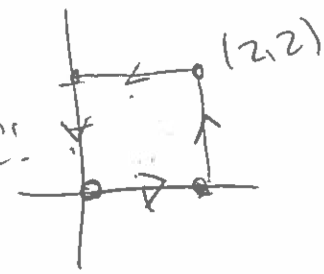
# Greens Thrm

$$\oint_C 2x+3y\,dx - 4y\,dy$$

$M\,dx - N\,dy$

C:

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$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dy\,dx$$

$$\int_0^2\int_0^2 -3\,dy\,dx = -3(4) = -12$$

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$$\iint_S xz\,dS$$

$z = 6x+2y$

$x = u$
$y = v$


$$\iint u(6u+2v)\sqrt{z_u^2+z_v^2+1}\,du\,dv$$

$z = 6u+2v$

$$\int_1^2\int_1^6 u(6u+2v)\sqrt{6^2+2^2+1}\,du\,dv$$

$$\sqrt{41}\int_1^2\int_1^6 6u^2+2uv\,du\,dv = \left.\frac{6u^3}{3}+u^2v\right|_1^6$$

$= 2\cdot 6^3+36v-2-v$

$$\int_1^2 35v+430\,dv$$

$452.5$

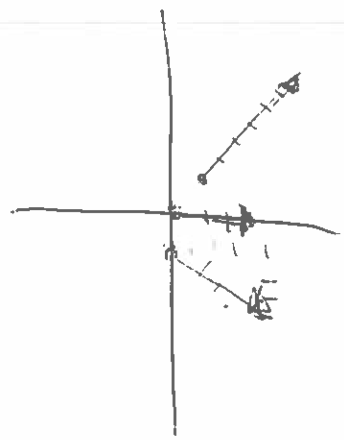
$$f(x,y) = 3x + y^2 e^x$$

$$\langle f_x, f_y \rangle = \langle 3 + y^2 e^x, 2y e^x \rangle$$

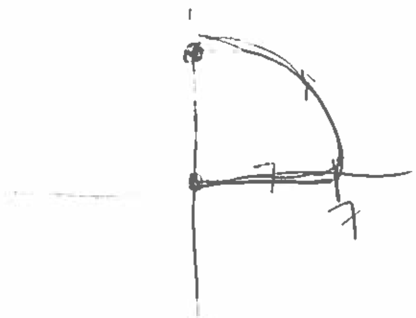
$$(0,0) \to F(0,0) = \langle 3, 0 \rangle$$

$$F(1,1) = \langle 3+e, 2e \rangle$$

$$F(0,-1) = \langle 4, -2 \rangle$$



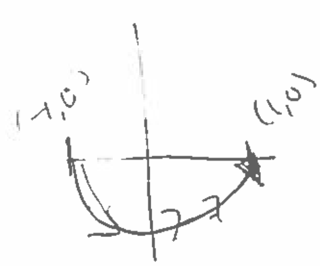
F.T.L.I




$$f(0,7) - f(0,0)$$

$$f(0,7) = 49 - 0 = 49$$

Ex $f(x,y) = 2x - 7y + 3xy$




$$\int_C 2x - 7y + 3xy \, ds$$

$$x = \cos t \quad \pi \le t \le 2\pi$$

$$y = \sin t$$

$$ds = \sqrt{x'^2 + y'^2} \, dt$$

$$\int_{\pi}^{2\pi} 2\cos t - 7\sin t + 3\cos t \sin t \, dt$$

$$F = \langle 6y^2, \cancel{4}z - \cos x, z^3 - x \rangle$$

$$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

$$\int_{-3}^{3}\int_{-3}^{3}\int_{-3}^{3} \nabla \cdot F$$

$$\iiint 3z^2 \, dz \, dy \, dx = z^3 \Big|_{-3}^{3} = 54$$

$$\int_{-3}^{3} 3z^2 \, dz \cdot \int_{-3}^{3} dy \int_{-3}^{3} dx \qquad 54 \cdot 6 \cdot 6 = 1944$$

$$\nabla \cdot F = \frac{\partial}{\partial x}F_1 + \frac{\partial}{\partial y}F_2 + \frac{\partial}{\partial z}F_3$$

$$\langle F_1, F_2, F_3 \rangle$$

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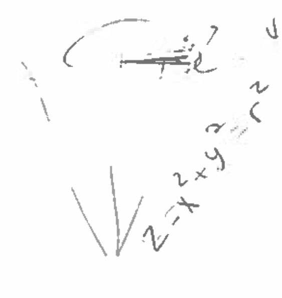
$$F = \langle 4x^3, 4y^3 - 2z, 2xy^2 \rangle$$



$$\nabla \cdot F = 12x^2 + 12y^2$$

$$\iiint 12x^2 + 12y^2 \, dV$$

$$12 \int_0^{2\pi}\int_0^{2}\int_{r^2}^{4} r^2 \cdot dz \, r \, dr \, d\theta = \int_0^{2\pi}\int_0^{2} r^2 \cdot (4 - r^2) \, r \, dr \, d\theta$$

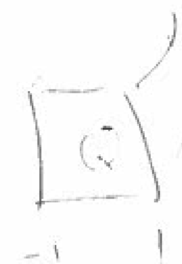
$$\int_0 r^4 - \frac{1}{5} \Big|_0^2 \quad d\theta$$

$$\int_0^{2\pi} 2^4 - \frac{2^5}{5} \, d\theta$$

$$2\pi\left(16 - \frac{32}{5}\right)$$

$$2\pi\left(\frac{80-32}{5}\right)$$

$$2\pi \frac{48}{5} \quad \frac{96\pi}{5}$$



$$F = \langle 7y^2, 6z - \cos x, z^3 - x \rangle$$

$$\nabla \cdot F = 0 + 0 + 3z^2$$

$$\iiint_{-1\,-1\,-1} 3z^2 \, dx\, dy\, dz$$

$$\int_{-1}^{1} 3z^2\, dz \cdot \int_{-1}^{1} dx \int_{-1}^{1} dy$$

$$z^3\Big|_{-1}^{1} \qquad 2 \cdot 2$$

$$2 \cdot 2 \cdot 2 = 8$$

#51

$$z = \sqrt{8 - x^2 - y^2}$$

$$x^2 + y^2 + z^2 = 8$$

$$z = \sqrt{x^2 + y^2}$$

$$x^2 + y^2 = z^2$$


$$0 \le \rho \le \sqrt{8}$$

$$0 \le \varphi \le \pi/4$$

$$0 \le \theta \le \pi$$

$$\sqrt{8 - x^2 - y^2} = \sqrt{x^2 + y^2}$$

$$8 = 2x^2 + 2y^2$$

$$4 = x^2 + y^2$$

$$4 = \rho^2 \sin^2\varphi$$

$$\sqrt{8}$$

$$\left(\frac{1}{\sqrt{2}}\right) = \sin^2\varphi$$

$$\frac{\sqrt{2}}{2} = \sin\varphi$$

$$\varphi = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ \quad \frac{\pi}{4}$$

$$\int_0^{\pi} \int_0^{\pi/4} \int_0^{\sqrt{8}} \rho^3 \cdot \underbrace{\rho^2 \sin\varphi}_{\text{Jacobian}} \, d\rho \, d\varphi \, d\theta$$

$$(x^2 + y^2 + z^2)^{3/2}$$

$$(\rho^2)^{3/2} = \rho^3$$