{\tau c}. \)
The friction force required to keep a car from skidding on a curve is given by
\[ F_N(t) = ma_N N(t). \]
Select the friction force needed to keep a car of mass \( m = 100 \) (slugs) from skidding.

\[ r(t) = \langle 300 \cos 2\pi t, 300 \sin 2\pi t \rangle \]

Since \( ||v(t)|| = 600\pi \), \( ||a(t)|| = 1200\pi^2 \), and \( a_T = \frac{d}{dt} ||v(t)|| = 0 \), we know

\[
a_N = \sqrt{||a(t)||^2 - a_T^2} = 1200\pi^2.
\]

Thus, \( F_N(t) = m a_N N(t) \)

\[ = 120000\pi^2 \langle -\cos 2\pi t, -\sin 2\pi t \rangle. \]
Identify the surface defined by the parametrically by \( x = 2 \sin u \cosh v \), \( y = 2 \sinh v \), \( z = 2 \cos u \cosh v \), \( 0 \leq u \leq 2\pi \) and \( -\infty < v < \infty \).

- A. The surface is a sphere centered at the origin with radius \( \sqrt{2} \).
- B. The surface is a hyperboloid of one sheet wrapped around the \( z \)-axis.
- C. The surface is a hyperboloid of one sheet wrapped around the \( y \)-axis.
- D. The surface is a sphere centered at the origin with radius 2.

\[
\begin{align*}
x^2 &= 4 \sin^2 u \cosh^2 v \\
z^2 &= 4 \cos^2 u \cosh^2 v \\
x^2 + z^2 &= 4 \cos^2 u (\sin^2 u + \cosh^2 u) \\
x^2 + z^2 &= 4 \cos^2 u \\
y^2 &= 4 \sinh^2 u \\
x^2 + z^2 - y^2 &= 4 (\cosh^2 u - \sinh^2 u) \\
\text{Hyp. of 1 sheet}
\end{align*}
\]
Find a parametric representation the portion of $z = \sqrt{x^2 + y^2}$ inside $2x^2 + 2y^2 = 32.$

- A. $x = r \cos \theta, \quad y = -r \sin \theta$ and $z = r,$ for $0 \leq r \leq 2$ and $0 \leq \theta \leq 2\pi.$
- B. $x = -r \cos \theta, \quad y = r \sin \theta$ and $z = r,$ for $0 \leq r \leq 2$ and $0 \leq \theta \leq 2\pi.$
- C. $x = r \cos \theta, \quad y = r \sin \theta$ and $z = -r,$ for $0 \leq r \leq 4$ and $0 \leq \theta \leq 2\pi.$
- D. $x = r \cos \theta, \quad y = -r \sin \theta$ and $z = -r,$ for $0 \leq r \leq 4$ and $0 \leq \theta \leq 2\pi.$
A graph indicating the cone and the cylinder is shown in below.

Dividing both sides of the equation $2x^2 + 2y^2 = 32$ by $2$ yields $x^2 + y^2 = 16$.

Notice that the equations for both surfaces include the term $x^2 + y^2$ and $x$ and $y$ appear only in this combination. This suggests the use of polar coordinates.

Taking $x = r \cos \theta$ and $y = r \sin \theta$, the equation of the cone $z = \sqrt{x^2 + y^2}$ becomes $z = r$ and the equation of the cylinder $x^2 + y^2 = 16$ becomes $r = 4$. 
Select the graph of the parametric surface.

\[ x = 2\sinh u, \quad y = v, \quad z = 2\cosh u \]

\[
\cosh(v) = \frac{e^v + e^{-v}}{2}
\]

\[
\sinh(v) = \frac{e^v - e^{-v}}{2}
\]

\[ \cosh > 0, \quad \text{only} \quad v > 0 \text{ or } v < 0 \]
Select the graph of the parametric surface.

\[ x = v \sinh u, \quad y = 4v^2, \quad z = v \cosh u \]
Select a parametric representation of the portion of $x^2 + y^2 = 36$ from $z = 0$ to $z = 3$.

- A. $x = 6 \cos \theta$ and $y = 6 \sin \theta$ for $0 \leq \theta \leq 2\pi$ and $0 \leq z \leq 3$.
- B. $x = 6 \cos \theta$ and $y = -6 \sin \theta$ for $0 \leq \theta \leq 2\pi$ and $0 \leq z \leq 3$.
- C. $x = 6 \cos \theta$ and $y = 6 \sin \theta$ for $0 \leq \theta \leq 2\pi$ and $0 \leq z \leq 6$.
- D. $x = 6 \cos \theta$ and $y = -6 \sin \theta$ for $0 \leq \theta \leq 2\pi$ and $0 \leq z \leq 6$. 

\[ r = 6 \]

\[ x = r \cos \theta, \quad y = r \sin \theta, \quad z = 2 \]

\[ f (z, \theta) = \frac{z^2}{2} + 6 \sin \theta + 6 \cos \theta \]

\[ 0 \leq \theta \leq 2\pi \]
Find a parametric representation of the portion of \( z = 576 - x^2 - y^2 \) above the \( xy \)-plane.

- **A.** \( x = r \cos \theta, \ y = r \sin \theta, \) and \( z = 24 - r^2 \) for \( 0 \leq r \leq 24 \) and \( 0 \leq \theta \leq 2\pi. \)
- **B.** \( x = r \cos \theta, \ y = r \sin \theta, \) and \( z = -576 + r^2 \) for \( 0 \leq r \leq 24 \) and \( 0 \leq \theta \leq 2\pi. \)
- **C.** \( x = r \cos \theta, \ y = r \sin \theta, \) and \( z = 576 - r^2 \) for \( 0 \leq r \leq 24 \) and \( 0 \leq \theta \leq 2\pi. \)
- **D.** \( x = r \cos \theta, \ y = r \sin \theta, \) and \( z = -24 + r^2 \) for \( 0 \leq r \leq 24 \) and \( 0 \leq \theta \leq 2\pi. \)
Select the parametric representation of the surface.

\[
\frac{x^2}{9} - \frac{y^2}{81} - \frac{z^2}{36} = 1
\]

- **A.** \(x = \pm \cosh u, \ y = 9 \sinh u, \ \text{and} \ z = 3 \sinh u \cos v\)
  where \(-\infty \leq u \leq \infty\) and \(0 \leq v \leq 2\pi\).

- **B.** \(x = \pm 3 \cosh u, \ y = 9 \sinh u \cos v, \ \text{and} \ z = 6 \sinh u \sin v\)
  where \(-\infty \leq u \leq \infty\) and \(0 \leq v \leq 2\pi\).

- **C.** \(x = \pm \cosh u, \ y = 9 \sinh u, \ \text{and} \ z = 6 \sinh u \sin v\)
  where \(-\infty \leq u \leq \infty\) and \(0 \leq v \leq 2\pi\).

- **D.** \(x = \pm 3 \cosh u, \ y = 9 \sinh u \cos v, \ \text{and} \ z = 3 \sinh u \cos v\)
  where \(-\infty \leq u \leq \infty\) and \(0 \leq v \leq 2\pi\).
Select the parametric equation for the surface.

A. \( x = u, \quad y = u \cos v, \quad z = u \sin v \)

B. \( x = u \cos v, \quad y = u \sin v, \quad z = v^2 \)

C. \( x = u, \quad y = u \cos v, \quad z = u \sin v \)

D. \( x = u, \quad y = \sin u \cos v, \quad z = \sin u \sin v \)
Relate to cylindrical coordinates defined by $x = r \cos \theta$, $y = r \sin \theta$ and $z = z$.

Select parametric equations for the wedge in the first octant bounded by $y = 0$, $y = x^2 + y^2 = 196$, $z = 0$ and $z = 8$.

- **A.** $x = r \cos \theta$, $y = r \sin \theta$, $z = z$ gives the desired region with $0 \leq \theta \leq \frac{\pi}{4}$, $0 \leq r \leq 14$ and $0 \leq z \leq 8$.
- **B.** $x = r \cos \theta$, $y = r \sin \theta$, $z = z$ gives the desired region with $0 \leq \theta \leq \frac{\pi}{3}$, $0 \leq r \leq 196$ and $0 \leq z \leq 8$.
- **C.** $x = r \cos \theta$, $y = r \sin \theta$, $z = z$ gives the desired region with $0 \leq \theta \leq \frac{\pi}{4}$, $0 \leq r \leq 8$ and $0 \leq r \leq 14$.
- **D.** $x = r \cos \theta$, $y = r \sin \theta$, $z = z$ gives the desired region with $0 \leq \theta \leq \frac{\pi}{2}$.
$x^2 + y^2 = 1.16$

$\sqrt{1.16} = 1.4$

$x = \cos \theta, \ y = \sin \theta, \ z = z$

gives the desired region

$0 \leq x \leq 1.4$

$0 \leq z \leq 8$