37. The solid below $x^2 + y^2 + z^2 = 4z$ and above $z = \sqrt{x^2 + y^2}$

$x^2+y^2=z^2$

$\rho^2 = 4\rho\cos\varphi$

$x^2+y^2+z^2=2z^2$

$\rho^2 = 2\rho^2\cos^2\varphi$

$\rho^2(1-2\cos^2\varphi)=0$

$1-2\cos^2\varphi=0$

$1=2\cos^2\varphi$

$\frac{1}{2}=\cos^2\theta$

$\sqrt{\frac{1}{2}}=\cos\varphi$

$\cos^{-1}(\sqrt{\frac{1}{2}})=\varphi$

$$\iiint_Q f(x,y,z)\,dV$$

$\rho-4\cos\phi=0$

$\rho=4\cos\varphi$

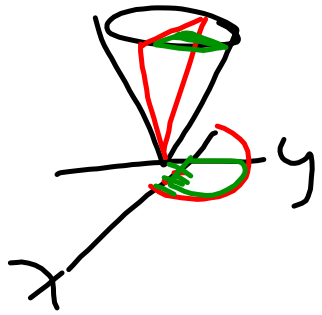
$$V=\int_{\theta=0}^{2\pi}\int_{\varphi=0}^{\pi/4}\int_{\rho=0}^{4\cos\varphi} \cancel{f(\rho\cos\theta\sin\varphi,\rho\sin\theta\sin\varphi,\rho\cos\varphi)}\ 1\ \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

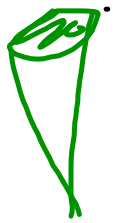
$$\int_0^{2\pi}d\theta\int_0^{\pi/4}\int_0^{4\cos\varphi}\rho^2\sin\varphi\,d\rho\,d\phi$$

$$2\pi\int_0^{\pi/4}\frac{\rho^3}{3}\Big|_0^{4\cos\varphi}\sin\varphi\,d\varphi$$

52. $\displaystyle\int_0^4 \int_0^{y=\sqrt{16-x^2}} \int_{z=\sqrt{x^2+y^2}}^{4} \underbrace{\sqrt{x^2+y^2+z^2}}_{\rho}\ \underbrace{dz\,dy\,dx}_{\rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta}$

$J.$





$\ldots\ z = 4 = \rho\cos\varphi$

$\rho = \dfrac{4}{\cos\varphi} = 4\sec\varphi$

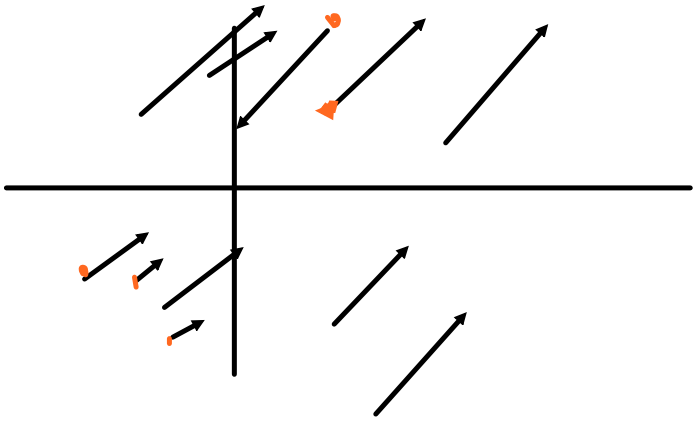
$$\int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{\pi/4} \int_{\rho=0}^{4\sec\varphi} \rho^3 \sin\varphi \, d\rho\, d\varphi\, d\theta$$

$$\int_0^{\pi/2} d\theta \int_0^{\pi/4} \frac{\rho^4}{4}\Big|_0^{4\sec\varphi} \sin\varphi\, d\varphi$$

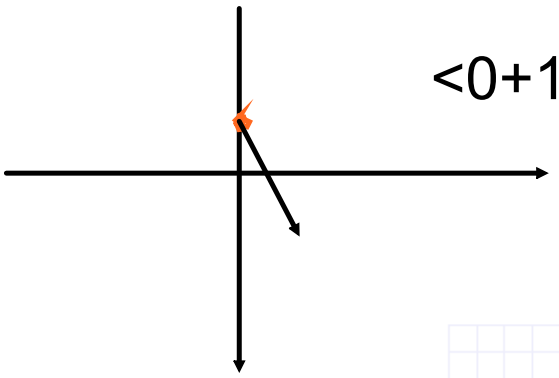
$$\left(\frac{\pi}{2}\right) \int_0^{\pi/4} \frac{4^4 \sec^4\varphi}{4} \sin\varphi\, d\varphi$$

VECTOR FIELDS

$\vec{F}(x,y)$

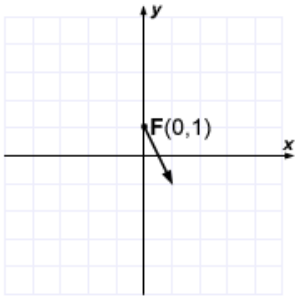


For the vector field $F(x, y) = \langle x + y, -2y + x \rangle$, evaluate $F(0, 1)$. Plot the vector $F(x, y)$ using the point $(0, 1)$ as the initial point.



<0+1,-2+0> <1,-2>

D.



Gradient Field <fx,fy>

$$\nabla f = \langle f_x, f_y \rangle$$

Find the gradient field corresponding to the function $f(x, y) = x^3 y - e^y$.

$$\langle 3x^2y, x^3 - e^y \rangle$$

Gradient Field is also called a

Conservative Vector Field

Determine whether or not the vector field $\langle 17, y \rangle$ is conservative. If it is, find a potential function.

If $\nabla f(x, y) = \langle 17, y \rangle$, then $\frac{\partial f}{\partial x} = 17$ and $\frac{\partial f}{\partial y} = y$.

$f(x, y) = \int 17\, dx = 17x + g(y)$

$\frac{\partial f}{\partial y} = g'(y) = y$

$g(y) = \frac{y^2}{2} + c$

$f(x, y) = 17x + \frac{y^2}{2} + c$

The vector field is conservative.

Determine whether or not the vector field $(x - 28xy)i + (y^2 - 14x^2)j$ is conservative. If it is, find a potential function.

$$\frac{\partial f}{\partial x} = x - 28xy$$

$$f = \frac{x^2}{2} - 14x^2y + g(y)$$

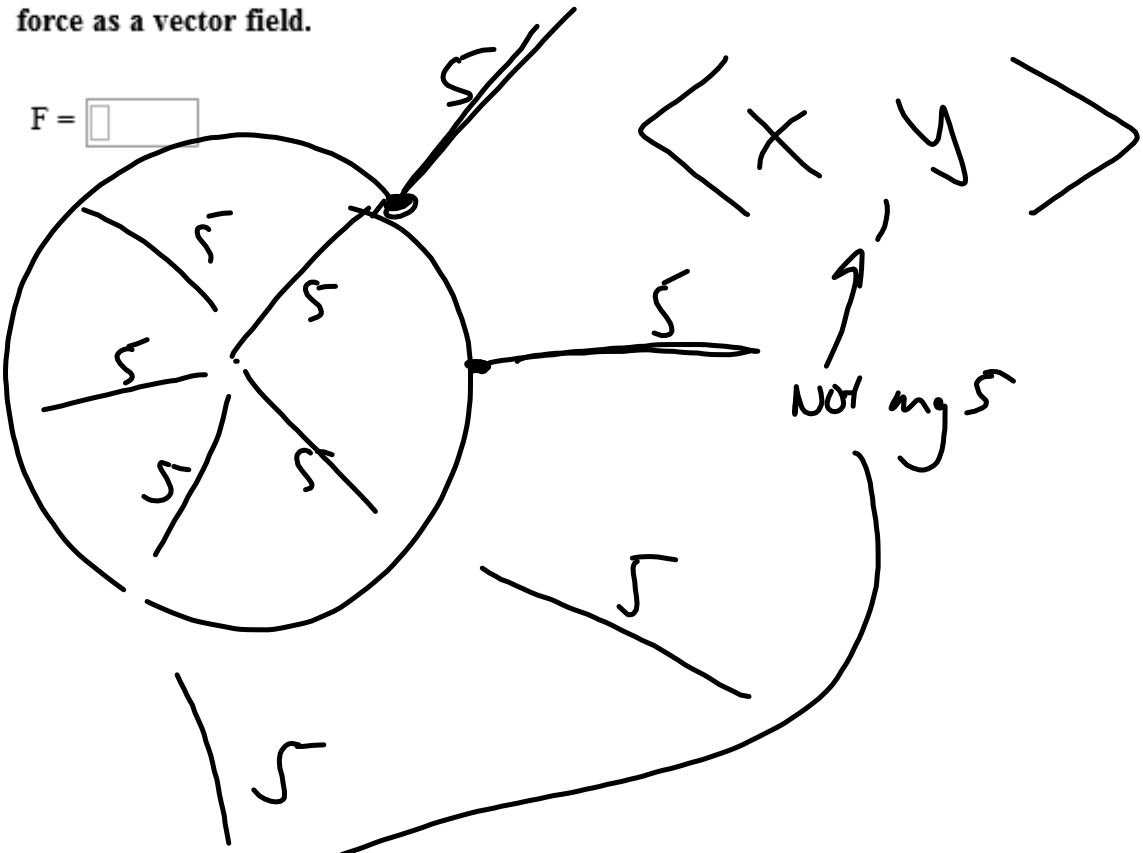
$$\frac{\partial f}{\partial y} = y^2 - 14x^2$$

$$f = \frac{y^3}{3} - 14x^2y + g(x)$$

$$f = \frac{x^2}{2} - 14x^2y + \frac{y^3}{3}$$

A two−dimensional force acts radially away from the origin with magnitude 5. Write the force as a vector field.

F =



$$\frac{\langle x, y\rangle}{|\langle x, y\rangle|} = \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right\rangle = \vec{u}$$

as unit vector

$$\left\langle \frac{5x}{\sqrt{x^2+y^2}}, \frac{5y}{\sqrt{x^2+y^2}} \right\rangle$$

Compute the exact work done by the force field $F(x, y, z) = \langle 2y, 2xz, 6y \rangle$ acting o
an object as it moves along the helix defined parametrically by $x = 4\cos t$,
$y = 4\sin t$ and $z = 6t$, from the point $(4, 0, 0)$ to the point $(-4, 0, 6\pi)$.

$$\int_0^{\pi} \vec{F} \cdot dr$$

$$\vec{r} = \langle \overset{x}{4\cos t}, \overset{y}{4\sin t}, \overset{z}{6t} \rangle \quad 0 \le t \le \pi$$

$$d\vec{r} = \langle -4\sin t, 4\cos t, 6 \rangle$$

$$\vec{F} = \langle 2y, 2xz, 6y \rangle$$

$$\vec{F}(t) \langle 8\sin t, 48t\cos t, 24\sin t \rangle$$

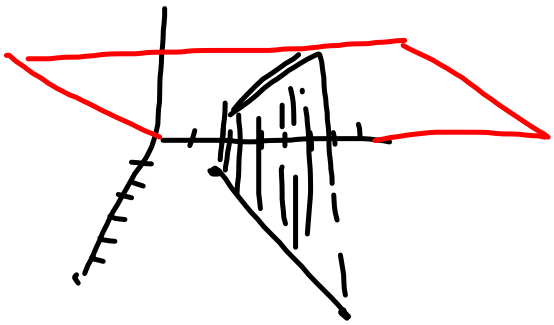
$$\int_0^{\pi} -32\sin^2 t + 192t\cos^2 t + 144\sin t \, dt$$

Line Integral

single integral over a curve in multiple dimensions

Evaluate the line integral.

$\int_C 2x \, ds$, where C is the line segment from (1, 2) to (7, 7)



$x(t) = 1 + 6t$

$y(t) = 2 + 5t$

$$\int_{t=0}^{1} \underbrace{2(1+6t)}_{x} \underbrace{\sqrt{61}}_{ds} \, dt$$

$ds = \sqrt{(x')^2 + (y')^2} \, dt$

$= \sqrt{(6)^2 + (5)^2} \, dt$

$ds = \sqrt{61} \, dt$

$2\sqrt{61}\left(t + 3t^2\right)\Big|_0^1$

$= 8\sqrt{61}$

Evaluate the line integral.

$\int_C 6z\ ds$, where C is the line segment from (5, 0, 1) to (6, −10, 10)

$$\int_0^1 6(1+9t)\sqrt{182}\,dt$$

$x = 5 + t$
$y = 0 - 10t$
$z = 1 + 9t$

$0 \le t \le 1$

$ds = \sqrt{1 + 100 + 81}\,dt$
$= \sqrt{182}\,dt$

Compute the work done by the force field F along the curve C.

$\mathbf{F}(x, y) = \langle 9x, 9y \rangle$, C is the line segment from (1, 1) to (3, 4)

$r = \langle x, y \rangle$

$dr = \langle dx, dy \rangle$

$x = 1 + 2t,\ y = 1 + 3t,\ 0 \le t \le 1$

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C 9x\ dx + 9y\ dy$$

dt dt

$$= \int_0^1 [9(1 + 2t)(2) + 9(1 + 3t)(3)]\ dt$$

$$= \int_0^1 (117t + 45)\ dt = \frac{207}{2}$$