

CURL  
DIV.

$$\nabla f = \langle f_x, f_y, f_z \rangle$$

$$F_{12} - F_{3x} = 0 \quad F_{1y} - F_{2x} = 0 \quad F_{2z} - F_{3y} = 0$$

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$$f_{xz} - f_{zx} = 0 \quad f_{xy} - f_{yx} \quad f_{yz} - f_{zy}$$

$$F = \langle F_1, F_2, F_3 \rangle$$

conservative?  
 $\text{curl } F = 0$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_{1x} & F_{2x} & F_{3x} \end{vmatrix}$$

$$\langle F_{3y} - F_{2z}, F_{3x} - F_{1z}, F_{1y} - F_{2x} \rangle = 0$$
$$\langle P_y - N_z, P_x - M_z, M_y - N_x \rangle = 0$$

$$\langle 2xz, 3z^2, x^2 + 6yz \rangle$$

$$f_x \quad f_y \quad f_z$$

$$\int 2xz \, dx \quad \int 3z^2 \, dy \quad \int (x^2 + 6yz) \, dz$$

$$x^2 z + \dots \quad 3yz^2 + (xz^2) \quad x^2 z + 3yz^2 + (yz^2)$$

$$f = x^2 z + 3yz^2$$

$$\nabla f = \mathbf{F}$$

Point  $(0,0,0) \rightarrow (1,1,1)$

or

$$f(1,1,1) - f(0,0,0)$$

$$1+3$$

$$4$$

$$0$$

$$= 4$$

$$x^2 + y^2 - z^2 = 9$$

$$x = 3 \cos u \cosh v$$

$$y = 3 \sin u \cosh v$$

$$z = 3 \sinh v$$

$$x^2 + y^2 =$$

$$9 \cos^2 u \cosh^2 v + 9 \sin^2 u \cosh^2 v$$

$$9 \cosh^2 v \left[ \underbrace{\cos^2 u + \sin^2 u}_{=1} \right]$$

$$x^2 + y^2 = 9 \cosh^2 v$$

$$z^2 = 9 \sinh^2 v$$

$$x^2 + y^2 = z^2$$

$$9 \cosh^2 v - 9 \sinh^2 v$$

$$9 (\cosh^2 v - \sinh^2 v)$$

$$9 \cdot 1$$

$$x^2 - y^2 - z^2 = 1$$

$$x = \cosh v \quad y = \cos u \sinh v \quad z = \sin u \sinh v$$

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$$x^2 \quad \cosh^2 v \quad \sinh^2 v$$

$$x^2 - y^2 - z^2$$

$$\cosh^2 v - \sinh^2 v = 1$$

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$$2 \leq z \leq 3$$

$$2 \leq \sinh v \leq 3$$

$$\sinh^{-1}(2) \leq v \leq \sinh^{-1}(3)$$